International Journal of Statistics and Applied Mathematics

**ISSN:** 2456-1452
Maths 2022; 7(1): 146-153

www.mathsjournal.com
Received: 10-09-2021
Accepted: 17-10-2021

**PK Tripathy**
P.G. Department of Statistics, Utkal University, Bhubaneswar, Odisha, India

**Anima Bag**
P.G. Department of Statistics, Utkal University, Bhubaneswar, Odisha, India

**Corresponding Author:**
**Anima Bag**
P.G. Department of Statistics, Utkal University, Bhubaneswar, Odisha, India

# Fuzzy inventory model with multilevel permissible delay in payments

**PK Tripathy and Anima Bag**

**DOI:** https://doi.org/10.22271/maths.2022.v7.i1b.785

**Abstract**
In the last few decades the study of deteriorating items has gained enormous importance. Most of the companies are facing cut throat competition due to the wastage of resources that drastically reduce their profit margins. So decision makers adopt various techniques for disposal of decaying items with minimum inventory cost. In the present paper a fuzzy inventory model for deteriorating items is considered with four levels of permissible delay in payments. Time and selling price induced quadratic demand is considered and fuzzy approach is introduced in certain parameters to count the uncertainty in the environment. The objective of the study is to optimize the total cost function to reduce economic losses. Empirical Investigation is demonstrated and sensitivity analysis is carried out to gain managerial insights.

**Keywords:** Deterioration, fuzzy approach, replenishment, inventory cost

## Introduction

Deterioration is a continuous process that decreases the effectiveness of goods when stored for a long time. So decision makers adopt various techniques for disposal of decaying items with minimum inventory cost. Managing the inventory of perishable items helps in smooth functioning of an enterprise or business organization. Goyal (1985) [8] was the pioneer who developed an inventory model considering delay in payment. Aggarwal and Jaggi (1995) [1] generalized Goyal's model by allowing credit period in the inventory model for deteriorating items. Chung and Huang (2005) [5] presented an EOQ model for inventory control in the presence of trade credit. Chung, Huang, and Huang (2002) [6] and Chung and Huang (2003) [4] considered permissible delay in payments to develop an optimal replenishment policy for EOQ models. Das *et al*. (2011) [7] allowed credit period to the buyer for time varying demand in developing the inventory model. Tripathy and Pradhan (2011) depicted an interactive inventory model between producer and buyer for extraordinary purchase and credit period with two parameter weibull demand rate. An economic production quantity model under permissible delay was introduced by Roy *et al*. (2013) with time proportional deterioration. Teng *et al*. (2014) [16] considered two level trade credits in developing the inventory model. Sarkar *et al*. (2015) [12] considered fixed lifetime products with trade credit policy in their inventory model. Tripathy and Pandey (2015) [17] introduced backlogging in permissible delay period with time dependent demand. Tayal *et al.* (2016) applied preservation technology in the integrated production inventory model for perishable products. Tripathi *et al*. (2018) [18] adorned their research work by using stock dependent demand under various trade credits. Bag and Tripathy (2018) [2] developed the inventory model by considering four levels of permissible delay periods.

To get a legalistic approach to the inventory parameters fuzziness is applied. It is used in situations when uncertainties prevail in the formulation of the model. Park (1987) [12] fuzzified the inventory carrying cost and developed the inventory model. Mishra and Singh (2011) [11], Yao and Chiang (2003) [4], Jaggi *et al*. (2012) [9], Jaggi *et al*. (2016) [10], Jing *et al*. (2003) [3], Tripathy and Bag (2019) [19] are some other researchers who developed the inventory models by introducing the fuzzy approach.

In this chapter a fuzzy inventory model is framed for deteriorating items with time and selling price induced quadratic demand.

Four levels of conditional deliveries are considered and individual cases are developed accordingly. The objective of the study is the optimization of the cost function to reduce economic losses.

The rest of the chapter is developed as follows. Notations and assumptions are placed in section 2. The mathematical model of the inventory problem is derived in section 3. Empirical investigation for the proposed model is placed in section 4. Further sensitivity analysis is carried out in section 5 to study the changes in the decision variables with a change in system parameters. Finally the concluding remarks with scope of future research are emphasized in section 6.

