International Journal of Statistics and Applied Mathematics 2022; 7(2): 57-62

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(2): 57-62

www.mathsjournal.com
Received: 19-12-2021
Accepted: 11-02-2022

Bhavana Shrivastava
Department of Mathematics, Rabindranath Tagore University, Bhopal, Madhya Pradesh, India

Bhawna Agrawal
Department of Mathematics, Rabindranath Tagore University, Bhopal, Madhya Pradesh, India

Sanjeet Kumar
Department of Mathematics Lakshmi Narain College of Technology & Science, Bhopal, Madhya Pradesh, India

Corresponding Author:
Bhawna Agrawal
Department of Mathematics, Rabindranath Tagore University, Bhopal, Madhya Pradesh, India

Fuzzy linear programming problem with α-cut and roubast ranking methods

Bhavana Shrivastava, Bhawna Agrawal and Sanjeet Kumar

Abstract
In this paper we are going to solve Fuzzy Linear Programming Problem with Triangular and Trapezoidal fuzzy numbers. Here we are using ranking method technique with α-Cut to get optimal solution for solving fuzzy linear programming problem with trapezoidal and triangular fuzzy numbers and also compare the existing methods. To illustrate the method numerical examples is solved and the obtained results are discussed.

Keywords: Fuzzy linear programming problem, ranking function, triangular fuzzy number, trapezoidal fuzzy number, roubast ranking method, α-optimal solution

Introduction

In today's highly economical market, the pressure on organizations to find better solutions to create and convey to customers becomes stronger. How and when to produce and send the products to the customers in the essential quantities, they also want in a cost-effective method, become more challenging. Linear programming problem provide a powerful modal to meet this challenge. They make sure the efficient production and movement timely availability of raw materials and finished goods. The first studies applying LP to diets were published between 1950 and 1960 [2]. This search for diet solutions started with Jerry Cornfield, who formulated "The Diet Problem" for the Army during World War II (1941–1945), in this search the provided modal of a low-cost diet that would meet the require nutrition of a soldier. The economist George Stigler, endeavoured optimization techniques to establish the cheapest diet delivering enough energy, proteins, vitamins, and minerals [1]. After this study many scholars work in this area like. Dantzib (1997) introduced linear programming, one of the most powerful tools in operations research.

Linear programming is an important tool in the arsenal of means at a decider's disposal. It is one of the most powerful tool of operational research to solve real world problems. Linear programming theoretical underpinning is now well established and as a result, a broader array of techniques including the simplex method [5] The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka *et al*. [9] in the framework of the fuzzy decision of Bellman and Zadeh [16] The first formulation of fuzzy linear programming (FLP) is proposed by Zimmermann [7]. Afterwards, many authors have considered various kinds of FLP problems and have proposed several approaches for solving these problems [1, 2, 3, 4, 5]. Fuzzy set theory has been applied to many fields such as control theory and management science, mathematical modelling and industrial applications. The concept of fuzzy linear programming (FLP) on general level was first proposed by Tanaka *et al*. [9] Afterwards, many authors considered various types of FLP problems and provided several methods for solving this problem. In particular, most convenient methods are based on the concept of comparison of fuzzy numbers by using linear ranking functions [8, 9, 11, 15]. Of course, linear ranking functions have been proposed by researchers to suit their requirements of the problem under consideration and conceivably there are no generally accepted criteria for application of ranking functions. Nevertheless, usually in such situations authors define a crisp model which is equivalent to an FLP problem and then use optimal solution of the model as the optimal solution of the FLP problem.

Mahdavi-Amiri and Nasseri [11] extended the concepts of duality in FNLP problems as a similar problem leading to the dual simplex algorithm [6] for solving such problems. Usually in such methods authors define a crisp model which is equivalent to the FLPP and then use optimal solution of the model as the optimal solution of the FLPP. In [9], by using a general linear ranking function we consider a fuzzy linear programming problem with trapezoidal numbers and solve by matrices, simplex with ranking and crisp method. Our main contribution here is the established of a new approach for solving the FLPP by using ranking function. Moreover, we illustrate our method with an example.

Preliminaries

In this section some basic definitions and arithmetic operations are reviewed.

