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Dynamical analysis of a juvenile-adult model

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Abstract

Because of the competition among populations in the world, cannibalism is practiced by a wide variety of populations. In most populations where cannibals part resources with their immolation, dependent of a size trait is observed. Using this perspective, a juvenile-adult model is developed under the assumption that adults prey juveniles and adults are bigger in size than juveniles. The model incorporates a Holling type I function that proper the intra-specific predation mount of the crowd at that the juvenile is used up, each adult, as a subordinate of the juvenile density making the model applicable to most population structures as compared to those in literature. Furthermore, the presented sufficient conditions in which the system reaches an equilibrium show the exibility of the model. In particular, we discussed the stability conditions of equilibriums.

Keywords: Global dynamics, model of juvenile-adult, cannibalism

1. Introduction

Cannibalism or intra-specific predation is an extensive event amid a variety of taxa ^[1, 2, 3]. Cannibalism is a behavioral trait found in several kinds, for instance, fish, bug, protozoa, amphibians, birds, mammals, and more ^[1, 2]. Although most references to this behavior are anecdotal or based on casual laboratory observations. Many cannibals are the outcome of genetic and environmental effects ^[1, 2]. Many kinds' face food shortages either threats to their survival because of whether or environmental changes ^[4]. Therefore, the phenomenon of adult intra-specific predation on juvenile does not turn to infrequent ^[5]. Mathematical modelling and the pertaining dynamic study have declared that it has significant gradually changing and environmental concepts ^[5, 6, 7] for one kind. As intra-specific predation is contain in inter-proper haunt-haunter models, the intra-specific predation can be one of two obvious illustrated with level-formed prey or predator populations ^[7, 8, 9], or unobvious engaged in prey or hunter populations outside level of form ^[10, 11]. Intra-specific predation was long considered and deviant's behavior. Mature of frame of witness now includes that cannibalism is both typical and significant in ecology of many species. Fox ^[1] discovered that cannibalism is a usual action in 147 kinds and Polis ^[2] reported cannibalism in almost 1300 sorts. Cannibalism can have strong impacts on the dynamics and life histories of crowd and as a result, it may be considered as an extraordinary element in the ecology species. Intra-specific predation plays significant environmental and gradually change function in crowd. It answer as a limitation on crowd measurement, favors the improvement of replace existence date tactics, entangle society dynamics, forms act of community, beget the progressive of relative choose, decrease genital achievement and possible conduct to complicate nonlinear population dynamics containing chaos ^[2, 3]. Some singles are biologically inclined to show this behavior and geographically different populations possible to show different cannibalism rates. In the process of times of low resource availability, cannibalism of juveniles by adults can redirect reproductive energy compared to times of higher resource access ^[3] and cannibalism of juveniles possible function as a "lifeboat" system when resources are low and adults and juveniles are challenge for the uniform or even various resources ^[12, 13].

The study of cannibalism includes gradually change and environmental trends. The recently have largely concentrated on two perspective of cannibalism on populations:

(1) Results on change in population figure in the process of time and (2) if cannibalism can role as a "life raft" system in accordance with cannibalistic populations can outlive stage of

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male deficiency that would differently outdate noncannibalistic populations [12, 14]. As a regard to the before, asked population dynamics of cannibals contain a different of regimes for instance equilibrium, serial cycles, intermittent cycles and chaotic dynamics [15, 16]. Cannibalism has been applied to confirm population dynamics [7], nevertheless this forecast is based on the hypothesis that cannibals do not achievement any authority from cannibalizing.

The size to that cannibalism impacts the dynamics of a specified population is such as rest on the co-occurrence of another causes for instance struggle from noncannibalistic relating to or interspecies struggle and haunt, due to several cannibals portion resources with their sufferers, the after that possible contest with cannibals for this parted resource [2, 11]. Polis recommended that this contest should be a significant factor in the growth of cannibalism in accordance with cannibals profit two from meal on sacrifice and dropping struggle for parted resources. Cannibals mostly have maximum and minimum victim amounts, which rest on their own body size [2] and the impacts of cannibalism [17]. Mathematical pattern offer us beneficial vision into the prospective result of intra-specific predation demeanor on population dynamics. The dynamic models for intra-specific predation type contain time of discrete variance equation model, progress-time ordinary differential equation (ODE) pattern and first-order hyperbolic partial differential equation (PDE) pattern, etc. Their pattern displays that intra-specific predation treatment can control number of population, improve the survival of the kinds, engender disability, etc.

The occurrence that matures prey on juveniles of the same kinds has been generally accepted in world [3], has been the subject of a broad mathematical written action and various kind of pattern approaches have been projected [19].

These studies are important for good reasons. The part of cannibalism in population dynamics is vague. Particularly, several attempts have been dedicated in these history to found the function of cannibalism in becoming stable or undermining population periods and the impact is not still totally admitted. The encasement discussed here are more influential: such as, Dieckmann *et al.* [15] discovered that population fluctuation possible upheld by egg cannibalism, Hastings [16], Tribolium, Desharnais and Liu discovered in a Tribolium model that cannibalism is destabilizing [18]. Cushing first showed two a constant and a destabilizing consequence of cannibalism in a separate pattern and then, for number- anatomy pattern, he decided that population fluctuations are cause principally to size base more than cannibalism [20].