**Table 1:** Contribution of some selected authors

| Authors | Demand | Demand type | Number of Stages | Deterioration | Fuzzification |
|---|---|---|---|---|---|
| Goyal (1985) [8] | Constant | Linear | 2 | Absent | Absent |
| Agarwal & Jaggi (1995) | Time dependent | Linear | 2 | Present | Absent |
| K.J. Chung, Y.F. Huang & C.K. Huang (2002) [6] | Constant | Constant | 2 | Absent | Absent |
| K.J. Chung & Y.F. Huang (2003) [4] | Constant | Constant | 3 | Absent | Absent |
| Das *et al*. (2011) [7] | Time dependent | Linear | 3 | Absent | Absent |
| C.K. Tripathy & L.M. Pradhan (2011) | Constant | Constant | 2 | Present | Absent |
| S. Tayal *et al*. (2016) | Time dependent | Exponential | 2 | Present | Absent |
| P.K. Tripathy & S. Pradhan (2012) [19] | Time dependent | Exponential (Ramp type) | 2 | Present | Absent |
| Tripathi *et al*. (2018) [18] | Stock dependent | Linear | 4 | Present | Absent |
| P.K. Tripathy & Anima Bag (2019) [19] | Time and selling price dependent | Quadratic | 4 | Present (time dependent) | Absent |
| Jaggi *et al*. (2016) [10] | Price dependent | Linear | 3 | Absent | Present |
| Present | Time and selling price dependent | Quadratic | 4 | Present (not time dependent) | Present |

## Notations and Assumptions

In this chapter, the following notations are used to develop the inventory model.

- $I(t)$ : Inventory level at any instant of time *t*
- $C_p$ : Purchase cost per unit
- $S$ : Selling price per unit
- $D(t)$ : Demand rate at any instant of time *t*
- $\theta$: Rate of deterioration, $0 < \theta < 1$
- $C_h$ : Holding cost per unit item.
- $k$ : Deterioration cost per unit item.
- $C_o$ : Ordering cost per order.
- $r$ : Rate of cash discount, 0 < r < 1
- $I_p, I_e$ : The interest paid and earned / dollar / year respectively
- $M_d, M_p$ : Cash discount and permissible delay periods respectively, $M_p > M_d$
- $T_1, T_2, T_3, T_4$ : The length of cycle time for cases I, II, III, IV respectively.
- $Q1, Q2, Q3, Q4$ : The order quantity of the inventory system for cases I, II, III, IV respectively.
- $TC1, TC2, TC3, TC4$ : The total cost of the inventory system per year for cases I, II, III, IV respectively.

The following assumptions are considered in this chapter.

1. The demand rate is both selling price and time induced.

$$D(t) = as^{-\beta}\left(1 + bt - ct^2\right)$$

2. The lead time is zero or negligible.
3. Shortage is not allowed.
4. The permissible delay period is more than the cash discount period.

## Mathematical Formulation and Optimal Solution

The inventory level for the depleting items at any instance of time *t* with selling price dependent demand is represented by the differential equation as below.

$$\frac{dI(t)}{dt} + \theta I(t) = -as^{-\beta}\left(1 + bt - ct^2\right),\ 0 \le \mathrm{t} \le \mathrm{T} \tag{1}$$

With the initial condition $I(T) = 0$, the solution of the above differential equation is

$$I(t) = \frac{-as^{-\beta}}{\theta^3}\left[\theta^2\left(1 + bt - ct^2\right) - \theta(b - 2cT) - 2c + e^{\theta(T-t)}\left(\theta^2\left(cT^2 - bT - 1\right) - 2\theta\left(-\frac{b}{2} + cT\right) + 2c\right)\right] \tag{2}$$

At the beginning, the supplier has '*Q*' units in the inventory system.

$$Q = I(0) = \frac{-as^{-\beta}}{\theta^3}\left[\theta^2 - \theta b - 2c + e^{\theta T}\left\{\theta^2\left(cT^2 - bT - 1\right) - 2\theta\left(-\tfrac{b}{2} + cT\right) + 2c\right\}\right] \tag{3}$$

Total cost/year consists of the following elements.

