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A class of ratio-cum-product estimators of population mean under extreme ranked set sampling (ERSS) and simple random sampling (SRS)

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Abstract
Extreme Ranked Set Sampling (ERSS) is a survey technique which seeks to improve the likelihood that collected sample data provides a good representation of the population and minimizes costs associated with obtaining them. The main goal of a statistical survey is to reduce sampling errors either by devising suitable sampling scheme or by formulating efficient estimator of the population parameters. In an attempt to address the problem of weak or loss of efficiency usually suffered in estimation of population mean under Simple Random Sampling (SRS), a class of ratio-cum-product estimators for population mean of the study variable Y is proposed based on ERSS using information on a single accompanying variable. Members of the proposed class of estimators were obtained by assigning various values to the scalars that helps in designing the estimators. These members were then transformed to a form that can be easily expanded using Taylor's series approximation, from where various properties such as biases, relative biases, Mean Square Errors (MSEs), and Optimal Mean Square Errors (OMSEs) were derived under large sample approximation. Empirical study was conducted using three natural population data sets in order to investigate the performances and efficiency of the proposed classes of estimators under ERSS over its corresponding counterpart's estimator based on SRS and some existing ratio and product estimators. This empirical study was followed up with a computer simulation study using R-software. The results revealed that the advocated class of estimators in ERSS produced smaller biases and MSEs which is an indicator of appreciable gain in efficiency and superiority over its corresponding counterpart estimator and some existing ratio type estimators in sample survey for all cases considered in this work and are therefore adjudged to provide a better alternative whenever efficiency is required.

Keywords: Estimators, extreme ranked set sampling, mean square errors, simple random sampling, simulation

Introduction

Under sampling techniques, the estimation of population variables is of utmost interest. This estimation most times seek to make use of better methods of estimation that would give an improved result. More often, interest is on the mean of a definite feature of a finite population on the ground of the portion taken from the population following a specific sampling scheme. This is so because the mean has a wider use in sampling and statistical analysis. Many sampling techniques depend on the possession of advance information about an auxiliary variate. Researches which adopt supplementary information in statistical survey (sampling) are broad and traceable to the pioneer work of Bowley (1926) [27] who carried out the groundwork of contemporary sampling theory involving stratified random sampling and Neyman (1934, 1938) [37, 38]. Nevertheless, application of supplementary knowledge in estimation technique to enhance the performances of estimators was introduced by Watson (1937) [51] and Cochran (1940, 1942) [28, 29].

McIntyre (1952) [35] initiated and put in the procedure of ranked-set-sampling (RSS) in approximating the average pasture output as a better/reliable approaches and inexpensive scheme than the procedure of Simple Random Sampling (SRS). This technique is functional in circumstances where units of interest are very simple and economical to order than to observe in relation to a variable under consideration.

Takahasi and Wakimoto (1968) [50] separately set out the procedure of RSS and revealed an impressive mathematical reasoning, which is in tandem with McIntyre's instinctive postulation. Dell and Clutter (1972) [30] proved that deviations in ordering lowers the accuracy of the RSS average comparative to SRS average. Nevertheless, RSS average is always superior over the SRS average till ordering is too substandard as to produce a probabilistic sample when its performances are akin to that of SRS average.
The techniques of Extreme-Ranked Set Sampling (ERSS) as first introduced by Samawi *et al.* (1996) [41] to estimate the population mean and showed that the mean based on ERSS though unbiased but is more efficient that the sample me due to SRS. Furthermore, Samawi (1996) [41] introduced the principle of Stratified Ranked Set Sampling (SRSS); to improve the precision of estimating the population means in case of SSRS.
Ali and Iqbal (2021) [3] proposed an efficient generalized family of estimators to estimate finite population mean of study variable under Ranked Set Sampling utilizing information on an auxiliary variable and concluded that when correlation between the study and auxiliary variables increases, the proposed generalized family of estimators proved to be efficient estimator of population mean of the study variable.
Further researches on RSS method include but not limited to Al-Omari el tal.(2009) [2], Al-Omari (2019) [1], Haq and Shabbir (2010) [31], Kaur *et al*. (1995) [33] Al Saleh and Al-Kadiri (2000) [18], Al-Saleh and Al-Omari (2002) [22], Abu Dayyeh *et al*. (2002) [4], Al-Saleh and-Zheng (2002) [25], Al-Saleh and Samawi (2000) [24], Ozturk and Wolfe (2000) [39], Ozturk (2002) [40], Al-Saleh and Ababneh (2015) [20], Zheng and Al-Saleh (2002) [25], Al-Saleh and Darabseh (2017)) [23]. The expeditious development in the area of RSS over the past twenty (20) years provided a boost for the uprising of other key connected methods to inferential statistics. In this paper, a class of ratio-cum-product estimators of population mean of the study variable is proposed by employing ERSS method following information on a single accompanying variable. The expressions of the biases and Mean Square Errors (MSEs) of the proposed estimators were calculated. Analytical and simulation study of performances and efficiencies of the estimators over the usual SRS method using their (MSEs) were carried out in an attempt to support the theoretical results with numerical illustration. Application to real life data was successfully done to illustrate the method, from where conclusion was drawn following the results obtained from the work.

Sampling methods

Here, we present the sampling scheme which is employed in the course of this work i.e Ranked Set Sampling (RSS), (ERSS), as well as the frequently used (SRS).

RSS Description

Step 1: Select m random samples each of size m bivariate units from the population under consideration.