As a result, it is hard to get general implications and a need of deeper research of cannibalism in several biologic order arises. Also these researches uncover very attractive mathematical complications. Between the others, it show our focus to improve a juvenile-mature pattern hardly supposing that adults haunt on juveniles. The original objective of my study is to offer the universal research of model (2.1).

This study contains of six parts. In part. 2, it improve a juvenile-adult model to portray the dynamics of a cannibalistic types. In part. 3, it do a primary study of the model, that contains nondimensionalization, focal stability of disappearance equilibrium, and the factor. In parts. 4 and 5, this do the universal study. In part. 6, we summarize our results.

2. Model Formulation: As powerful tools, mathematical models play important roles in accounting for positive and negative feedbacks inherent in cannibalism. Literature search shows that most of dynamical models for cannibalistic populations are in other-time difference equation models or age/size structured first order hyperbolic equation models. In my study, we improve a juvenile-adult model by hardly supposing that adults haunt on juveniles, based on the fact that cannibals are almost most of the time greater than their sacrifices. Let $x(t)$ and $y(t)$ indicate the figure or density of juveniles and adults, separately at time t . The mathematical model describing population reproduction, growth, and mortality is specified by

$$\begin{cases} \frac{dx}{dt} = by - gx - mx - \min\{cx, M\}y, \\ \frac{dy}{dt} = gx - \mu y - ky^2 + e \min\{cx, M\}y. \end{cases} \tag{2.1}$$

with a positive initial condition, it specifies the initial density of adolescents and matures. The model parameters b, g, m, k, c, M, μ, e are all affirmative coefficients. b is the breeding rate; g is the puberty rate at that juveniles develop into adolescents; m and μ are mortality rates of juveniles and adults, individually; k is the contest coefficient; The term $\min\{cx, M\} := p(x)$ (where c and M are affirmative stables) shows the Holling kind I function reply that determines the rate of cannibalism for the population in that adolescents are use up, per mature, as a function of the juvenile density. The function $p(x)$ indicates that the assumption rate is a linearly incrementing function accordance to the juvenile density when the density of juveniles is lower than the threshold value $\frac{M}{c}$ and that the using up rate is a stable M once the juvenile density go beyond the threshold value $\frac{M}{c}$. The term $ep(x)y$ means that cannibalism decreases adult mortality by given that a further source of energy, where $0 < e \leq 1$ is the cause of change.

3. Elementary model of study

To make things easier the challenge and facilitate of study, we nondimensionalize system (2.1) by introducing the below aspect less quantities:

$$\tilde{t} = bt, \tilde{x} = \frac{c}{b}x, \tilde{y} = \frac{c}{b}y, \tilde{m} = \frac{m}{b}, \tilde{g} = \frac{g}{b}, \tilde{k} = \frac{k}{c}, \tilde{\mu} = \frac{\mu}{b}, \tilde{M} = \frac{M}{b}$$

We drop the tildes for notational easily, and our system reduces to

$$\begin{cases} \frac{dx}{dt} = y - (m + g)x - \min\{x, M\}y, \\ \frac{dy}{dt} = gx - \mu y - ky^2 + e \min\{x, M\}y. \end{cases} \tag{3.1}$$

Evidently, system (3.1) has a unique boundary equilibrium $E_0(0,0)$, at which the population extirpates. The linearized matrix of system (3.1) at E_0 is and its eigenvalues are.

$$J(E_0) = \begin{pmatrix} -(m + g) & 1 \\ g & -\mu \end{pmatrix},$$

$$\lambda_{1,2} = \frac{1}{2} \left[-m - g - \mu \pm \sqrt{(m + \mu + g)^2 - 4((g + m)\mu - g)} \right].$$

Thus, if $\mu > \frac{g}{m+g}$, then $\lambda_1 < \lambda_2 < 0$, it means that E_0 is a stable node; if $\mu < \frac{g}{m+g}$, after that $\lambda_1 < 0 < \lambda_2$, it means that E_0 is an unstable saddle and the unstable path is $(1, \lambda_2 + m + g)$; if $\mu = \frac{g}{m+g}$, then $\lambda_1 < \lambda_2 = 0$, the stability of E_0 relates on high discipline terms, will be debated future

After to the unique aspect parameters, the situation $\mu < \frac{g}{m+g}$ is equivalent to $b > \mu(\frac{g+m}{g})$, which suggests that for fixed population mortality rates and puberty rate, the population perseveres if the reproductive rate is high enough.

Let

$$f_1(x, y) = y - (m + g)x - \min\{x, M\}y = \begin{cases} (1 - x)y - (m + g)x, & 0 \leq x \leq M; \\ (1 - M)y - (m + g)x, & x > M, \end{cases} \tag{3.2}$$

And

$$f_2(x, y) = gx - \mu y - ky^2 + e \min\{x, M\}y = \begin{cases} gx - \mu y - ky^2 + exy, & 0 \leq x \leq M; \\ gx - \mu y - ky^2 + eMy, & x > M. \end{cases} \tag{3.3}$$

If $M \geq 1$ then $\frac{dx}{dt} = f_1(x, y) < 0$ for $x \geq 1$ and $y \geq 0$, that is, any answer $(x(t), y(t))$ with $x(0) \geq 1, y(0) \geq 0$ will cross through the line $x = 1$ from right to left and ultimately enter into the domain $(x, y): 0 \leq x \leq 1, y \geq 0$, that is affirmatively stable for the flow totally by system (3.1).