i. Ordering cost $= \dfrac{C_0}{T}$ (4)

ii. Purchase cost $= \dfrac{C_p Q}{T} = \dfrac{-as^{-\beta}C_p}{\theta^3 T}\left[\theta^2 - \theta b - 2c + e^{\theta T}\left\{\theta^2\left(cT^2 - bT - 1\right) - 2\theta\left(-\tfrac{b}{2} + cT\right) + 2c\right\}\right]$ (5)

iii. Holding cost =

iv. $\dfrac{C_h}{T}\int_0^T I(t)dt = \dfrac{C_h}{T}\left(\dfrac{-as^{-\beta}}{\theta^3}\right)\left[\theta^2 T + \dfrac{b}{2}\theta^2 T^2 - \dfrac{c}{3}\theta^2 T^3 + \theta - b - \dfrac{2c}{\theta} + e^{\theta T}\left(c\theta T^2 - b\theta T - \theta + b - 2cT + \dfrac{2c}{\theta}\right)\right]$ (6)

v. Deterioration cost = $K\theta\int_0^T I(t)dt = K\left(\dfrac{-as^{-\beta}}{\theta^2}\right)\left[\theta^2 T + \dfrac{b}{2}\theta^2 T^2 - \dfrac{c}{3}\theta^2 T^3 + \theta - b - \dfrac{2c}{\theta} + e^{\theta T}\left(c\theta T^2 - b\theta T - \theta + b - 2cT + \dfrac{2c}{\theta}\right)\right]$ (7)

Four possible cases are considered. In case-I $(M_d \le T)$ and case-II $(M_d > T)$, cash discount is offered. In case-I $(M_d \le T)$ and case-III $(M_p \le T)$ some interest is paid and also some interest is earned. In case-II $(M_d > T)$ and Case-IV $(M_p > T)$ no interest is paid, only interest is earned.

**Case I:** $(M_d \le T)$

The cash discount gained by the customer per unit time = $\dfrac{rC_p Q}{T}$

$$= \frac{rC_p}{T}\left(\frac{-as^{-\beta}}{\theta^3}\right)\left[\theta^2 - \theta b - 2c + \left(\theta^2(cT^2 - bT - 1) - 2\theta(-\frac{b}{2} + cT) + 2c\right)e^{\theta T}\right]$$

The interest paid per year is $\dfrac{C_p(1-r)I_p}{T}\int_{M_d}^{T} I(t)dt$

$$= \left(\frac{-as^{-\beta}}{\theta^3}\right)\frac{C_p(1-r)I_p}{T}\left[\begin{array}{l}\theta^2 T + \frac{b}{2}\theta^2 T^2 - \frac{c}{3}\theta^2 T^3 + \theta - b - \frac{2c}{\theta} - \theta^2 M_d - \frac{b}{2}\theta^2 M_d{}^2 + \frac{c}{3}\theta^2 M_d{}^3 + b\theta M_d - c\theta M_d{}^2 + 2cM_d \\ -\left(cT^2\theta - b\theta T - \theta + b - 2cT + \frac{2c}{\theta}\right)\left(1 + \theta T - \theta M_d + \theta^2\left(T^2 - 2TM_d + M_d{}^2\right)\right)\end{array}\right] \tag{9}$$

During $(0, M_d)$ the customer sells products and deposits the revenue into an account that earns $I_e$ per dollar per year.
Thus interest earned per unit time is

$$\frac{sI_e}{T}\int_0^{M_d} as^{-\beta}(1 + bt - ct^2)tdt = as^{1-\beta}\frac{I_e}{T}M_d{}^2\left[\frac{1}{2} + \frac{bM_d}{3} - \frac{cM_d{}^2}{4}\right] \tag{10}$$

The total relevant cost per unit time $TC1$ is calculated by $TC1$= Cost of placing order + Cost of purchasing units after discount rate (r) + Holding cost + Deterioration cost + Interest paid per unit time – Interest earned per unit time (11)

The parameters $s, \theta, C_h, b, c$ vary within certain limit due to environmental uncertainty. Hence fuzzy approach is implemented to the model to deal with such uncertainties. Triangular fuzzification is implemented for this purpose. The defuzzification of the total cost is carried out by using Graded Mean Representation Method, Signed Distance Method and Centroid Method.
Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (s_1, s_2, s_3), \tilde{\theta} = (\theta_1, \theta_2, \theta_3), \tilde{C}_h = (C_{h1}, C_{h2}, C_{h3}), \tilde{b} = (b_1, b_2, b_3), \tilde{c} = (c_1, c_2, c_3)$$

