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Fuzzy inventory model with multilevel permissible delay in payments

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Abstract

In the last few decades the study of deteriorating items has gained enormous importance. Most of the companies are facing cut throat competition due to the wastage of resources that drastically reduce their profit margins. So decision makers adopt various techniques for disposal of decaying items with minimum inventory cost. In the present paper a fuzzy inventory model for deteriorating items is considered with four levels of permissible delay in payments. Time and selling price induced quadratic demand is considered and fuzzy approach is introduced in certain parameters to count the uncertainty in the environment. The objective of the study is to optimize the total cost function to reduce economic losses. Empirical Investigation is demonstrated and sensitivity analysis is carried out to gain managerial insights.

Keywords: Deterioration, fuzzy approach, replenishment, inventory cost

Introduction

Deterioration is a continuous process that decreases the effectiveness of goods when stored for a long time. So decision makers adopt various techniques for disposal of decaying items with minimum inventory cost. Managing the inventory of perishable items helps in smooth functioning of an enterprise or business organization. Goyal (1985) ^[8] was the pioneer who developed an inventory model considering delay in payment. Aggarwal and Jaggi (1995) ^[1] generalized Goyal's model by allowing credit period in the inventory model for deteriorating items. Chung and Huang (2005) ^[5] presented an EOQ model for inventory control in the presence of trade credit. Chung, Huang, and Huang (2002) ^[6] and Chung and Huang (2003) ^[4] considered permissible delay in payments to develop an optimal replenishment policy for EOQ models. Das *et al.* (2011) ^[7] allowed credit period to the buyer for time varying demand in developing the inventory model. Tripathy and Pradhan (2012) ^[19] depicted an interactive inventory model between producer and buyer for extraordinary purchase and credit period with two parameter weibull demand rate. An economic production quantity model under permissible delay was introduced with time proportional deterioration. Teng *et al.* (2014) ^[16] considered two level trade credits in developing the inventory model. Sarkar *et al.* (2015) ^[12] considered fixed lifetime products with trade credit policy in their inventory model. Tripathy and Pandey (2015) ^[17] introduced backlogging in permissible delay period with time dependent demand. Tayal *et al.* (2016) ^[21] applied preservation technology in the integrated production inventory model for perishable products. Tripathi *et al.* (2018) ^[18] adorned their research work by using stock dependent demand under various trade credits. Bag and Tripathy (2018) ^[2] developed the inventory model by considering four levels of permissible delay periods.

To get a legalistic approach to the inventory parameters fuzziness is applied. It is used in situations when uncertainties prevail in the formulation of the model. Park (1987) ^[12] fuzzified the inventory carrying cost and developed the inventory model. Mishra and Singh (2011) ^[11], Yao and Chiang (2003) ^[4], Jaggi *et al.* (2012) ^[9], Jaggi *et al.* (2016) ^[10], Jing *et al.* (2003) ^[3], Tripathy and Bag (2019) ^[19] are some other researchers who developed the inventory models by introducing the fuzzy approach.

In this chapter a fuzzy inventory model is framed for deteriorating items with time and selling price induced quadratic demand.

Four levels of conditional deliveries are considered and individual cases are developed accordingly. The objective of the study is the optimization of the cost function to reduce economic losses.

The rest of the chapter is developed as follows. Notations and assumptions are placed in section 2. The mathematical model of the inventory problem is derived in section 3. Empirical investigation for the proposed model is placed in section 4. Further sensitivity analysis is carried out in section 5 to study the changes in the decision variables with a change in system parameters. Finally the concluding remarks with scope of future research are emphasized in section 6.

