International Journal of Statistics and Applied Mathematics 2022; 7(1): 159-162

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(1): 159-162

www.mathsjournal.com
Received: 17-09-2021
Accepted: 23-10-2021

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Application of integral equations using numerical wavelet methods

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DOI: https://doi.org/10.22271/maths.2022.v7.i1b.791

Abstract
The mother wavelet is a prototype function that is adjusted during the wavelet analysis process. A contracted, high frequency version of the prototype wavelet is used for temporal analysis, while a larger, low frequency version of a comparable wavelet is used for frequency analysis. Data activities can be accomplished using the wavelet coefficients since the first signal can be spoken to as a wavelet extension. If we also chose the best wavelet as indicated by the data, the data is meagerly spoken to. Astronomy, nuclear engineering, turbulence, earthquake predictions, acoustics, sub and coding, magnetic resonance, imaging, optics, fractals, speech discrimination, neurophysiology, radar, human vision, signal and image processing, and pure mathematics applications, such as understanding partial differential equations, are all influencing the use of wavelets.

Keywords: Acoustics, sub and coding, magnetic resonance, imaging, optics, fractals, speech discrimination, neurophysiology

1. Introduction

Integral equations are used as mathematical models for a variety of physical circumstances, and they can also be used to reformulate other mathematical issues. In recent years, there has been a lot of interest in using wavelet methods to solve integral equations. The first work that used the Haar wavelet approach to solve an integral equation was published by Beylkin, G 1991 [14]. After that, there has been a slew of questions regarding using this method to illustrate various types of integral equations. In Haar wavelet method is connected to explain various types of linear integral equations (Fredholm, Volterra, integro differential, and pitifully solitary integral equations), as well as the Eigen value issue, while connected the Haar wavelet method to unravel the nonlinear Fredholm integral equation, and used the wavelet for settling Fredholm integral equation demonstrated the application of the Haar wavelet change to fathoming integral and differential equations Kelmanson MA *et al*. 2018 [1].

2. Integral equation

An integral equation is one in which at least one integral sign represents an unknown function. For instance, for $a \le x \le b$, $a \le t \le b$, the equations

$$\int_a^b K(x,t)y(t)dt = f(x) \tag{1.1}$$

$$y(x) - \lambda \int_a^b K(x,t)y(t)dt = f(x) \tag{1.2}$$

And

$$y(x) = \int_a^b K(x,t)[y(t)]^2 dt \tag{1.3}$$

The unknown function y(x) lies on the whole integral equations, while f(x) and K (x, t) are known functions and "a" and "b" are constants. In x and t, the previously described functions might be real or complex valued functions. We have only considered actual valued functions in our research Das P *et al*. 2015 [2].

2.1 Classification of integral equations
In ordinary and partial differential equations, an integral equation is referred to as a linear or nonlinear integral equation. We've seen how the integral equation can speak to the differential equation in a similar way. As a result, these two equations have a reasonable relationship. When only linear operations are done on the unknown function, an integral equation is called linear. Nonlinear integral equations are integral equations that are not linear. The most often used integral equations can be divided into two categories: Volterra and Fredholm integral equations. We must also classify them as either homogeneous or non-homogeneous integral equations Wazwaz AM 2011 [3].

1) Fredholm integral equation
The Fredholm integral equation is an integral equation whose solution leads to the study of Fredholm kernels and operators, as well as Fredholm theory. The shape is the broadest type of Fredholm linear integral equations Maleknejad K *et al*. 2002 [4]

$$g(x)y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt, \qquad (1.4)$$

The Fredholm integral equation of third kind is defined as follows: a, b are two constants, f(x), g(x), and K(x, t) are known functions, whereas y(x) is an unknown function and is a non-zero genuine or complex parameter. The kernel of the integral equation is known as K (x, t).

2) Volterra integral equation
The Volterra integral equations are a type of integral equation that isn't found anywhere else. They are divided into two groups, referred known as the first and second kinds. The form is the broadest form of Volterra linear integral equations Zou H *et al.* 2019 [5]

$$g(x)y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt, \qquad (1.5)$$

The Volterra integral equation of third kind is defined as follows: an is constant, f(x), g(x), and K (x, t) are known functions, whereas y(x) is an unknown function and is a non-zero real or complex parameter. The kernel of the integral equation is known as K (x, t).

