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Pathway fractional integral operator associated by Struve function and generalized Struve function

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Abstract
The aim of this paper is to establish two theorems using pathway fractional integral operator via Struve function and generalized Struve function. Our results are quite general in nature. Some special cases are also obtained here. We also point out their Riemann-Liouville fractional integral operator results in special cases. Our results will help to extend some classical statistical distributions to wider classes of distributions, these are useful in practical applications.

Keywords: Pathway fractional integral operator/Struve function/generalized Struve function mathematics subject classification: 33C60, 26A33

Introduction

Let f(x) ∈ L(a,b), $\alpha \in C, R(\alpha) > 0$, then left side Riemann-Liouville fractional integral operator is defined as [7].

$$(I_{0+}^{\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt \qquad (1.1)$$

Where, R$(\alpha) > 0$
Let f(x) ∈ L(a,b) $\eta \in C$, R $(\eta) > 0$, a > 0,and a "Pathway parameter " $\alpha < 1$. Then the pathway fractional integral operator is defined by [14], also see [6]

$$\left(P_{0+}^{(\eta,\alpha)} f\right) = x^{\eta} \int_0^{\frac{x}{a(1-\alpha)}} \left[1-\frac{a(1-\alpha)t}{x}\right]^{\eta/1-a} f(t)dt \qquad (1.2)$$

When $\alpha = 0$, a = 1 and η is replaced by $\eta - 1$ in (1.2) it yields

$$\left(I_{0+}^{\eta} f\right)(x) = \frac{1}{\Gamma(\eta)} \int_0^x (x-t)^{\eta-1} f(t)dt \;\ldots. \qquad (1.3)$$

Which is the left-sided Riemann-Liouville fractional integral defined in (1.1)
Fractional integration operators play an important role in the solution of several problems of diversified fields of science and engineering. Many fractional integral operators like Riemann-Liouville, Weyl, Kober, Erdely-Kober and Saigo operators are studied by various workers due to their applications in the solution of integral equations arising in several problem of many areas of physical, engineering and Technological sciences. A detailed description of these operators can be found in the survey paper by Srivastava and Saxena [3].
In this paper, we consider a function defined as follows
The Struve function of order p is given by

$$H_p(z) = \left(\frac{z}{2}\right)^{p+1} \sum_{k=0}^{x} \frac{(-1)^k}{\Gamma\left(k+\frac{3}{2}\right)\Gamma\left(k+p+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^{2k} \qquad (1.4)$$

The Struve function and its more generalization are found in many papers [1, 2, 4, 8, 5, 9, 10, 11, 13].
The generalized Struve function given by [7].

$$H_l^{\lambda}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma\left(\lambda k+l+\frac{3}{2}\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{z}{2}\right)^{2k+l+1}, \lambda > 0 \tag{1.5}$$

And by [4]

$$H_l^{\lambda,\alpha}(z) = \sum_{k=0}^{n} \frac{(-1)^k}{\Gamma\left(\lambda k+l+\frac{3}{2}\right)\Gamma\left(\alpha k+\frac{3}{2}\right)}\left(\frac{z}{2}\right)^{2k+l+1}, \lambda > 0, \alpha > 0 \tag{1.6}$$

Another generalized form studied by [8] as follows:

$$H_{l,\xi}^{\lambda}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma\left(\lambda k+\frac{1}{\xi}+\frac{3}{2}\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{z}{2}\right)^{2k+l+1} \xi > 0, \lambda > 0 \tag{1.7}$$

Where $(\lambda)_n$ is the Pochhammer symbol defined for $(\lambda \in \mathbb{C})$ by Srivastava and Choi (see [12])

$$(\lambda)_n = \begin{cases} 1 & (n=0) \\ \lambda(\lambda+1)\ldots(\lambda+n-1) & (n \in N) \end{cases} \lambda \neq 0$$

The beta function is defined by

$$B(n,m) = \int_0^1 x^{n-1}(1-x)^{m-1}dx \tag{1.8}$$

Main Results

Theorem 1: Let $\eta, \rho_i, \lambda, \gamma_i \in C, R\left(1+\frac{\eta}{1-\alpha}\right) > 0$, min {Re (ρ_i), Re (λ), Re (γ_i), Re (η)} > 0 and $\alpha < 1$.