Hereafter this study system (3.1) in the below 2 cases: $M \geq 1$ and $0 < M < 1$.

4. The case $M \geq 1$

In this case system (3.1) becomes

$$\begin{cases} \frac{dx}{dt} = f_1(x, y), \\ \frac{dy}{dt} = f_2(x, y), \end{cases} \tag{4.1}$$

Where

$$f_1(x, y) = y - (m + g)x - xy,$$

$$f_2(x, y) = gx - \mu y - ky^2 + exy.$$

According to define the survival equilibria of system (4.1), this calculate the nullclines. A survival equilibrium exists if and just if the x -nullcline and the y -nullcline interchange in the inner of the first quadrant on the xy -plane. By setting $f_1(x, y) = 0$, it find the x -nullcline in the first quadrant:

$$y - (m + g)x - xy = 0$$

$$x = \frac{y}{m + g + y} := \phi_1(y), y \geq 0$$

For $y \geq 0$, the function $\phi_1(y)$ has the below feature:

$$\phi_1(0) = 0, \lim_{y \rightarrow +\infty} \phi_1(y) = 1, \phi_1'(y) > 0.$$

Hence feature tell us that $\phi_1(y)$ is a monotonically rising function with horizontal asymptote $x = 1$. The same, the y -nullcline in the first quadrant may this cover by setting $f_2(x, y) = 0$, shown by

$$gx - \mu y - ky^2 + exy = 0 \Rightarrow x = \frac{y(ky + \mu)}{g + ey} := \phi_2(y), y \geq 0.$$

When $y \geq 0$, the function $\phi_2(y)$ satisfies

$$\phi_2(0) = 0, \lim_{y \rightarrow +\infty} \phi_2(y) = +\infty, \phi_2'(y) > 0,$$

Which suggests that $\phi_2(y)$ is a monotonically incrementing function with tangent asymptote $= \frac{k}{e}(y - g + \frac{\mu}{k})$.

4.1 The case $\mu > e$

It first display that in this case the explanations of system (4.1) with positive primary data are at all times positive and finally bounded.

Theory 4.1. All explanations of system (4.1) with primary situations $x(0) \geq 0$ and $y(0) \geq 0$ will approach, enter and remain in the rectangular area,

$$\Omega_0 = \left\{ (x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq \frac{g}{\mu - e} \right\}.$$

Proof. Since $\frac{dx}{dt}\bigg|_{x=0} = y > 0, \frac{dx}{dt}\bigg|_{x=1} = y < 0$, for $0 \leq x \leq 1$ and $\frac{dy}{dt}\bigg|_{y=0} = gx > 0$.

If $y > \frac{g}{\mu - e}$, then $\frac{dy}{dt} = gx - \mu y - ky^2 + exy \leq g - \mu y + ey = g - (\mu - e)y < 0$, so, $0 \leq y \leq \frac{g}{\mu - e}$.

Thus, we will just study solutions of system (4.1) with primary circumstances $x(0), y(0) \in \Omega_0$. Note that $\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} = -(m + g) - y - \mu - 2ky + ex < 0$ in Ω_0 , since $\mu > e$, there is no closed orbit lying and staying in Ω_0 by Bendixson-Dulec criteria. Therefore combining theorem (4.1), it have the bellowing consequence.

Theory 4.2. The structure (4.1) has no any closed orbit.

Currently it have to find the interior equilibria of the system (4.1), for the reason it have to set:

$$f_1(x, y) = f_2(x, y) = 0$$

so that implies, $\phi_1(y) = \phi_2(y)$ we will define thus the y -coordinate of an inner equilibrium of system (4.1) is an affirmative solution of the equation $\phi(y) = 0, y \in (0, +\infty)$, that is possible to write it again as

$$\begin{aligned} \phi(y) &= \phi_1(y) - \phi_2(y) = \frac{y}{m + g + y} - \frac{y(ky + \mu)}{g + ey} \\ \phi(y) &= \frac{y}{m + g + y} - \frac{y(ky + \mu)}{g + ey} \\ \phi(y) &= \frac{-yh(y)}{(g + ey)(m + g + y)}, y \in [0, +\infty), \end{aligned} \tag{4.2}$$

Where

$$h(y) = ky^2 + d_1y + d_2, d_1 = k(g + m) + \mu - e, d_2 = (g + m)\mu - g. \tag{4.3}$$

It have

$$\phi_1(y) = \frac{y}{m + g + y} := x, y \geq 0$$

$$\phi_2(y) = \frac{y(ky + \mu)}{g + ey} := x, y \geq 0$$

It have found x and it is part of the resolution, we have to fined y by solving the quadratic equation this will get

$$\phi_1(y) = \phi_2(y)$$

$$\phi_1(y) - \phi_2(y) = 0 \Rightarrow \phi(y) = 0$$

$$\phi(y) = \frac{-y[ky^2 + (km + kg + \mu - e)y + (m + g)\mu - g]}{(g + ey)(m + g + y)}$$