3) Singular integral equation
A singular integral equation is defined as an integral that extends as long as possible or when the kernel of the integral becomes unbounded at least once inside the integration interval. The equations below, for example, are singular integral equations Maleknejad K *et al*. 2021 [6].

$$y(x) = f(x) + \lambda \int_{-\infty}^{\infty} e^{-|x-t|}\, y(t)dt \qquad (1.6)$$

And

$$f(x) = \int_0^x \frac{1}{(x-t)^a} y(t)dt, 0 < a < 1 \qquad (1.7)$$

4) Integro-differential equation
Vito Volterra investigated the subject of population growth in the mid-1900s, and new types of equations were created and dubbed integra-differential equations. The unknown function y(x) appears as a blend of the ordinary derivative and the integral sign in these equations. For occurrence Lin XY *et al*. 2009 [7].

$$y^n(x) = f(x) + \lambda \int_0^x \frac{1}{(x-t)^a} y(t)dt, y(0) = 0, y'(0) = 1 \qquad (1.8)$$

$$y'(x) = f(x) + \lambda \int_0^x (xt)y(t)dt, y(0) = 1,$$

Both the second order Volterra integro-differential equation and the first order Fredholm integro-differential equation are used in the preceding equations.

5) Special kind of kernels
The kernel function is a critical component of the integral equation. The following is a classification of kernel functions Mouley J *et al*. 2021 [8]:

- **Symmetric kernel**: A kernel *K* (*x*, *t*) is symmetric (or complex symmetric) if

$$K(x,t) = \overline{K}(x,t)$$

Where the bar implies the complex conjugate A real kernel $K(x,t)$ is symmetric if

$$K(x,t) = K(x,t)$$

- **Separable or degenerate kernel:** A kernel $K(x,t)$ is entitled dissimilar or decline on the off chance that it very well can be linked as the whole of a finite quantity of terms, every one of which is the consequence of a function of x just and a function of t just, i.e.

$$K(x,t) = \sum_{i=0}^{n} g_i(x)h_i(t)$$

- **Non-degenerate kernel:** A kernel $K(x,t)$ is termed non-degenerate if it cannot be isolated as the purpose of x and function of$t$. For instance, $e^{xt}\sqrt{x,t}$,are the non-degenerate kernels?

3. Wavelets
Wavelets are now regarded as a game-changing new mathematical tool in signal and image processing, time series analysis, geophysics, approximation theory, and a variety of other fields. First and foremost, wavelets were used in seismology to provide a time measurement to seismic analysis, where Fourier analysis fails. Fourier analysis is ideal for concentrating stationary data (data whose factual properties are invariant over time), but it isn't appropriate for considering data with transient events that can't be measurably predicted from the data past. Wavelets were created with such no stationary data in mind; their all-encompassing statement and solid outcomes have quickly proven to be beneficial to a variety of disciplines. Wavelets philosophy is a abstemiously new and emerging territory in mathematical research Liu CS et al. 2009 [9].
We study, in this section, the space $L^2(\mathbb{R})$ of calculable functions$f$, considered on the real line ℝ, that fulfill

$\int_{-\infty}^{\infty} |f(t)|^2 dt \leq \infty$ (1.9)

Certainly, we exploration for such "waves" that produce $L^2(\mathbb{R})$, these waves must rot to zero at ±∞; and for all pragmatic aim, the rot must to be fast. That is, we exploration for small waves, or "wavelets", to yield $L^2(\mathbb{R})$. For this aim, we rise toward a solitary function ψ that makes all $L^2(\mathbb{R})$. Since, ψ is quick rot, to cover whole real line, we shift ψ along$\mathbb{R}$. For computational efficiency, we have used integral powers of 2 for frequency allocating. That is, think about the small waves

$\psi(2^j t - k), j, k \in \mathbb{Z}$ (1.10)

$\psi(2^j t - k)$ Is gotten from a solitary wavelet occupation ψ (t) by a parallel enlargement (expansion by 2j) and a dyadic interpretation (of k/2j) any wavelet function $\psi \in L^2(\mathbb{R})$ has two contentions as ψj, k and characterized by

$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^j t - k), j, k \epsilon \mathbb{Z}$ (1.11)

Where the quantity $2^{j/2}$ is for normality.