Table 1: Contribution of some selected authors

Authors	Demand	Demand type	Number of Stages	Deterioration	Fuzzification
Goyal (1985) ^[8]	Constant	Linear	2	Absent	Absent
Agarwal & Jaggi (1995) ^[11]	Time dependent	Linear	2	Present	Absent
K.J. Chung, Y.F. Huang & C.K. Huang (2002) ^[6]	Constant	Constant	2	Absent	Absent
K.J. Chung & Y.F. Huang (2003) ^[4]	Constant	Constant	3	Absent	Absent
Das <i>et al.</i> (2011) ^[7]	Time dependent	Linear	3	Absent	Absent
C.K. Tripathy & L.M. Pradhan (2012) ^[19]	Constant	Constant	2	Present	Absent
S. Tayal <i>et al.</i> (2016) ^[21]	Time dependent	Exponential	2	Present	Absent
P.K. Tripathy & S. Pradhan (2012) ^[19]	Time dependent	Exponential (Ramp type)	2	Present	Absent
Tripathi <i>et al.</i> (2018) ^[18]	Stock dependent	Linear	4	Present	Absent
P.K. Tripathy & Anima Bag (2019) ^[19]	Time and selling price dependent	Quadratic	4	Present (time dependent)	Absent
Jaggi <i>et al.</i> (2016) ^[10]	Price dependent	Linear	3	Absent	Present
Present	Time and selling price dependent	Quadratic	4	Present (not time dependent)	Present

Notations and Assumptions

In this chapter, the following notations are used to develop the inventory model.

- $I(t)$: Inventory level at any instant of time t
- C_p : Purchase cost per unit
- s : Selling price per unit
- $D(t)$: Demand rate at any instant of time t
- θ : Rate of deterioration, $0 < \theta < 1$
- C_h : Holding cost per unit item.
- k : Deterioration cost per unit item.
- C_o : Ordering cost per order.
- r : Rate of cash discount, $0 < r < 1$
- I_p, I_e : The interest paid and earned / dollar / year respectively
- M_d, M_p : Cash discount and permissible delay periods respectively, $M_p > M_d$
- T_1, T_2, T_3, T_4 : The length of cycle time for cases I, II, III, IV respectively.
- $Q1, Q2, Q3, Q4$: The order quantity of the inventory system for cases I, II, III, IV respectively.
- $TC1, TC2, TC3, TC4$: The total cost of the inventory system per year for cases I, II, III, IV respectively.

The following assumptions are considered in this chapter.

1. The demand rate is both selling price and time induced.

$$D(t) = as^{-\beta}(1 + bt - ct^2)$$
2. The lead time is zero or negligible.
3. Shortage is not allowed.
4. The permissible delay period is more than the cash discount period.

Mathematical Formulation and Optimal Solution

The inventory level for the depleting items at any instance of time t with selling price dependent demand is represented by the differential equation as below.

$$\frac{dI(t)}{dt} + \theta I(t) = -as^{-\beta}(1 + bt - ct^2), 0 \leq t \leq T \tag{1}$$

With the initial condition $I(T) = 0$, the solution of the above differential equation is

$$I(t) = \frac{-as^{-\beta}}{\theta^3} \left[\theta^2(1 + bt - ct^2) - \theta(b - 2cT) - 2c + e^{\theta(T-t)} \left(\theta^2(cT^2 - bT - 1) - 2\theta\left(-\frac{b}{2} + cT\right) + 2c \right) \right] \tag{2}$$

At the beginning, the supplier has ‘Q’ units in the inventory system.

$$Q = I(0) = \frac{-as^{-\beta}}{\theta^3} \left[\theta^2 - \theta b - 2c + e^{\theta T} \left\{ \theta^2 (cT^2 - bT - 1) - 2\theta \left(-\frac{b}{2} + cT\right) + 2c \right\} \right] \tag{3}$$

Total cost/year consists of the following elements.

i. Ordering cost = $\frac{C_0}{T}$ (4)

ii. Purchase cost = $\frac{C_p Q}{T} = \frac{-as^{-\beta} C_p}{\theta^3 T} \left[\theta^2 - \theta b - 2c + e^{\theta T} \left\{ \theta^2 (cT^2 - bT - 1) - 2\theta \left(-\frac{b}{2} + cT\right) + 2c \right\} \right]$ (5)

iii. Holding cost =

iv. $\frac{C_h}{T} \int_0^T I(t) dt = \frac{C_h}{T} \left(\frac{-as^{-\beta}}{\theta^3} \right) \left[\theta^2 T + \frac{b}{2} \theta^2 T^2 - \frac{c}{3} \theta^2 T^3 + \theta - b - \frac{2c}{\theta} + e^{\theta T} \left(c\theta T^2 - b\theta T - \theta + b - 2cT + \frac{2c}{\theta} \right) \right]$ (6)

v. Deterioration cost = $K\theta \int_0^T I(t) dt = K \left(\frac{-as^{-\beta}}{\theta^2} \right) \left[\theta^2 T + \frac{b}{2} \theta^2 T^2 - \frac{c}{3} \theta^2 T^3 + \theta - b - \frac{2c}{\theta} + e^{\theta T} \left(c\theta T^2 - b\theta T - \theta + b - 2cT + \frac{2c}{\theta} \right) \right]$ (7)