We can see that the denominator is positive for every $y \geq 0$. Now we need to find the positive solution of this equation:

$$ky^2 + (km + kg + \mu - e)y + (m + g)\mu - g = 0, y \geq 0$$

We discuss the existence or nonexistence of the affirmative answers of the above equation:

$$h(y) = 0, y \geq 0,$$

Hence, we find

$\lim_{y \rightarrow +\infty} h(y) \rightarrow +\infty > 0, k > 0$ so, $h(y)$ is a quadratic function the above equation. If $h(0) = d_2 < 0$ (i. e., $\mu < \frac{g}{m+g}$), there is only one $y_1 \in [0, +\infty)$ so that $h(y_1) = 0$, hence, there exists a unique interior equilibrium point $E_1(x_1, y_1)$ with $x_1 = \phi_1(y_1) = \phi_2(y_1)$.

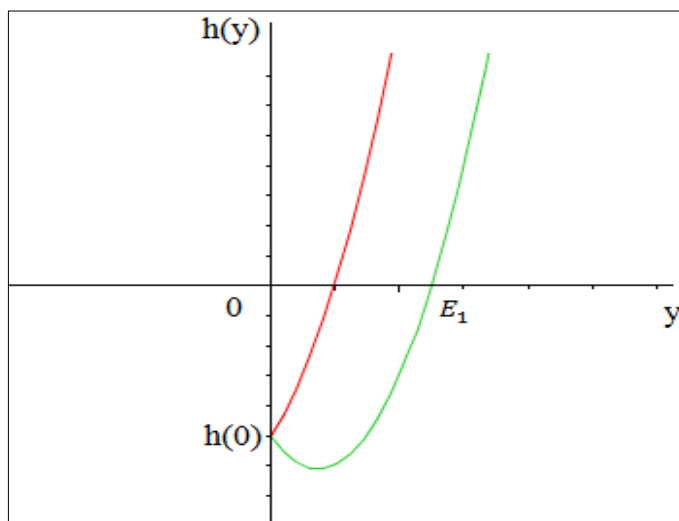


Fig 4.1: When $h(0) < 0$.

If $h(0) = d_2 > 0$ (i. e., $\mu > \frac{g}{m+g}$), we have two cases:

If $d_1 > 0$, axis of symmetry, $y = -\frac{d_2}{2k} < 0$ so if $y > 0$, then $h(y) < 0$, that suggests the system has no interior equilibrium point.

If $d_1 < 0, y = -\frac{d_2}{2k} > 0$,

If $\Delta > 0$, then exist $0 < y_1 < y_2 < +\infty$, as that $h(y_1) = h(y_2) = 0$, has two interior equilibrium $E_1(x_1, y_1)$ and $E_2(x_2, y_2)$.

If $\Delta < 0$, then not interior equilibrium point. If $\Delta = 0$, only exist uniqueness interior equilibrium point $E_1(x_1, y_1)$.

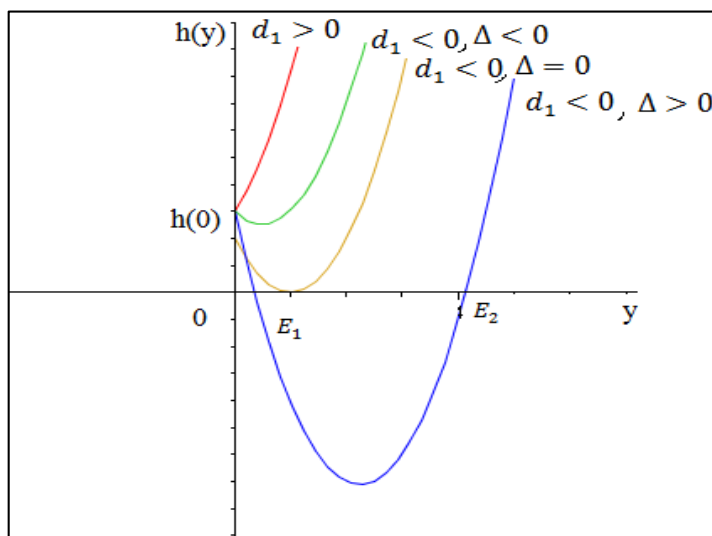


Fig 4.2: When $h(0) > 0$.

Finally, we remark that if $h(x) = d_2 = 0$ (i. e., $\mu = \frac{g}{m+g}$), we have also two cases:

- I. If $d_1 > 0$, axis of symmetry, $y = -\frac{d_2}{2k} < 0$, so if $y > 0$, then $h(y) > 0$, that suggests the system has no interior equilibrium point.
- II. If $d_1 < 0$, $y = -\frac{d_2}{2k} > 0$, there exists a unique interior equilibrium point $E_1(x_1, y_1)$.

