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A study of applications of graph colouring in various fields

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Abstract
Graph theory is an important branch of applied mathematics with a lot of applications in many fields. Graph theory has a broad scale of applications in many practical situations. Graph coloring is one decent approach which can deal with many problems of graph theory. The main aim of this paper is to present the importance of graph coloring ideas in various fields for researchers that the concept of graph theory can be used by them. In this paper an overview is presented especially to project the idea of graph theory and applications of graph colouring.

Keywords: Graph, graph coloring, vertex coloring, applications of graph colouring

Introduction

Graph the theory is generally regarded as most enjoyable branch of discrete mathematics. This is because it has twin nature; it has the brilliant proofs in all the abstract reasoning, and it has a lot of applications in every field. Graphs are important because it is a way of expressing information in pictorial form. A graph can show information that equivalent to many words. Today graph theory a developed theory with a large collection of challenging games and interesting puzzles. Characteristic of Graph theory is that it depends very little on other branches of Mathematics that is independent in itself. Graph theory is rapidly becoming into the mainstream of mathematics mainly because of its applications in various fields which include physics, biology, chemistry, electrical engineering, computer science, operation research etc. In computer science the ideas of graph theory are highly utilized (Daniel M, 2004) [1].

A graph is drawn up of vertices and edges. In other words we can say that graph is an ordered pair G = (V, E) consisting a collection of vertices is V with a collection of edges E. This paper has been divided into two sections. First section gives the historical background of graph theory and a few definitions used in graph theory. Second section emphasized graph coloring and its applications in various fields of life (Formanowicz P, 2012) [2].

History of Graph theory: Many mathematicians have contributed to the growth of graph theory. Graph theory is originated with the problem of Konigsberg Bridge, in 1735. This problem conduct to the concept of Eulerian graph. Euler studied the problem of Konigsberg Bridge and give a structure to resolve the problem called Eulerian graph. The idea of a complete graph and bipartite graph have presented by A.F Mobius, in 1840. The concept of a tree was enacted by Gaustav Kirchhoff in the year 1845, and he enlisted graph theoretical concept in the calculation of currents in electrical networks or circuits. Thomas Gutherie founded the famous four color problem in 1852. In 1969, Heinrich has solved the four color problem by using a computer. Many problems that are considered hard to determine can easily solve use of graph theory (Vedavathi N, 2013) [3].

Basic concept of Graph Theory

Graph theory includes different kinds of graphs, each having basic graph properties. There are different operations that may be performed over various types of graph. Graphs are utilized in more applications as a powerful tool to solve large and complicated problems. Therefore, we need to know some definitions that are part of graph theory (Chong Leung JY, 2014) [4].

Graph: Generally graph G consists of two things, the set of vertices V and also the set of edges E.

Self-loop: An edge having the identical vertex as both its end vertices is called a self-loop.

Parallel edges: More than one edge related to a given pair of vertices is called parallel edges.

Directed graph: A graph in which a set of vertices and edges that are connected together, where all the edges are directed from one vertex to another is called directed graph (Ufuktepe U, 2015) [5].

Undirected graph: A graph where all the edges are bidirectional is called an undirected graph.

Simple graph: A graph which contains neither self-loop nor parallel edges is called a simple graph.

Adjacent vertices: Two vertices are said to be adjacent if they are the end vertices of the identical edge.

Degree of vertices: The number of edges incident on a vertex, with self-loop counted twice is called degree of that vertex.

Isolated vertex: A vertex with zero degree is called isolated vertex.

Pendant vertex: A vertex of degree one is called Pendant vertex.

Regular graph: A graph is said to be regular graph if degree of each vertex is equal. A graph is said to be K regular graph if degree of each vertex is K (Arya S, 2016) [6].

Complete graph: A graph in which each pair of vertices is connected by an edge is said to be complete graph.

Bipartite graph: If vertices of a graph can be divided into two independent sets, X and Y such that every edge (x, y) either connects a vertex from X to Y or vertex from Y to X then the graph is said to be a bipartite graph (Bincy AK, 2017) [7].

Euler Path: In a graph G a simple path is called Euler path if it traverses every edge of graph exactly once.

Euler Circuit: A circuit in a graph G which traverses every edge of graph exactly once is called Euler circuit.

Eulerian Graph: Eulerian graph is a graph which contains either Euler Path or Euler circuit.