Basic definitions

1. **Fuzzy set:** A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval [0, 1] i.e. $A = \{(x, \mu_A(x); x \in X\}$, Here $\mu_A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of xε X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2. **Triangular Fuzzy Numbers**
   A number $\tilde{A}$ is a triangular fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3)$. where a_1, a_2, a_3 are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ a < x_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\ a_2 \le x \le a_3 \\ 0 & for\ x > a_3 \end{cases}$$

By using min and max, we have an alternative expression for the proceeding equation:

triangle (x; a, b, c) = max $\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c},\right)\right)$

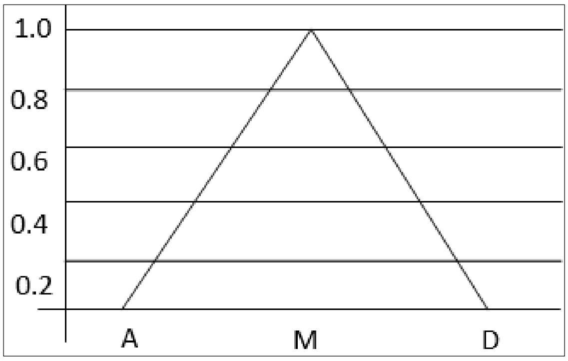

Fig 1: Graphical Representation of Triangular Fuzzy Number

3. **Trapezoidal fuzzy number**

A fuzzy number $\tilde{A}$ is a trapezoidal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$. where a_1, a_2, a_3, a_4 are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \dfrac{a_3 - x}{a_4 - a_3} & for\ a_3 \le x \le a_4 \\ 0 & for\ x > a_4 \end{cases}$$

An alternative concise expression using min and max is:

Trapezoidal (x; a, b, c, d) = Max$\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c},\right)\right)$

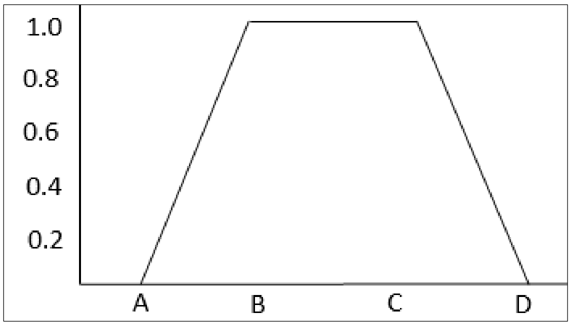

Fig 2: Graphical Representation of Trapezoidal Fuzzy Number

4. Operations of Triangular Fuzzy Numbers

The following are the four operations that can be performed on triangular fuzzy numbers: let $\widetilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then,

- **Addition:** $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- **Subtraction:** $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
- **Multiplication:** $\tilde{A} \times \tilde{B} = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$
- **Division:** $\frac{\tilde{A}}{\tilde{B}} = (\min\left(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}\right), \frac{a_2}{b_2}, \max(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}))$

5. Operations of Trapezoidal Fuzzy Numbers

The following are the four operations that can be performed on trapezoidal fuzzy numbers: let $\widetilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ then,

- **Addition:** $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
- **Subtraction:** $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
- **Multiplication:** $\tilde{A} \times \tilde{B} = (t_1, t_2, t_3, t_4)$

Where $t_1 = \min(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$
$t_2 = \min(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$
$t_3 = \max(a_2b_2, a_2b_3, a_3b_2, a_3b_3)$
$t_4 = \max(a_1b_1, a_1b_4, a_4b_1, a_4b_4)$

- **Division:** $\frac{\tilde{A}}{\tilde{B}} = (\min\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right), \min\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right),$

$\max\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right), \max\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right))$

Robust Ranking Technique

Roubast ranking technique which satisfy compensation, linearity, and additives properties and provides results which are consist human intuition. If ã is a fuzzy number then the Roubast Ranking is defined by R(ã) = $\int_0^1 (0.5)(a_\alpha^L a_\alpha^U) d\alpha$, where $(a_\alpha^L a_\alpha^U)$ is the α level cut of the fuzzy number ã.

In this paper we use this method for ranking the objective values. The Roubast ranking index R(ã) gives the representative value of fuzzy number ã.