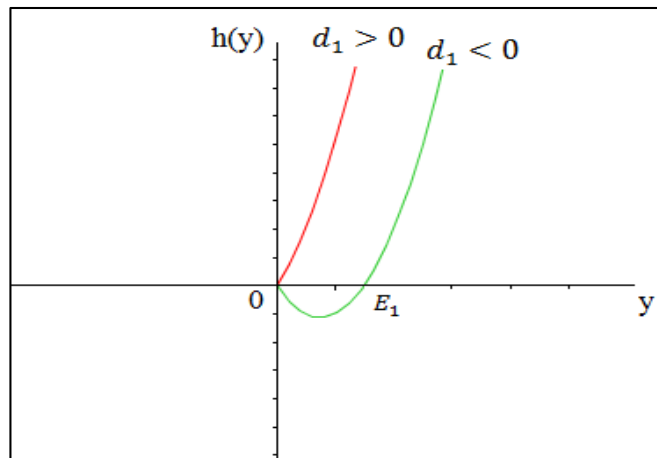


Fig 4.3: When $h(0) = 0$.

Brief the top of argument, it arrive at the below consequence.

Theory 4.3

- 1. whether $\mu \geq \frac{g}{m+g}$ and $k(m + g) + \mu > e$, or $\mu > \frac{g}{m+g}$ and $\Delta < 0$, then system (4.1) only has a stable trivial equilibrium E_0 .
- 2. If $\mu < \frac{g}{m+g}$, or $\mu = \frac{g}{m+g}$ and $k(m + g) + \mu < e$, thus system (4.1) has an unstable trivial equilibrium E_0 and a unique positive stable node E_1 .
- 3. If $\mu > \frac{g}{m+g}$, $k(m + g) + \mu < e$, and $\Delta > 0$, thus system (4.1) has a stable trivial equilibrium E_0 and two affirmative equilibrium E_1 and E_2 with E_1 being unstable saddle and E_2 being stable node.
- 4. If $\mu > \frac{g}{m+g}$, $k(m + g) + \mu < e$ and $\Delta = 0$, thus system (4.1) has a stable trivial equilibrium E_0 , and a unique affirmative saddle-node $E_{1,2}$.

Proof. According to Theorem 4.2, we will explain and prove the stable solution we got, the linearized matrix of system (4.1) at $E_1(x_1, y_1)$ is

$$J(E_1) = \begin{pmatrix} \frac{\partial f_1}{\partial x}(x_1, y_1) & \frac{\partial f_1}{\partial y}(x_1, y_1) \\ \frac{\partial f_2}{\partial x}(x_1, y_1) & \frac{\partial f_2}{\partial y}(x_1, y_1) \end{pmatrix}.$$

The trace of $J(E_1)$ is,

$$f_1(x, y) = y - (m + g)x - xy,$$

$$f_2(x, y) = gx - \mu y - ky^2 + exy,$$

$$TrJ(E_1) = \frac{\partial f_1}{\partial x}(x_1, y_1) + \frac{\partial f_2}{\partial y}(x_1, y_1) = -(m + g) - y_1 - \mu - 2ky_1 + ex_1,$$

meanwhile, the determinant of $J(E_1)$ is

$$DetJ(E_1) = \frac{\partial f_1}{\partial x} \cdot \frac{\partial f_2}{\partial y} - \frac{\partial f_1}{\partial y} \cdot \frac{\partial f_2}{\partial x}$$

$$= \frac{\partial f_1}{\partial x} \cdot \frac{\partial f_2}{\partial x} \left[\phi_1'(y) - \phi_2'(y) \right]$$

Here we omit the variable (x_1, y_1) .
Since

$$f_1(x, y) = y - (m + g)x - xy,$$

$$f_2(x, y) = gx - \mu y - ky^2 + exy,$$

$$TrJ(E_i) = -(m + g) - y_i - \mu - 2ky_i + ex_i = \frac{y_i}{x_i} - \frac{gx_i}{y_i} - ky_i < 0,$$

x_i, y_i are solution to the equations,

$$\begin{cases} y_i - (m + g)x_i - x_i y_i = 0, \\ gx_i - \mu y_i - ky_i^2 + ex_i y_i = 0, \end{cases}$$

$$\dot{\phi}(y) = \dot{\phi}_1(y) - \dot{\phi}_2(y) = - \frac{\left[h(y) + y h'(y) \right] (g + m + y)(g + ey) - yh(y)[g + ey + e(m + g + y)]}{(g + m + y)^2 (g + ey)^2}$$

$$\phi'(y) = \phi'_1(y) = - \frac{y_i h'(y_1) (m + g + y_i)(g + ey_i)}{(m + g + y)^2 (g + ey)^2}$$

So $DetJ(E_i) = y_i h'(y_1)$

We will discuss the stability of interior equilibrium points,

1. If $\mu < \frac{g}{m+g}$, then $h'(y_1) > 0$ and $detJ_{E_1} > 0$, so E_1 is stable node.
2. If $\mu > \frac{g}{m+g}$, we have two cases:
 - i. If $k(m + g) + \mu > e$, no positive equilibria.
 - ii. If $k(m + g) + \mu < e$, then we discussion about Δ ,

If $\Delta > 0$ has two interior equilibria $E_i(x_i, y_i), x_1 < x_2$, and $h'(y_1) < 0, h'(y_2) > 0$, then E_1 is an unstable saddle, E_2 is a stable node.

If $\Delta < 0$, no positive equilibria.

If $\Delta = 0$, then $E_1 = E_2$ is a saddle-node.