Using Graded Mean Representation Method of defuzzification, the total cost will be

$$(TC1)_{gd} = \frac{1}{6}\left[TC1_{gm1} + 4TC1_{gm2} + TC1_{gm3}\right]$$

Using Signed Distance Method of defuzzification, the total cost will be

$$(TC1)_{sd} = \frac{1}{4}\left[TC1_{sd1} + 2TC1_{sd2} + TC1_{sd3}\right]$$

Using Centroid Method of defuzzification, the total cost will be

$$(TC1)_{dc} = \frac{1}{3}\left[TC1_{dc1} + TC1_{dc2} + TC1_{dc3}\right]$$

**Case-II (*$M_1$>T*)**
In this case no interest is payable. Cash discount is same as that of case - I.
The interest earned per unit time is

$$\frac{sI_e}{T}\int_0^T as^{-\beta}(1+bt-ct^2)tdt + (M_d - T)\int_0^T as^{-\beta}(1+bt-ct^2)dt = -as^{-\beta}T\left[sI_e\left(\frac{1}{2}+\frac{bT}{3}-\frac{cT^2}{4}\right)+(M_d - T)\left(1+\frac{bT}{2}-\frac{cT^2}{3}\right)\right] \quad (12)$$

Total relevant cost per year $TC2$ is calculated by $TC2$ = Cost of placing order + Cost of purchasing units after discount rate (r) + Holding cost + Deterioration cost – Interest earned per unit time (13)

In the similar manner using graded mean integration method, signed distance method and centroid method of defuzzification, the total costs will be

$$(TC2)_{gd} = \frac{1}{6}\left[TC2_{gm1} + 4TC2_{gm2} + TC2_{gm3}\right], (TC2)_{sd} = \frac{1}{4}\left[TC2_{sd1} + 2TC2_{sd2} + TC2_{sd3}\right], (TC2)_{dc} = \frac{1}{3}\left[TC2_{dc1} + TC2_{dc2} + TC2_{dc3}\right]$$

**Case-III ($(M_p \le T)$)**
In this case, the amount is paid at time $M_p$, there is no cash discount.
The interest paid per unit time is

$$\frac{C_p I_p}{T}\int_{M_p}^T I(t)dt = \frac{-as^{-\beta}C_p I_p}{\theta^3 T}\left[\begin{array}{l}\theta^2 T + \frac{b}{2}\theta^2 T^2 - \frac{c}{3}\theta^2 T^3 + \theta - b - \frac{2c}{\theta} - \theta^2 M_p - \frac{b}{2}\theta^2 M_p^{\ 2} + \frac{c}{3}\theta^2 M_p^{\ 3} + b\theta M_p - c\theta M_p^{\ 2} + 2cM_p \\ -\left(cT^2\theta - b\theta T - \theta + b - 2cT + \frac{2c}{\theta}\right)\left(1+\theta T - \theta M_p + \theta^2\left(T^2 - 2TM_p + M_p^{\ 2}\right)\right)\end{array}\right] \quad (14)$$

The interest earned per unit time is

$$\frac{sI_e}{T}\int_0^{M_p} as^{-\beta}(1+bt-ct^2)tdt = as^{1-\beta}\frac{I_e}{T}M_p^{\ 2}\left[\frac{1}{2}+\frac{bM_p}{3}-\frac{cM_p^{\ 2}}{4}\right] \quad (15)$$

Total relevant cost per unit time $TC3$ is calculated by
$TC3$ = Cost of placing order + Cost of purchasing units+ Holding cost + Deterioration cost + Interest payable per unit time – Interest earned per unit time (16)

In the similar manner using graded mean integration method, signed distance method and centroid method of defuzzifications, the total costs will be

$$(TC3)_{gd} = \frac{1}{6}\left[TC3_{gm1} + 4TC3_{gm2} + TC3_{gm3}\right],$$

$$(TC3)_{sd} = \frac{1}{4}\left[TC3_{sd1} + 2TC3_{sd2} + TC3_{sd3}\right], (TC3)_{dc} = \frac{1}{3}\left[TC3_{dc1} + TC3_{dc2} + TC3_{dc3}\right],$$