Four possible cases are considered. In case-I ($M_d \leq T$) and case-II ($M_d > T$), cash discount is offered. In case-I ($M_d \leq T$) and case-III ($M_p \leq T$) some interest is paid and also some interest is earned. In case-II ($M_d > T$) and Case-IV ($M_p > T$) no interest is paid, only interest is earned.

Case I: ($M_d \leq T$)

The cash discount gained by the customer per unit time = $\frac{rC_p Q}{T}$

$$= \frac{rC_p}{T} \left(\frac{-as^{-\beta}}{\theta^3} \right) \left[\theta^2 - \theta b - 2c + \left(\theta^2 (cT^2 - bT - 1) - 2\theta \left(-\frac{b}{2} + cT\right) + 2c \right) e^{\theta T} \right]$$

The interest paid per year is $\frac{C_p (1-r) I_p}{T} \int_{M_d}^T I(t) dt$

$$= \left(\frac{-as^{-\beta}}{\theta^3} \right) \frac{C_p (1-r) I_p}{T} \left[\theta^2 T + \frac{b}{2} \theta^2 T^2 - \frac{c}{3} \theta^2 T^3 + \theta - b - \frac{2c}{\theta} - \theta^2 M_d - \frac{b}{2} \theta^2 M_d^2 + \frac{c}{3} \theta^2 M_d^3 + b\theta M_d - c\theta M_d^2 + 2cM_d - \left(cT^2\theta - b\theta T - \theta + b - 2cT + \frac{2c}{\theta} \right) \left(1 + \theta T - \theta M_d + \theta^2 (T^2 - 2TM_d + M_d^2) \right) \right] \tag{9}$$

During $(0, M_d)$ the customer sells products and deposits the revenue into an account that earns I_e per dollar per year.

Thus interest earned per unit time is

$$\frac{sI_e}{T} \int_0^{M_d} as^{-\beta} (1 + bt - ct^2) t dt = as^{1-\beta} \frac{I_e}{T} M_d^2 \left[\frac{1}{2} + \frac{bM_d}{3} - \frac{cM_d^2}{4} \right] \tag{10}$$

The total relevant cost per unit time $TC1$ is calculated by $TC1 = \text{Cost of placing order} + \text{Cost of purchasing units after discount rate } (r) + \text{Holding cost} + \text{Deterioration cost} + \text{Interest paid per unit time} - \text{Interest earned per unit time}$ (11)

The parameters s, θ, C_h, b, c vary within certain limit due to environmental uncertainty. Hence fuzzy approach is implemented to the model to deal with such uncertainties. Triangular fuzzification is implemented for this purpose. The defuzzification of the total cost is carried out by using Graded Mean Representation Method, Signed Distance Method and Centroid Method. Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (s_1, s_2, s_3), \tilde{\theta} = (\theta_1, \theta_2, \theta_3), \tilde{C}_h = (C_{h1}, C_{h2}, C_{h3}), \tilde{b} = (b_1, b_2, b_3), \tilde{c} = (c_1, c_2, c_3)$$

Using Graded Mean Representation Method of defuzzification, the total cost will be

$$(TC1)_{gd} = \frac{1}{6} [TC1_{gm1} + 4TC1_{gm2} + TC1_{gm3}]$$

Using Signed Distance Method of defuzzification, the total cost will be

$$(TC1)_{sd} = \frac{1}{4} [TC1_{sd1} + 2TC1_{sd2} + TC1_{sd3}]$$

Using Centroid Method of defuzzification, the total cost will be

$$(TC1)_{dc} = \frac{1}{3} [TC1_{dc1} + TC1_{dc2} + TC1_{dc3}]$$

Case-II ($M_I > T$)

In this case no interest is payable. Cash discount is same as that of case - I.