If $\mu = \frac{g}{m+g}$, we have two cases:

If $k(m + g) + \mu > e$, no positive equilibria.

If $k(m + g) + \mu < e, h'(y_1) > 0$, then E_1 is a stable node.

5. The case $0 < M < 1$

It revision system (3.1) as

$$\begin{cases} \frac{dx}{dt} = f_1(x, y), \\ \frac{dy}{dt} = f_2(x, y), \end{cases} \tag{5.1}$$

where $f_1(x, y)$ and $f_2(x, y)$ are showed by (3.2) and (3.3), in order . Solving the equation $f_1(x, y) = 0$,

$$y = \begin{cases} \frac{(m+g)x}{1-x}, 0 \leq x \leq M; \\ \frac{(m+g)x}{1-M}, x > M; \end{cases} \tag{5.2}$$

solving the equation $f_2(x, y) = 0$,

$$x = \begin{cases} \frac{ky^2 + \mu y}{g + ey}, 0 \leq x \leq M; \\ \frac{\mu y + ky^2 - eMy}{g}, x > M; \end{cases} \tag{5.3}$$

so that,

$$\begin{cases} y = \frac{(m+g)x}{1-x}, 0 \leq x \leq M; \\ ky^2 + (km + kg + \mu - e)y + (m+g)\mu - g = 0; \end{cases}$$

and

$$\begin{cases} y = \frac{(m+g)x}{1-M}, x > M; \\ k(g+m)y + (\mu - eM)(m+g) - (1-M)g = 0; \end{cases}$$

Then we have

$$(em + eg - g)x^2 + (km + kg - \mu - e)(m + g)x + 2gx + \mu(m + g) - g = 0, 0 \leq x \leq M$$

and

$$k(m + g)^2x + (\mu - eM)(m + g)(1 - M) - (1 - M)^2g = 0, x > M;$$

Where

$$h^-(x) = (em + eg - g)x^2 + a_1x + a_2,$$

$$a_1 = (km + kg - \mu - e)(m + g) + 2g, a_2 = \mu(m + g) - g; \tag{5.4}$$

$$h^+(x) = k(m + g)^2x + (\mu - eM)(m + g)(1 - M) - (1 - M)^2g = 0.$$

We need analyze the intersects of the $h^-(x)$, $h^+(x)$ and the x -axis, $h^-(0) = \mu(m + g) - g$;

$$h^-(M) = (em + eg - g)M^2 + a_1M + a_2;$$

$$\Delta = a^2 - 4(em + eg - g)a_2 \text{ Let} \tag{5.5}$$

$$h(x) = \begin{cases} h^-(x), 0 \leq x \leq M; \\ h^+(x), x > M; \end{cases}$$

Thus the x -coordinate of an interior equilibrium of system (4.1) is a affirmative explanation of the equation $h(x) = 0$.

It is easy to see that $h^-(M) = h^+(M)$ for the reason that the function $h(x)$ is progressive for $x \geq 0$. Also if the discriminant,

If $\Delta > 0$ thus the quadratic equation $h^-(x)$ has two real roots:

$$\alpha_1 := \frac{-a_1 - \sqrt{\Delta}}{2(em + eg - g)}, \alpha_2 := \frac{-a_1 + \sqrt{\Delta}}{2(em + eg - g)} \tag{5.6}$$

If $\Delta = 0$, then $\alpha_1 = \alpha_2 = -\frac{a_1}{2(em + eg - g)}$, if $\Delta < 0$, no real root.

In terms of the sign of $h^-(0)$, it attend the bellowing three cases.

Case (i) $h^-(0) = a_2 > 0$, that is, $\mu > \frac{g}{m+g}$ and if $em + eg - g < 0$. In this case $\Delta > 0$ and $\alpha_1 < 0$, thus the figure of equilibria of system (4.1) related on the magnitude of α_2 and M .

(ia) If $0 < M < \alpha_2$, then $h(M) > 0$, so $h(x) > 0$ for all $x \geq 0$, it denote that system (4.1) has no affirmative equilibria and the extirpation equilibrium E_0 is GAS hence E_0 is stable when $\mu > \frac{g}{m+g}$.

(ib) If $M > \alpha_2$, then $h^-(M) = h^+(M) < 0$, since there exist $x_1 \in (0, M)$ and $x_2 \in (M, +\infty)$ such that $h^-(M) = h^+(M) = 0$, which suggests that system (4.1) has two affirmative equilibria $E_1(x_1, y_1)$ and $E_2(x_2, y_2)$. Therefore, E_1 is unstable saddle and E_2 is stable node.

(ic) If $M = \alpha_2$, then $h(M) = 0$, then the saddle E_1 and the node E_2 stilly approach together in the case $M > \alpha_2$ and collide together, become a saddle-node.

$h^-(0) = a_2 > 0$, that is, $\mu > \frac{g}{m+g}$ if $em + eg - g > 0$, the discriminant Δ perhaps affirmative, or negative, even zero.

If $\Delta > 0$, we have two cases:

(i) If $a_1 > 0$, no affirmative equilibrium.

(ii) If $a_1 > 0$, then $0 < \alpha_1 < \alpha_2$,

If $M < \alpha_1$, no affirmative equilibrium.