**Case-IV** $(M_p > T)$
In this case no interest is payable.

The interest earned per unit time is

$$\frac{sI_e}{T}\int_0^T as^{-\beta}(1+bt-ct^2)tdt+\left(M_p-T\right)\int_0^T as^{-\beta}(1+bt-ct^2)dt=-as^{-\beta}T\left[sI_e\left(\frac{1}{2}+\frac{bT}{3}-\frac{cT^2}{4}\right)+\left(M_p-T\right)\left(1+\frac{bT}{2}-\frac{cT^2}{3}\right)\right] \quad (17)$$

Total relevant cost per unit time $TC4$ is calculated by $TC4$ = Cost of placing order + Cost of purchasing units + Holding cost + Deterioration cost + Interest payable per unit time – Interest earned per unit time (18)

In the similar manner using graded mean integration method, signed distance method and centroid method of defuzzification, the total costs will be

$$(TC4)_{gd}=\frac{1}{6}\left[TC4_{gm1}+4TC4_{gm2}+TC4_{gm3}\right],$$

$$(TC4)_{sd}=\frac{1}{4}\left[TC4_{sd1}+2TC4_{sd2}+TC4_{sd3}\right],(TC4)_{dc}=\frac{1}{3}\left[TC4_{dc1}+TC4_{dc2}+TC4_{dc3}\right],$$

The optimal values of T for cases I, II, III, and IV are determined by solving $\frac{\partial TCi}{\partial T}=0$, for i = 1, 2, 3, 4 and total costs are determined accordingly.

For convexity of the total cost function, the necessary and sufficient conditions $\frac{\partial^2 TCi}{\partial T^2}>0$, i=1,2,3,4 should be satisfied,

In fuzzy model, the values of T for different cases are also worked out and the total costs are calculated accordingly. Convexity conditions are also checked.

**Empirical Investigation**

To illustrate the effect of the proposed model, the following empirical investigation is performed with four possible cases. Both crisp and fuzzy models are considered for the purpose to obtain more realistic result. Mathematica 5.1 software is used to obtain the relevant results and convexity test is also performed.

**Numerical illustration-1**

Let the parameters be set as follows

**Case: 1** $(M_d\le T)$

Crisp Model

$$a=100, C_h=\$4/unit/year, I_p=\$0.08/year, I_e=\$0.005/year, C_p=\$40/unit, s=\$65/unit, \theta=0.1/year, r=0.08$$
$$M_d=1month, C_0=\$5000/order, b=0.2, c=0.3, \beta=0.1, k=\$0.001/unit$$

**Result:** $T$ = 5.44013*years*, Total Cost $TC1=\$393140$ , Purchase Quantity $Q1=844.854units$

**Fuzzy Model**

Implementing triangular fuzzification, let the parameters be

$$\tilde{s}=(60,65,70),\tilde{\theta}=(0.05,0.1,0.15),\tilde{C}_h=(3,4,5),\tilde{b}=(0.1,0.2,0.3),\tilde{c}=(0.2,0.3,0.4)$$

Using Graded Mean Representation Method of defuzzification,

$$T_{gd}=5.30765years,(TC1)_{gd}=\$394320, Q1_{gd}=746.428units$$

Using Signed Distance Method of defuzzification,

$$T_{sd}=5.29666years,(TC1)_{sd}=\$396620, Q1_{sd}=738.587units$$

Using Centroid Method of defuzzification,

$$T_{cd}=5.27785years,(TC1)_{cd}=\$397320, Q1_{cd}=725.281units$$

**Case: 2** $(M_d > T)$

**Crisp Model**

$a = 100, h = \$4/unit/year, I_p = \$0.08/year, I_e = \$0.005/year, C_p = \$40/unit, s = \$65/unit, \theta = 0.1, r = 0.08/year,$
$M_d = 9months, C_0 = \$5000/order, b = 0.2, c = 0.3, \beta = 0.1, k = \$0.001/unit$

**Result:** $T$ = 8.84653$years$, Total Cost $TC2 = \$43925.7$, Purchase Quantity, $Q2 = 7093.02units$

**Fuzzy Model**

Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (60,65,70), \tilde{\theta} = (0.05,0.1,0.15), \tilde{C}_h = (3,4,5), \tilde{b} = (0.1,0.2,0.3), \tilde{c} = (0.2,0.3,0.4)$$