Then for the pathway fractional operator $P_{0+}^{(\eta,\alpha)}$ defined by (1.2) the following formula holds:

$$\left(P_{0+}^{(\eta,\alpha)} t^{\rho-1} H_p(t)\right)(x) = \frac{x^{\eta+1} r\left(1+\frac{\eta}{1-\alpha}\right)}{a(1-\alpha)} H_{p+1}\left(\frac{x}{a(1-\alpha)}\right) \cdot {}_1\Psi_1$$

$$\left[\left(\frac{x}{a(1-\alpha)} \middle| \begin{matrix} (p+2,2) \\ \left(p+\frac{\eta}{1-\alpha}+3,2\right) \end{matrix}\right)\right] \tag{2.1}$$

Proof: Making use of (1.2) and (1.4) in LHS of the theorem 1 and then interchanging the order of integration and summation, we evaluate the inner integral by making use of beta function(1.8) to arrive at the desired result in (2.1).

Theorem 2: Let $\eta, \rho_i, \beta, \gamma_i, \delta_i \in C, R\left(1+\frac{\eta}{1-\alpha}\right) > 0$, min {Re$(\rho_i)$, Re$(\beta)$, Re$(\gamma_i)$, Re$(\delta_i)$, Re$(\eta)$} > 0 and $\rho_i, \delta_i > 0$, $\alpha < 1$. Then for the pathway fractional operator $P_{0+}^{(\eta,\alpha)}$ defined by (1.2) then the following formula holds:

$$P_{0+}^{(\eta,\alpha)}\left[t^{\rho-1} H_{l,\xi}^{\lambda}(t)\right](x) = \frac{x^{\eta+1}/\left(1+\frac{\eta}{1-\alpha}\right)}{a(1-\alpha)} H_{l,\xi}^{\lambda}\left(\frac{x}{a(1-\alpha)}\right) \cdot {}_1\Psi_1\left[\left(\frac{x}{a(1-\alpha)} \middle| \begin{matrix} (l+2,2) \\ \left(l+\frac{\eta}{1-\alpha}+3,2\right) \end{matrix}\right)\right] \tag{2.2}$$

Proof: Making use of (1.2) and (1.7) in LHS of the theorem 2 and then interchanging the order of integration and summation, we evaluate the inner integral by making use of beta function(1.8) to arrive at the desired result in (2.2).

Special Cases

1. If we take $\alpha = 0$, a =1 and η is replaced by $\eta - 1$ in (2.1) then Pathway fractional integral operator will reduce Riemann-Liouville fractional integral defined in (1.1). Then we get following result

$$\left(I_{0+}^{a} t^{p-1} H_p(t)\right)(x) = x^{\eta}\Gamma(\eta) H_{p+1}(x).\ {}_1\Psi_1\left[\left(x \middle| \begin{matrix} (p+2,2) \\ (p+\eta+3,2) \end{matrix}\right)\right] \tag{3.1}$$

2. If we take $\alpha = 0$, a =1 and η is replaced by $\eta - 1$ in (2.2). Then Pathway fractional integral operator will reduce Riemann-Liouville fractional integral defined in (1.1). Then we get following result

$$\left(I_{0+}^{a} t^{p-1} H_{l,\xi}^{\lambda}(t)\right)(x) = x^{\eta}\Gamma(\eta) H_{l,\xi}^{\lambda}(x)\ .{}_1\Psi_1\left[\left(x \middle| \begin{matrix} (l+2,2) \\ (l+\eta+3,2) \end{matrix}\right)\right]. \tag{3.2}$$

Conclusion

In this paper, we have presented Struve function and generalized struve *via* pathway fractional integral operator. As in this operator parameter α establishes a path of going from one distribution to another and to different classes of distributions. We conclude that our results will help to extend some classical statistical distributions to wider classes of distributions, useful in practical applications.

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