Hamiltonian Path: A path in a connected graph G that contains each vertex of graph exactly once is said to be a Hamiltonian graph.

Hamiltonian Circuit: A circuit that contains each vertex of graph exactly once except for the first vertex, which is also the last said to be a Hamiltonian circuit (Bharathi SN, 2017) [8].

Hamiltonian Graph: A graph that contains either a Hamiltonian path or a Hamiltonian circuit is called a Hamiltonian graph.

Graph Coloring: Graph coloring is a major topic of graph theory with many applications and unsolved problems. Graph coloring is the procedure of assignment of colors to each vertex of a graph such that no adjacent vertices get same color. In other words, Graph coloring is a way of coloring the vertices of a graph such that no two adjacent vertices have the same color. It is called a vertex coloring. If we paint all the vertices of a graph with colors such that no two adjacent vertices have the same color then it is called the proper coloring of a graph. A graph in which every vertex has been assigned a color according to the proper coloring is called a properly colored graph. Similarly, if we give a color to each edge so that no two incident edges share the same color then it is called edge coloring. A graph can be properly colored in many ways. A proper coloring in which the least number of colors needed to color the graph is called its chromatic number. It is denoted by the symbol Ψ (G), where G is a graph. A graph that can be assigned (proper) with k colors is called a k colorable graph. And its chromatic number is exactly k (Kristoforus R, 2017) [9].

Applications of graph coloring

Graphs are used to model many problems of the various real field. Graphs are very powerful, important and flexible tool to model. There are many numbers of applications.

GPS or Google Maps: GPS or Google Maps are used to find the shortest route from one destination to another. The destinations are presented by vertices and their connections are presented by edges consisting distance. The best route is determined by the software. To pick up students from their stop to school Schools/Colleges are also using this technique. Each stop shows a vertex and the route shows an edge. A Hamiltonian path represents the efficiency of including every vertex with in the route (Vinutha MS, 2017) [10].

Making Schedule or Time Table

- **Nurse Schedule problem:** Nurse Schedule problem could be a major problem faced by many hospitals everywhere the planet that is a subclass of scheduling problems that are hard to resolve. The work is difficult for the duty planner because the duty planner should make sure that every scheduling decision made complies with a mix of hard hospital rules and soft nurse preference rules. This problem will be solved by graph coloring (Bjorken JD, 1965) [11].
- **Aircraft Scheduling:** Aircraft scheduling is another great problem. This problem is often solved by graph coloring. Assuming that there are k aircrafts which they need assigned n flights. The i^{th} flight should be during the number (ai, bi). If two flights overlap, then the identical aircraft cannot be assigned to both the flights. This problem can be solved by graph coloring. The vertices of a graph correspond to the flights. Two vertices are connected when the corresponding time intervals overlap. Therefore, the graph is an interval graph which is able to

be colored optimally in polynomial time (Biggs N, 1986) [12].

- **Exam Timetable Scheduling:** Exam time-table is a planned schedule for the examinations. Time-tables are so planned to give students a healthy window for preparations every educational institution has two c academic commonest scheduling problems are course timetabling and exam timetabling. Effective timetable is important to the performance of any educational institute.
- It impacts their ability to fulfill changing and evolving subject demands and their combinations in a cost-effective manner satisfying various constraints we are able to solve this problem by using graph coloring concept (Deo N, 2003) [13].

Sudoku

A single player logic based puzzle is called sudoku. This puzzle is a grid of 81 cells, which is split into 9 rows, columns and regions (or blocks). The aim is to put the numbers from 1-9 into empty cells in such way, that in every row, every column and each region (3 x 3 block) each number appears just the once. Sudoku Graph is a graph with 81 vertices (or nodes). Each cell in the Sudoku is seen as a node of the graph. Each node (or cell) has an edge to every other node (cell) in its respective column, row, and 3 x 3 grid. Sudoku can be viewed as Graph and thus can be solved by Graph Coloring with a Chromatic Number, G = 9. It is no different to using 9 different color to paint the vertices in a way that no two adjacent vertices have the identical color (Rosen KH, 2011) [14].

Conclusion

Graph concepts are used to model many kinds of relations and processes in physical, biological, social and information systems. The concept of Graph theory is also seen in several research areas of computer science. This paper mainly concentrates on applications that uses graph coloring concept. The main aim of this paper is to explore role of Graph coloring in various fields of life. Graph coloring is powerful tool that makes things easier.

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