Using Graded Mean Representation Method of defuzzification,

$$T_{gd} = 8.30765 years, (TC2)_{gd} = \$45432, Q2_{gd} = 5471.35 units$$

Using Signed Distance Method of defuzzification,

$$T_{sd} = 6.66546 years, (TC2)_{sd} = \$46544, Q2_{sd} = 2155.77 units$$

Using Centroid Method of defuzzification,

$$T_{cd} = 6.30765 years, (TC2)_{cd} = \$48998, Q2_{cd} = 1689.87 units$$

**Case: 3** $(M_p \leq T)$

**Crisp Model**

$a = 100, h = \$4/unit, I_p = \$0.08/year, I_e = \$0.005/year, C_p = \$40/unit, s = \$65/unit, \theta = 0.1, r = 0.08/year,$
$M_d = 2months, C_0 = \$5000/order, b = 0.2, c = 0.3, \beta = 0.1, k = \$0.001/unit$

**Result**

$T$ = 5.49334$years$, Total Cost $TC3 = \$382540$, Purchase Quantity $Q3 = 886.467units$

**Fuzzy Model**

Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (60,65,70), \tilde{\theta} = (0.05,0.1,0.15), \tilde{C}_h = (3,4,5), \tilde{b} = (0.1,0.2,0.3), \tilde{c} = (0.2,0.3,0.4)$$

Using Graded Mean Representation Method of defuzzification,

$$T_{gd} = 5.30765 years, (TC3)_{gd} = \$384320, Q3_{gd} = 741.885 units$$

Using Signed Distance Method of defuzzification,

$$T_{sd} = 5.00765 years, (TC3)_{sd} = \$387889, Q3_{sd} = 549.369 units$$

Using Centroid Method of defuzzification,

$$T_{cd} = 4.66544 years, (TC3)_{cd} = \$409908, Q3_{cd} = 364.499 units$$

**Case: 4** $(M_p > T)$

**Crisp Model**

$a = 100, h = \$4/unit, I_p = \$0.08/year, I_e = \$0.005/year, C_p = \$40/unit, s = \$65/unit, \theta = 0.1, r = 0.08/year,$
$M_d = 10months, C_0 = \$5000/order, b = 0.2, c = 0.3, \beta = 0.1, k = \$0.001/unit$

**Result:** $T$ =7.76267, Total Cost $TC4 = 25235.2$, Purchase Quantity, $Q4 = 4125.73$

**Fuzzy Model**

Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (60,65,70), \tilde{\theta} = (0.05,0.1,0.15), \tilde{C}_h = (3,4,5), \tilde{b} = (0.1,0.2,0.3), \tilde{c} = (0.2,0.3,0.4)$$

Using Graded Mean Representation Method of defuzzification,

$$T_{gd} = 6.30760 years, (TC4)_{gd} = \$26344.6, Q4_{gd} = 1689.81 units$$

Using Signed Distance Method of defuzzification,

$$T_{sd} = 5.34456 years, (TC4)_{sd} = \$28665.7, Q4_{sd} = 773.12 units$$

Using Centroid Method of defuzzification,

$$T_{cd} = 5.30765 years, (TC4)_{cd} = \$30998, Q4_{cd} = 746.428 units$$

**Sensitivity Analysis**

Sensitivity analysis is performed on the parameters $\theta, r$ *and* $M$ to analyse the effect of the changes in their values on the optimal solutions. Graded Mean Integration method of defuzzification is considered for the purpose as it gives minimum cost as compared to signed distance method and centroid method.

**Sensitivity analysis reveals that**

- The increase in rate of deterioration '$\theta$' results an increase in the values of Total Cost *TC* and Order Quantity Q.
- The increase in rate of cash discount '*r*' results a decrease in the total cost but results more Order Quantity.
- The more the period of cash discount $M_d$ offered the less is the Total Cost and Order Quantity. But after certain period the Total Cost increases.
- Similarly the more the permissible delay period is offered, the less is the Total Cost and Order Quantity. But after certain period the total cost increases.