3.1 Wavelet transform

Wavelet is an abbreviation for 'small wave.' As a result, wavelet analysis is linked to the examination of signals with small span finite energy functions. They transform the signal under consideration into another portrayal that alters the signal in a more beneficial manner. This signal transformation is known as the wavelet transform. The first is interpreting (change of position). We shift the wavelet's focal point along the time hub. The second is scalability. The wavelet transform fundamentally evaluates the wavelet's neighborhood coordinating with the signal. If the wavelet matches the signal well at a scale and location, an expanded transform value is obtained. The transform value is then shown in the transform plane in two dimensions. The wavelet transform was recorded at several points in the signal and for various wavelet sizes. Continuous wavelet transform is used when the technique is carried out in a smooth and continuous manner. Also, keep in mind that the spectrum is affected by the wavelet type used in the study. Mathematically, we specify a wavelet as Kaneko H *et al*. 2003 [10]:

$\varphi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\varphi\left(\frac{t-b}{a}\right)\ a, b \epsilon R, a \neq 0,$ (1.12)

Where b is site parameter and 'a' is scaling parameter. The function must be time limited in order to be wavelet. We interpret the wavelet by varying the parameter b for a given scaling parameter a. We characterize wavelet transform as:

$W(a, b) = \int_t f(t)\frac{1}{\sqrt{|a|}}\varphi\left(\frac{t-b}{a}\right)dt$ (1.12 (a))

For each *(a, b)*, we have wavelet transform coefficient, if |a| < 1, at that point the wavelet in is compressed version (smaller assistance in time domain) of the mother wavelet and relates essentially to higher frequencies. Then again, when |a| > 1, at that point $\psi a, b\ (t)$ has a bigger time-width than ψ (t) and associates to bring down frequencies consequently, wavelets have time widths adjusted to their frequencies. This is the opinion purpose behind the accomplishment of Morlet wavelets in signal dispensation and time-frequency signal analysis.

3.2 Continuous wavelet transform

Let f (x) be any square integrable function. Then the incessant wavelet transforms W ψf of $f \in L^2(\mathbb{R})$ with respect to ψ is distinct as Delves LM *et al*. 1988 [11]

$W_\varphi(b, a) = |a|^{-\frac{1}{2}}\int_{-\infty}^{\infty} f(x)\overline{\varphi\left(\frac{x-b}{a}\right)}$ (1.13)

Where 'a' and 'b' are real, and bar indicates the complex conjugation. Consequently, the wavelet transform is a function of two variables. There normalizing factor $\frac{1}{\sqrt{a}}$ a ensure that the energy breaks the similar for all a and b; that is

$\int_{-\infty}^{\infty}\left|\varphi_{a,b}(x)\right|^2 dt = \int_{-\infty}^{\infty}|\varphi\ (x)|^2\ dt$ (1.14)

The finiteness of this continuous (admissibility condition) restrain the class of L^2 ($\mathbb{R}$) function that can be utilized as wavelets this suggests.

$\int_{\infty}^{\infty} \varphi(x) = 0$ (1.15)

See for more details. With the constant Cψ, we have the subsequent reconstruction formula an f. At the point when the suitable response is negative, the utilization of a discrete subset seems a sensible aim.

$f(x)=\frac{1}{C_\varphi}\iint_{R^2} W_\varphi(b, a)\overline{\varphi\left(\frac{x-b}{a}\right)}\frac{dadb}{a^2}, f\in L_2(R)$ (1.16)

Notice that the likelihood of reform is ensured by the admissibility condition. Currently we move from CWT to discrete wavelet transform.

3.3 Continuous to discrete wavelet transforms

It is legitimate to ponder whether it is important to know Cψ anywhere to progress the thought is as per the subsequent: we think about separate subset of R+∗ and R. Give us a accidental to settle $a_0 > 1$ and $b_0 > 0$ and take $a\in \{a_0^j\}_{j\in z}$ and $b\in \{ka_0^j b_0\}_{j,k\in z}$ Instead of using the family of wavelets Richards C 2016 [16]:

$\varphi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\varphi\left(\frac{t-b}{a}\right)\ a\epsilon R^{+*}, b \in R.$ (1.17)

For the discrete wavelet transform we use the family of wavelets well-ordered by Z'