The interest earned per unit time is

$$\frac{SI_e}{T} \int_0^T as^{-\beta} (1 + bt - ct^2) dt + (M_d - T) \int_0^T as^{-\beta} (1 + bt - ct^2) dt = -as^{-\beta} T \left[SI_e \left(\frac{1}{2} + \frac{bT}{3} - \frac{cT^2}{4} \right) + (M_d - T) \left(1 + \frac{bT}{2} - \frac{cT^2}{3} \right) \right] \tag{12}$$

Total relevant cost per year $TC2$ is calculated by $TC2 =$ Cost of placing order + Cost of purchasing units after discount rate (r) + Holding cost + Deterioration cost – Interest earned per unit time (13)

In the similar manner using graded mean integration method, signed distance method and centroid method of defuzzification, the total costs will be

$$(TC2)_{gd} = \frac{1}{6} [TC2_{gm1} + 4TC2_{gm2} + TC2_{gm3}] \quad (TC2)_{sd} = \frac{1}{4} [TC2_{sd1} + 2TC2_{sd2} + TC2_{sd3}] \quad (TC2)_{dc} = \frac{1}{3} [TC2_{dc1} + TC2_{dc2} + TC2_{dc3}]$$

Case-III ($M_p \leq T$)

In this case, the amount is paid at time M_p , there is no cash discount.

The interest paid per unit time is

$$\frac{C_p I_p}{T} \int_{M_p}^T I(t) dt = \frac{-as^{-\beta} C_p I_p}{\theta^3 T} \left[\theta^2 T + \frac{b}{2} \theta^2 T^2 - \frac{c}{3} \theta^2 T^3 + \theta - b - \frac{2c}{\theta} - \theta^2 M_p - \frac{b}{2} \theta^2 M_p^2 + \frac{c}{3} \theta^2 M_p^3 + b\theta M_p - c\theta M_p^2 + 2cM_p \right. \\ \left. - \left(cT^2\theta - b\theta T - \theta + b - 2cT + \frac{2c}{\theta} \right) \left(1 + \theta T - \theta M_p + \theta^2 (T^2 - 2TM_p + M_p^2) \right) \right] \tag{14}$$

The interest earned per unit time is

$$\frac{SI_e}{T} \int_0^{M_p} as^{-\beta} (1 + bt - ct^2) dt = as^{1-\beta} \frac{I_e}{T} M_p^2 \left[\frac{1}{2} + \frac{bM_p}{3} - \frac{cM_p^2}{4} \right] \tag{15}$$

Total relevant cost per unit time $TC3$ is calculated by

$TC3 =$ Cost of placing order + Cost of purchasing units+ Holding cost + Deterioration cost + Interest payable per unit time – Interest earned per unit time (16)

In the similar manner using graded mean integration method, signed distance method and centroid method of defuzzifications, the total costs will be

$$(TC3)_{gd} = \frac{1}{6} [TC3_{gm1} + 4TC3_{gm2} + TC3_{gm3}]$$

$$(TC3)_{sd} = \frac{1}{4} [TC3_{sd1} + 2TC3_{sd2} + TC3_{sd3}] \quad (TC3)_{dc} = \frac{1}{3} [TC3_{dc1} + TC3_{dc2} + TC3_{dc3}]$$

Case-IV ($M_p > T$)

In this case no interest is payable.

The interest earned per unit time is

$$\frac{sI_e}{T} \int_0^T as^{-\beta} (1 + bt - ct^2) dt + (M_p - T) \int_0^T as^{-\beta} (1 + bt - ct^2) dt = -as^{-\beta} T \left[sI_e \left(\frac{1}{2} + \frac{bT}{3} - \frac{cT^2}{4} \right) + (M_p - T) \left(1 + \frac{bT}{2} - \frac{cT^2}{3} \right) \right] \quad (17)$$

Total relevant cost per unit time $TC4$ is calculated by $TC4 = \text{Cost of placing order} + \text{Cost of purchasing units} + \text{Holding cost} + \text{Deterioration cost} + \text{Interest payable per unit time} - \text{Interest earned per unit time}$ (18)

In the similar manner using graded mean integration method, signed distance method and centroid method of defuzzification, the total costs will be

$$(TC4)_{gd} = \frac{1}{6} [TC4_{gm1} + 4TC4_{gm2} + TC4_{gm3}]$$

$$(TC4)_{sd} = \frac{1}{4} [TC4_{sd1} + 2TC4_{sd2} + TC4_{sd3}], (TC4)_{dc} = \frac{1}{3} [TC4_{dc1} + TC4_{dc2} + TC4_{dc3}]$$

The optimal values of T for cases I, II, III, and IV are determined by solving $\frac{\partial TC_i}{\partial T} = 0$, for $i = 1, 2, 3, 4$ and total costs are determined accordingly.