**Conclusion**

This paper develops a fuzzy inventory model with multilevel permissible delay in payments. Cash discount period is less than the permissible delay period. The total cost is minimized by applying optimization technique. Through empirical investigation and sensitivity analysis it is concluded that the rate of cash discount should be more so as to minimize the total cost as it promotes the early sale of the inventory. The increased rate of deterioration results a rise in total cost. So it is advised to the seller to sell the inventory as early as possible to avoid deterioration by providing multilevel permissible delay in payments so as to minimize the total cost. The results in this paper provide a valuable reference to the decision makers in planning procurement and controlling inventories. The model can be applied in the inventory system that deals with perishable items like packaged foods, Grocery items, confectionery etc.

This paper can be generalized by allowing shortage in the model. Advertisement frequency and entropy can also be included in the model for future study.

**References**


1. Aggarwal SP, Jaggi CK. "Ordering policies of deteriorating items under permissible delay in payment". Journal of Operational Research Society. 1995;46:658-662.
2. Bag A. Tripathy PK. "Decision support model with price and time induced demand under multilevel conditional deliveries", Revista Investigacion Operacional, 2018;40(4):441-451.
3. Jing Yao S, Jershan Chiang. "Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance, European Journal of Operational Research. 2003;148:401-409.
4. Chung KJ, Huang YF. "The optimal cycle time for EPQ inventory model under permissible delay in payments". International Journal of Production Economics. 2003;84:307-318.
5. Chung KJ, Huang TS. "The algorithm to the EOQ model for Inventory control and trade credit". Journal of the Operational Research Society, 2005;42:16-27.
6. Chung KJ, Huang YF, Huang CK. "The replenishment decision for EOQ inventory model under permissible delay in payments". Opsearch, 2002;39:327–340.
7. Das D, Ray A, Kar S. "Optimal payment time for retailer under permitted delay in payments by the wholesaler with dynamic demand and hybrid number cost parameters", Opsearch. 2011;48:171-196.
8. Goyal SK. "Economic order quantity under conditions of permissible delay in payments", Journal of Operational Research Society. 1985;36:335-338.
9. Jaggi CK, Pareek S. "Fuzzy inventory model for deteriorating items with time varying demand and shortages", American Journal of Operations Research. 2012;2(6):81-92.
10. Jaggi CK, Yadavalli VSS, Sharma A, Tiwari S. "A fuzzy EOQ Model with allowable shortage under different trade credit terms". Applied Mathematics and Information Science. 2016;10(2):785-805.
11. Mishra VK, Singh LS. "Deteriorating inventory model for time dependent demand and holding cost with partial backlogging". International Journal of Management Science and Engineering Management. 2011;6(4):267-271.

12. Park K. “Fuzzy-set theoretic interpretation of economic order quantity”. IEEE Transactions on Systems, Man, and Cybernetics SMC. 1987;17:1082-1084.
13. Sarkar B, Saren S. “An inventory model with trade credit policy and variable deterioration for fixed lifetime products”, Annals of Operation Research. 2015;229:677-702.
14. Shah NH, Shah D, Patel DG. “An EOQ Model for perishable inventory under credit period dependent quadratic demand”, Mexican Journal of Operations Research. 2014;3:2-12.
15. Shah NH, Jani MY, Shah DB. “Economic Order Quantity Model under trade credit and customer returns for price sensitive quadratic demand”, Revista Investigación Operacional. 2015;36:240-248.
16. Teng JT, Yang HL, Chern MS. “An inventory model for increasing demand under two levels of trade credit linked to order quantity”, Applied Mathematical Modeling. 2014;37:7624-7632.
17. Tripathi RP, Pandey HS. “Inventory model with Weibull time-dependent demand rate and completely backlogged permissible delay in payment”, International Journal of Industrial Engineering Computations, Uncertain Supply chain Management. 2015;3:321-332.
18. Tripathi RP, Singh D, Aneja S. “Inventory models for stock dependent demand and time varying holding cost under different trade credits”, Yugoslav Journal of Operations Research. 2018;28:139-151.
19. Tripathy CK, Pradhan LM. “An EOQ model for three parameter weibull deterioration with permissible delay in payments and associated salvage value”, International Journal of Industrial Engineering Computations. 2012;3:115-122.
20. Tripathy PK, Bag A. “Optimal disposal strategy with controllable deterioration and shortage”, International Journal of Agricultural and Statistical Sciences. 2019;15(1):271-279.