$\varphi_{j,k}(x) = a_0^{-\frac{j}{2}}\varphi(a_0^{-j}x - kb_0)a_0 > 1, b_0 > 0 fixedf\ or\ j, k \in Z$ (1.18)

For $f \in L^2$ we define the separate wavelet transform of the function f by:

$C_f(j,k) = \int_R f(x)\overline{\varphi_{j,k}(x)}dx = \langle f, \varphi_{j,k}\rangle_{L^2}$ (1.19)

Where j, k ∈ Z When value of $a_0 = 2$, $b_0 = 1$ construct separate wavelet transform as

$\varphi_{j,k}(x) = 2^{-j/2}\varphi(2^{-j}x - k)$ (1.20)

This is used in multi-resolution analysis including an ortho-normal basis for L^2 ($\mathbb{R}$)

4. Basic scaling function and basic wavelet

The set $\{\phi(x-k)\}_{k\epsilon z}$is an orthonormal basis for V_0 by axiom. Currently, it pursues by rehashed request of aphorism that is $\{\phi(2^j x-k)\}_{k\epsilon z}$ an orthonormal basis for V_j. Note that the function $\phi(2^j x)$ is decoded by $\frac{k}{2j}$ i.e. it changes toward becoming narrower and translations get littler as j increments. The L^2- standard of one of these basis functions is as per the following Mouley J *et al.* 2020 [15].

Therefore the set $\{2^{\frac{1}{2}}\phi(2^j x-k)\}_{k\epsilon z}$ is an ortho-noraml basis for V_j.We call ϕ as the basic scaling function, since we make a whole bunch of basis function by dilation and translation of ϕ. Likewise it is exposed that there exists a function $\psi(x)$ such that $\{2^{\frac{1}{2}}\psi(2^j x=k)\}_{k\epsilon z}$ is an ortho-normal basis forW_j. We call ψ as the basis wavelet or mother wavelet. Note that iot might not be possible to fast either of them (ϕ or ψ) obviously but there are efficient approaches for calculating the values of ϕ or ψ at some dyadic rational points. For continence, we now present the following notation

$$\phi_{j,k}(x)=2^{\frac{1}{2}}\phi(2^j x-k) \quad (1.21)$$

$$\psi_{j,k}(x)=2^{\frac{1}{2}}\psi(2^j x-k) \quad (1.22)$$

And

$$\phi k(x)=\phi_{ok}(x) \quad (1.23)$$

$$\psi k(x)=\psi_{ok}(x) \quad (1.24)$$

Since it pursues that is orthogonal to ask and note that all are mutual orthogonal, and so the wavelets are orthogonal crosswise over scales. All composed we have the associated orthogonally relations.

$$\int_{-\infty}^{\infty}\Phi_{j,k}(x)\,\Phi_{j,l}(x)dx=\delta_{k,l} \quad (1.25)$$

$$\int_{-\infty}^{\infty}\Psi_{i,k}(x)\,\Psi_{j,l}(x)dx=\delta_{i,j,k,l} \quad (1.26)$$

$$\int_{-\infty}^{\infty}\Phi_{i,k}(x)\,\Psi_{j,l}(x)dx=0, j\geq i, \quad (1.27)$$

Where $i,j-k,l\,£\,Z$ and$k$,' I' is the Kronecker delta defined as:

$$\delta_{k,l}=\begin{cases}0, & k\neq l\\ 1, & k=l\end{cases}$$

5. Conclusion

Higher order nonlinear boundary value problems are effectively solved using the wavelet approach. In addition, innovative wavelet-based numerical approaches for dealing with various types of boundary conditions are being developed. The proposed wavelet approaches, which include the Haar wavelet collocation method, Haar wavelet quasi-linearization method, wavelet-Galerkin method, and wavelet-multigrid method, have shown to be extremely beneficial in a wide range of real-world issues. In fact, wavelet methods are gaining traction as a viable alternative to classic numerical and semi-numerical approaches. The developed wavelet methods can be used to analyses any nonlinear boundary value problems with finite or infinite domains emerging in engineering and science due to inherent features such as Ortho normalization, compact support, simple explicit expression (in the case of Haar wavelets), and the hierarchy of solution approximation. In fact, the method's speed of convergence is boosted by the method's simplicity, sparsity of wavelet matrices, and solution representation with a substantially reduced number of wavelet coefficients.

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