For convexity of the total cost function, the necessary and sufficient conditions $\frac{\partial^2 TC_i}{\partial T^2} > 0$, $i=1, 2, 3, 4$ should be satisfied,

In fuzzy model, the values of T for different cases are also worked out and the total costs are calculated accordingly. Convexity conditions are also checked.

Empirical Investigation

To illustrate the effect of the proposed model, the following empirical investigation is performed with four possible cases. Both crisp and fuzzy models are considered for the purpose to obtain more realistic result. Mathematical 5.1 software is used to obtain the relevant results and convexity test is also performed.

Numerical illustration-1

Let the parameters be set as follows

Case: 1 ($M_d \leq T$)

Crisp Model

$$a = 100, C_h = \$4 / unit / year, I_p = \$0.08 / year, I_e = \$0.005 / year, C_p = \$40 / unit, s = \$65 / unit, \theta = 0.1 / year, r = 0.08$$

$$M_d = 1month, C_0 = \$5000 / order, b = 0.2, c = 0.3, \beta = 0.1, k = \$0.001 / unit$$

Result: $T = 5.44013years$, Total Cost $TC1 = \$393140$, Purchase Quantity $Q1 = 844.854units$

Fuzzy Model

Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (60,65,70), \tilde{\theta} = (0.05,0.1,0.15), \tilde{C}_h = (3,4,5), \tilde{b} = (0.1,0.2,0.3), \tilde{c} = (0.2,0.3,0.4)$$

Using Graded Mean Representation Method of defuzzification,

$$T_{gd} = 5.30765 years, (TC1)_{gd} = \$394320, Q1_{gd} = 746.428units$$

Using Signed Distance Method of defuzzification,

$$T_{sd} = 5.29666 years, (TC1)_{sd} = \$396620, Q1_{sd} = 738.587units$$

Using Centroid Method of defuzzification,

$$T_{cd} = 5.27785 years, (TC1)_{cd} = \$397320, Q1_{cd} = 725.281units$$

Case: 2 ($M_d > T$)**Crisp Model**

$a = 100, h = \$4/\text{unit}/\text{year}, I_p = \$0.08/\text{year}, I_e = \$0.005/\text{year}, C_p = \$40/\text{unit}, s = \$65/\text{unit}, \theta = 0.1, r = 0.08/\text{year},$
 $M_d = 9\text{months}, C_0 = \$5000/\text{order}, b = 0.2, c = 0.3, \beta = 0.1, k = \$0.001/\text{unit}$

Result: $T = 8.84653\text{years}$, Total Cost $TC2 = \$43925.7$, Purchase Quantity, $Q2 = 7093.02\text{units}$ **Fuzzy Model**

Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (60, 65, 70), \tilde{\theta} = (0.05, 0.1, 0.15), \tilde{C}_h = (3, 4, 5), \tilde{b} = (0.1, 0.2, 0.3), \tilde{c} = (0.2, 0.3, 0.4)$$

Using Graded Mean Representation Method of defuzzification,

$$T_{gd} = 8.30765\text{years}, (TC2)_{gd} = \$45432, Q2_{gd} = 5471.35\text{units}$$

Using Signed Distance Method of defuzzification,

$$T_{sd} = 6.66546\text{years}, (TC2)_{sd} = \$46544, Q2_{sd} = 2155.77\text{units}$$

Using Centroid Method of defuzzification,

$$T_{cd} = 6.30765\text{years}, (TC2)_{cd} = \$48998, Q2_{cd} = 1689.87\text{units}$$

Case: 3 ($M_p \leq T$)**Crisp Model**

$a = 100, h = \$4/\text{unit}, I_p = \$0.08/\text{year}, I_e = \$0.005/\text{year}, C_p = \$40/\text{unit}, s = \$65/\text{unit}, \theta = 0.1, r = 0.08/\text{year},$
 $M_d = 2\text{months}, C_0 = \$5000/\text{order}, b = 0.2, c = 0.3, \beta = 0.1, k = \$0.001/\text{unit}$