If $\alpha_1 < M < \alpha_2$, then exist $x_1 \in (0, M)$ and $x_2 \in (M, +\infty)$ like that $h^-(x_1) = h^+(x_2) = 0$, so, the system has two positive equilibrium point $E_1(x_1, y_1)$ and $E_2(x_2, y_2)$. Therefore E_1 is unstable saddle and E_2 is stable node.

If $M > \alpha_2$, $h^-(M) > 0$, exist $0 < x_1 < x_2 < M$, so that $h^-(x_1) = h^+(x_2) = 0$ the system has two positive equilibrium point $E_i(x_i, y_i)$, ($i = 1, 2$). Therefore E_1 is unstable saddle and E_2 is stable node.

If $\Delta = 0$, also we have two cases: (i) If $a_1 > 0$, no affirmative equilibrium.

(ii) whether $a_1 < 0$, then $0 < \alpha_1 = \alpha_2$,

If $M < \alpha_1$, no positive equilibrium.

If $M > \alpha_1$, only one positive equilibrium. Such that it is unstable saddle.

If $\Delta < 0$, we do not have any equilibrium

Case (ii) $h^-(0) = a_2 < 0$, it is, $\mu < \frac{g}{m+g}$ and if $em + eg - g < 0$, then $a_1 > 0$, the discriminant Δ it is possible affirmative, or negative, even zero.

(iia) in the beginning we consider the case $\Delta > 0$, $0 < \alpha_1 < \alpha_2$. Then the number of equilibria of system (4.1) related on the magnitude of α_1 , α_2 and M . If $0 < M < \alpha_1$, then $h(M) < 0$ and there exist $x_2 \in (M, +\infty)$ such that $h^-(x_1) = 0$ then the system has a unique equilibrium point. Thus it is stable node.

Also if $\alpha_1 < M < \alpha_2$ then $h(M) > 0$ the system has a unique equilibrium.

If $M > \alpha_2$ then $h(M) < 0$, then there exist $0 < x_1 < x_2 < M$ and $x_2 \in (M, +\infty)$, $h^-(x_1) = h^-(x_2) = h^+(x_3) = 0$, which suggests that system (4.1) has three positive equilibria $E_i(x_i, y_i)$, ($i = 1, 2, 3$), E_1 and E_3 are stable nodes, but E_2 is an unstable saddle. If $M = \alpha_1$, then $h(M) = 0$, system (4.5) has a unique positive equilibrium which is GAS.

If $M = \alpha_2$, then $h^-(M) = 0$, thus there exists $x_1 \in (0, M)$ and $x_2 = M$ like that $h^-(x_1) = h^-(x_2) = 0$, which show that system (4.5) have two affirmative equilibria $E_1(x_1, y_1)$ and $E_2(x_2, y_2)$. (iib) We at that moment suppose that the discriminant $\Delta < 0$. In this case there exists a unique $x_1 \in (M, +\infty)$ like that $h^+(x_1)$, which denote that system (4.1) has a unique affirmative equilibrium $E_1(x_1, y_1)$. Therefore E_1 is unstable saddle.

(iic) Currently we consider the case: If $\Delta = 0$ and $0 < M < \frac{-a_1}{2(em+eg-g)}$, system (4.5) has a unique positive equilibria $E_1(x_1, y_1)$.

If $\Delta = 0$ and $M > \frac{-a_1}{2(em+eg-g)}$, the equilibria E_1 and E_2 in the case $M > \alpha_2$ collide together, become an equilibrium $E_{1,2}$, this is a saddle-node and E_3 is also a stable node.

Finally, if $\Delta = 0$ and $M = \frac{-a_1}{2(em+eg-g)}$, the equilibria $E_{1,2}$ and E_3 in the case $\Delta = 0$ and $M > \frac{-a_1}{2(em+eg-g)}$ continue to collide together.

If $h^-(0) = a_2 < 0$, $\mu < \frac{g}{m+g}$ and if $em + eg - g > 0$, we have two cases:

(i) If $h^-(M) < 0$, then exists a unique $x_1 \in (M, +\infty)$, so that $h^+(x_1) = 0$, only one positive equilibrium $E_1(x_1, y_1)$. Therefore E_1 is stable node.

(ii) If $h^-(M) > 0$, then exists a unique $x_1 \in (0, M)$, so that $h^-(x_1) = 0$, only one positive equilibrium $E_1(x_1, y_1)$. Therefore, E_1 is unstable saddle.

Case (iii) $h^-(0) = a_2 = 0$, that is, $\mu = \frac{g}{m+g}$. In this case the quadratic equation $h^-(x) = 0$ has two real roots

$$\alpha_1 = 0, \alpha_2 = \frac{-a_1}{em+eg-g}.$$

(i) If $em + eg - g < 0$, then $a_1 > 0$ and $\alpha_2 > 0$,

If $0 < M < \alpha_2$ then $h^-(M) > 0$, no positive equilibrium.

If $M > \alpha_2$, then $h^-(M) < 0$, exists $x_1 \in (0, M)$ and $x_2 \in (M, +\infty)$, such that $h^-(x_1) = h^+(x_2) = 0$, the system has two positive equilibrium point $E_1(x_1, y_1)$ and $E_2(x_2, y_2)$. So E_1 is stable node and E_2 is unstable saddle.