Fuzzy Linear Programing Problem

$$\left.\begin{array}{ll} \text{Minimize or Maximize:} \quad Z = \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j & \\ \text{Subject to constraint:} & \\ \sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j \ (\leq, =, \geq) \ \tilde{b}_i & i = 1, 2, \ldots \ldots \ldots . m \\ \tilde{x}_j \ \geq \ 0 & j = 1, 2, \ldots \ldots \ldots . n \end{array}\right\} \quad (1)$$

Here,
$\tilde{c}_j$ = Per unit contribution of decision variable $\tilde{x}_j$
$\tilde{a}_{ij}$ = Input-output coefficient
$\tilde{b}_i$ = Total availability of the i^{th} resource

Numerical 1

$Max\ Z = (4,6,7,9)x + (3,5,7,10)y + (5,7,10,12)z$

$(3,4,6,9)x + (2,3,5,9)y + (5,7,9,13)z \leq (4,6,9,12)$

$(5,6,7,10)x + (7,9,10,12)y + (6,7,9,10)z \leq (7,9,10,13)$

$(6,7,10,13)x + (4,5,7,9)y + (5,7,12,15)z \leq (7,9,13,15)$

$x, > 0, y > 0 \text{ and } z > 0$

by Robust Ranking Function, we have

$$R(\bar{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U) d\alpha$$
$$R(4,6,7,9) = \int_0^1 (0.5)(2\alpha + 4, 9 - 2\alpha)\, d\alpha$$
$$R(4,6,7,9) = \int_0^1 (0.5)\,(2\alpha + 4 + 9 - 2\alpha) d\alpha = 6.5$$

Similarly

$R(4,6,7,9) = 6.5$ $R(3,5,7,10) = 6.25$ $R(5,7,10,12) = 8.5$

$R(3,4,6,9) = 5.75$ $R(2,3,5,9) = 4.75$ $R(5,7,9,13) = 8.5$ $R(4,6,9,12) = 7.75$

$R(5,6,7,10) = 7$ $R(7,9,10,12) = 9.5$ $R(6,7,9,10) = 8$ $R(7,9,10,13) = 9.75$

$R(6,7,10,13) = 9$ $R(4,5,7,9) = 6.25$ $R(5,7,12,15) = 9.75$ $R(7,9,13,15) = 11$

Now our numerical becomes after applying robust ranking function

$Max\ Z = 6.5x + 6.25y + 8.5z$

$5.75x + 4.75y + 8.5z \leq 7.75$

$7x + 9.5y + 8z \leq 9.75$

$9x + 7y + 10z \leq 11$

$x, > 0, y > 0 \text{ and } z > 0$

Method 1: After Applying Simplex Method by Tora Software we get following basic variables are: $x_1 = 0.67$, $x_2 = 0.28$, $x_3 = 0.31$ and basic feasible solution to get maximum profit is $Max\ Z = 8.74$.

Method 2: After Applying Row Echelon Methods we get the following values of basic variables are $x_1 = 0.67$, $x_2 = 0.28$, $x_3 = 0.31$ and basic feasible solution to get maximum profit is $Max\ Z = 8.74$.