Result**Result:** $T = 5.49334\text{years}$, Total Cost $TC3 = \$382540$, Purchase Quantity $Q3 = 886.467\text{units}$ **Fuzzy Model**

Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (60, 65, 70), \tilde{\theta} = (0.05, 0.1, 0.15), \tilde{C}_h = (3, 4, 5), \tilde{b} = (0.1, 0.2, 0.3), \tilde{c} = (0.2, 0.3, 0.4)$$

Using Graded Mean Representation Method of defuzzification,

$$T_{gd} = 5.30765\text{years}, (TC3)_{gd} = \$384320, Q3_{gd} = 741.885\text{units}$$

Using Signed Distance Method of defuzzification,

$$T_{sd} = 5.00765\text{years}, (TC3)_{sd} = \$387889, Q3_{sd} = 549.369\text{units}$$

Using Centroid Method of defuzzification,

$$T_{cd} = 4.66544\text{years}, (TC3)_{cd} = \$409908, Q3_{cd} = 364.499\text{units}$$

Case: 4 ($M_p > T$)**Crisp Model**

$a = 100, h = \$4/\text{unit}, I_p = \$0.08/\text{year}, I_e = \$0.005/\text{year}, C_p = \$40/\text{unit}, s = \$65/\text{unit}, \theta = 0.1, r = 0.08/\text{year},$
 $M_d = 10\text{months}, C_0 = \$5000/\text{order}, b = 0.2, c = 0.3, \beta = 0.1, k = \$0.001/\text{unit}$

Result: $T = 7.76267$, Total Cost $TC4 = 25235.2$, Purchase Quantity, $Q4 = 4125.73$

Fuzzy Model

Implementing triangular fuzzification, let the parameters be

$$\tilde{s} = (60,65,70), \tilde{\theta} = (0.05,0.1,0.15), \tilde{C}_h = (3,4,5), \tilde{b} = (0.1,0.2,0.3), \tilde{c} = (0.2,0.3,0.4)$$

Using Graded Mean Representation Method of defuzzification,

$$T_{gd} = 6.30760 \text{ years}, (TC4)_{gd} = \$26344.6, Q4_{gd} = 1689.8 \text{ units}$$

Using Signed Distance Method of defuzzification,

$$T_{sd} = 5.34456 \text{ years}, (TC4)_{sd} = \$28665.7, Q4_{sd} = 773.12 \text{ units}$$

Using Centroid Method of defuzzification,

$$T_{cd} = 5.30765 \text{ years}, (TC4)_{cd} = \$30998, Q4_{cd} = 746.428 \text{ units}$$

Sensitivity Analysis

Sensitivity analysis is performed on the parameters θ , r and M to analyse the effect of the changes in their values on the optimal solutions. Graded Mean Integration method of defuzzification is considered for the purpose as it gives minimum cost as compared to signed distance method and centroid method.

Sensitivity analysis reveals that

- The increase in rate of deterioration ' θ ' results and increase in the values of Total Cost TC and Order Quantity Q .
- The increase in rate of cash discount ' r ' results a decrease in the total cost but results more Order Quantity.
- The more the period of cash discount M_d offered the less is the Total Cost and Order Quantity. But after certain period the Total Cost increases.
- Similarly the more the permissible delay period is offered, the less is the Total Cost and Order Quantity. But after certain period the total cost increases.

Conclusion

This paper develops a fuzzy inventory model with multilevel permissible delay in payments. Cash discount period is less than the permissible delay period. The total cost is minimized by applying optimization technique. Through empirical investigation and sensitivity analysis it is concluded that the rate of cash discount should be more so as to minimize the total cost as it promotes the early sale of the inventory. The increased rate of deterioration results a rise in total cost. So it is advised to the seller to sell the inventory as early as possible to avoid deterioration by providing multilevel permissible delay in payments so as to minimize the total cost. The results in this paper provide a valuable reference to the decision makers in planning procurement and controlling inventories. The model can be applied in the inventory system that deals with perishable items like packaged foods, Grocery items, confectionery etc.

This paper can be generalized by allowing shortage in the model. Advertisement frequency and entropy can also be included in the model for future study.

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