(ii) If $em + eg - g > 0$,

If $a_1 < 0$, then $\alpha_2 > 0$, if $0 < M < \alpha_2$, then $h^-(M) < 0$, exists $x_1 \in (M, +\infty)$, such that $h^+(x_1) = 0$, only one positive equilibrium point $E_1(x_1, y_1)$. Therefore E_1 is stable node.

If $M > \alpha_2$, exists unique $x_1 \in (0, M)$, such that $h^-(x_1) = 0$, only one positive equilibrium point $E_1(x_1, y_1)$. Therefore E_1 is unstable saddle.

If $a_1 > 0$, then $\alpha_2 < 0$, no positive equilibrium.

6. Conclusion

In various cannibalistic populations, cannibals portion resources with its sacrifices, resulting in a size-dependent mixing of cannibalistic and contest exchanges, and is an extended behavioral trait in many species. Most of dynamical models for cannibalistic populations are separate-time difference equation models or age/size structured first order hyperbolic equation models.

In this study, we developed a model for juvenile-adult cannibalism under the assumption that the adults prey juveniles. Different from what has been presented in literature. We also considered that most juveniles are smaller in size as compared to adults. Hence the model combines both juvenile-adult and size structured models. Using the basic model by ^[21], model (2.1) presented in this paper incorporates Holling type I function that proper the cannibalism rate of the population at that the juvenile is used up, per adult, as a function of the juvenile density. In particular, the sample is designed to describe the following vitals, reproduction, development and mortality of the population. Under the assumption that reproduction is greater than 1, we derived conditions under which the system has a unique equilibrium point. The presented pattern can be applied to many species as the restricting conditions are relaxed as compared to those in literature.

7. References

1. Fox LR. Cannibalism in natural populations. *Annual review of ecology and systematics*. 1975;6(1):87-106.
2. Polis GA. The evolution and dynamics of intraspecific predation. *Annual Review of Ecology and Systematics*. 1981;12(1):225-251.
3. Elgar MA, Crespi BJ. Cannibalism: ecology and evolution among diverse taxa, 1992, No. 591.53 E53.
4. Li J, Zhu X, Lin X, Li J. Impact of cannibalism on dynamics of a structured predator-prey system. *Applied Mathematical Modelling*. 2020;78:1-19.
5. Veprauskas A, Cushing JM. A juvenile-adult population model: climate change, cannibalism, reproductive synchrony, and strong Allee effects. *Journal of biological dynamics*. 2017;11(sup1):1-24.
6. Cushing JM. A simple model of cannibalism. *Mathematical biosciences* 1991;107(1):47-71.
7. Van Den Bosch F, Gabriel W. Cannibalism in an age-structured predator-prey system. *Bulletin of Mathematical Biology*. 1997;59(3):551-567.
8. Buonomo B, Lacitignola D, Rionero S. Effect of prey growth and predator cannibalism rate on the stability of a structured population model. *Nonlinear Analysis: Real World Applications*. 2010;11(2):1170-1181.
9. Rault J, Benoît E, Gouzé JL. Stabilizing effect of cannibalism in a two stages population model. *Acta biotheoretica*. 2013;61(1):119-139.
10. Biswas S, Samanta S, Chattopadhyay J. Cannibalistic predator-prey model with disease in predator a delay model. *International Journal of Bifurcation and Chaos*. 2015;25(10):1550130.
11. Anholt BR. Cannibalism and early instar survival in a larval damselfly. *Oecologia*. 1994;99(1-2):60-65.
12. Van den Bosch F, De Roos AM, Gabriel W. Cannibalism as a life boat mechanism. *Journal of Mathematical Biology*. 1988;26(6):619-633.
13. Cushing JM. Some delay models for juvenile vs. adult competition. In *Differential Equations Models in Biology, Epidemiology and Ecology*, 1991, 177-188.
14. Orr BK, Murdoch WW, Bence JR. Population regulation, convergence, and cannibalism in *Notonecta* (Hemiptera). *Ecology*. 1990;71(1):68-82.
15. Diekmann O, Nisbet RM, Gurney WSC, Van den Bosch F. Simple mathematical models for cannibalism: a critique and a new approach. *Mathematical Biosciences*. 1986;78(1):21-46.
16. Hastings A. Cycles in cannibalistic egg-larval interactions. *Journal of Mathematical Biology*. 1987;24(6):651-666.
17. Claessen D, De Roos AM, Persson L. Population dynamic theory of size-dependent cannibalism. *Proceedings of the Royal Society of London. Series B: Biological Sciences*. 2004;271:333-340.
18. Desharnais RA, Liu L. Stable demographic limit cycles in laboratory populations of *Tribolium castaneum*. *The Journal of Animal Ecology*, 1987, 885-906.
19. Cushing JM. A juvenile-adult model with periodic vital rates. *Journal of mathematical biology*. 2006;53(4):520-539.
20. Cushing JM. A size-structured model for cannibalism. *Theoretical Population Biology*. 1992;42(3):347-361.
21. Cushing JM. An introduction to structured population dynamics, 1998, 71.