Numerical 2

$Max\ Z = (5,10,15)x + (5,10,20)y + (5,15,20)z$

$(5,10,15)x + (5,10,20)y + (5,15,20)z \leq (5,10,15)$

$(5,10,20)x + (5,10,20)y + (10,15,20)z \leq (10,15,20)$

$(5,10,15)x + (10,15,25)y + (5,10,15)z \leq (10,20,30)$

$x, > 0, y > 0 \text{ and } z > 0$

By using α - Cut Ranking Technique

R($\tilde{a}$) = $\int_0^1 (0.5)(a_\alpha^L + a_\alpha^U) d\alpha$

$where\ (a_\alpha^L, a_\alpha^U) = \{(b - a)\alpha + a, c - (c - b)\alpha\}$

Then

$$R(\tilde{a}) = \int_0^1 (0.5)\{(b-a)\alpha + a, c - (c-b)\alpha\} d\alpha$$

$$R(5{,}10{,}15) = \int_0^1 (0.5)(5\alpha + 5{,}15 - 5\alpha) d\alpha$$

$$R(5{,}10{,}15) = \int_0^1 (0.5)(20) d\alpha$$

$$R(5{,}10{,}15) = \int_0^1 (10) d\alpha$$

$R(5{,}10{,}15) = 10$

Similarly

$R(5{,}10{,}20) = 11.25;\ R(5{,}15{,}20) = 13.75;\ R(10{,}15{,}20) = 15;$

$R(10{,}15{,}20) = 15\ R(10{,}15{,}25) = 16.25$

Now our numerical becomes after applying robust ranking function

$Max\ Z = 10x + 11.25y + 13.75z$

$10x + 11.25y + 13.75z \leq 10$

$11.25x + 11.25y + 16.25z \leq 16.25$

$10x + 16.25y + 10z \leq 20$

$x, > 0, y > 0\ and\ z > 0$

Method 1: After Applying Simplex Method by Tora Software we get following basic variables are: $x_1 = 0,\ x_2 = 0,\ x_3 = 0.73$ and basic feasible solution to get maximum profit is $Max\ Z = 10$

Method 2: After Applying Row Echelon Methods we get the following values of basic variables are $x_1 = 62.14,\ x_2 = -19.43,$ $x_3 = -28.57$ and basic feasible solution to get maximum profit is $Max\ Z = 10$

Conclusion

In this paper we have considered the optimal solution as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. More over the fuzzy linear programing problems of trapezoidal and triangular numbers has been transformed into crisp numbers by using alpha cut ranking technique. Numerical examples show that by this method we can have the optimal solution by simplex method as well as row echelon method. By using ranking technique, we have shown that the total profit obtained is optimal moreover one can conclude that the solution of fuzzy linear programming problem can be obtained by this ranking method effectively. This technique can also be used in solving other types of problems like project schedules, assignment problems and network flow problems.

References

1. Ebrahimnejad A, Nasseri SH. Using complementary slackness property to solve linear programming with fuzzy parameters, Fuzzy Information and Engineering. 2009;3:233-245.
2. Ebrahimnejad A, Nasseri SH, Hosseinzadeh Lotfi FM. Soltanifar, A primal–dual method for linear programming problems with fuzzy variables, European Journal of Industrial Engineering. 2010;4(2):189-209.
3. Ebrahimnejad A, Nasseri SH, Hosseinzadeh Lotfi F. Bounded linear programs with trapezoidal fuzzy numbers, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems. 2010;18(3):269-286.
4. Ebrahimnejad A, Nasseri SH. A dual simplex method for bounded linear programmes with fuzzy numbers, International Journal of Mathematics in Operational Research. 2010;2(6):762-779.
5. Ebrahimnejad A, Nasseri SH, Mansourzadeh SM. Bounded primal simplex algorithm for bounded linear programming with fuzzy cost coefficients, International Journal of Operations Research and Information Systems. 2011;2(1):100-124.
6. Dantzig GB, Linear Programming and Extensions, Princeton University Press. 1963.
7. Zimmermann HJ. Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Systems. 1978;1:45-55.

8. Tanaka H, Okuda T, Asai K. On fuzzy mathematical programming, The Journal of Cybernetics. 1974;3:37-46.
9. Campos L, Verdegay JL. Linear programming problems and ranking of fuzzy numbers, Fuzzy Sets and Systems. 1989;32:1-11.
10. Mahdavi-Amiri N, Nasseri SH. Duality results and a dual simplex method for linear programming problems with trapezoidal fuzzy variables. Fuzzy Sets and Systems. 2007;158:1961-1978.
11. Mahdavi-Amiri N, Nasseri SH, Yazdani A. Fuzzy primal simplex algorithms for solving fuzzy linear programming problems, Iranian Journal of Operations Research. 2009;1:68-84.
12. Fortemps P, Roubens M. Ranking and defuzzification methods based on area compensation, Fuzzy Sets and Systems. 1996;82:319-330.
13. Bellman RE, Zadeh LA. Decision making in a fuzzy environment, Management Science. 1970;17:141-164.
14. Yager RR. A procedure for ordering fuzzy subsets of the unit I.