Step 2: Rank the units within each set-in relation to the variable of interest by eyeball approach or any cost-free method.

Step 3: From the first set of m units, the smallest ranked unit X is selected together with the corresponding Y, and from the second set of m units the second smallest ranked unit X is selected together with the corresponding Y. The process is continued until from the m^{th} set of m units the largest ranked unit X is selected with the associated Y. This process can be repeated r times to increase the sample size to rm RSS bivariate units.
In this work we assume that the ranking is done on the variable X for estimating the population mean of the study variable Y. Nevertheless, the entire process can be carried out again while the ranking can be done on the variable Y.

ERSS Description

Let $(X_{i(1)}, Y_{i[1]})$, $(X_{i(2)}, Y_{i[2]})$, … $(X_{i(m)}, Y_{i[m]})$ be the order Statistics of X_{i1}, X_{i2}, X_{i3}, … X_{im} and the judgment order of Y_{i1}, Y_{i2}, Y_{i3}, … Y_{im}, $(i = 1, 2, \dots m)$. Then the RSS units are: $(X_{1(1)}, Y_{1[1]})$, $(X_{2(2)}, Y_{2[2]})$, … $(X_{m(m)}, Y_{m[m]})$, here, $(\cdot)$ and $[\cdot]$ implies that the ordering of X is faultless or without error and the ordering of Y has error, be m independent random samplesof size m and assume that each member $(X_{i(j)}, Y_{i[j]})$ in the sample has the same bivariate distribution-function $F(x, y)$ with mean μ_X, μ_Y variance σ_X, σ_Y, and ρ_{XY}.
The ERSS method, as suggested by Samawi *et al*. (1996) [41], can be described as given below:

a) Select m random samples, each of size m units, from an infinite population and order the units within each sample with respect to a variable under consideration by impressionistic method or any other cost-free procedure. For exact quantification, if the sample size m is even, from the first $\frac{m}{2}$ sets, select the smallest ordered units and from the other $\frac{m}{2}$ sets select the largest ranked unit. Such a sample shall be represented by ERSSe.
b) If the sample size m is odd, then there are two options:
(i) From the first $\frac{m-1}{2}$ sets we choose the average of the observation of the smallest units in the $\frac{m-1}{2}$ sets, and from the other $\frac{m-1}{2}$ sets, we take the mean of the measures of the largest ranked unit. Such a sample shall be represented by $ERSS_{0(a)}$.
(ii) From the remaining measure of the m^{th} unit we take the median. Such a sample will be represented by $ERSS_{0(m)}$.

e: is even
$0(a)$: is odd average
$0(m)$: is odd median

The procedure can be continued r times, if need be, to get a sample of size rm units. The choices of (a) and $b(ii)$ is usually less difficult in application than the choice of $b(i)$. In this work, we considered the choices of (a) and $b(ii)$, (i.e the even case and the case of taking the median from the m^{th} sample if m is odd).

If m is even, then the observed ERSSe units are:

$(X_{1(1)}, Y_{1[1]})$, $(X_{2(1)}, Y_{2[1]})$, … $(X_{\frac{m}{2}(1)}, Y_{\frac{m}{2}[1]})$, $(X_{\frac{m+2}{2}(m)}, Y_{\frac{m+2}{2}[m]})$, $(X_{\frac{m+4}{2}(m)}, Y_{\frac{m+4}{2}[m]})$, …,$(X_{m(m)}, Y_{m[m]})$, then under ERSSe the sample means and variances of the study and accompanying variables are defined as:

$$\left.\begin{aligned} E(\bar{X}^{ERSSe}) &= \frac{1}{m}\left(\mu_{x(1)} + \mu_{x(m)}\right) \\ E(\bar{Y}^{ERSSe}) &= \frac{1}{m}\left(\mu_{y[1]} + \mu_{y[m]}\right) \end{aligned}\right\} \quad (1)$$

With variances

$$\left.\begin{aligned} Var(\bar{X}^{ERSSe}) &= \frac{1}{2m}(\sigma^2_{X(1)} + \sigma^2_{X(m)}) \\ Var(\bar{Y}^{ERSSe}) &= \frac{1}{2m}(\sigma^2_{Y[1]} + \sigma^2_{Y[m]}) \end{aligned}\right\} \quad (2)$$

If m odd, then measured $ERSS_{0(m)}$ units are:

$(X_{1(1)}, Y_{1[1]})$, $(X_{2(1)}, Y_{2[1]})$, … $(X_{\frac{m-1}{2}(1)}, Y_{\frac{m-1}{2}[1]})$, $(X_{\frac{m+1}{2}\left(\frac{m+1}{2}\right)}, Y_{\frac{m+1}{2}\left[\frac{m+1}{2}\right]})$, $(X_{\frac{m+3}{2}(m)}, Y_{\frac{m+3}{2}[m]})$ …, $(X_{m(m)}, Y_{m[m]})$, then under $ERSS_{0(m)}$ the sample means and variances of the study and accompanying variables are defined as:

$$\left.\begin{aligned} E\left(\bar{X}^{ERSS_{0(m)}}\right) &= \frac{m-1}{2m}\left(\mu_{x(1)}, + \mu_{x(m)}\right) + \frac{1}{m}\mu_{x\left(\frac{m+1}{2}\right)} \\ E\left(\bar{Y}^{ERSS_{0(m)}}\right) &= \frac{m-1}{2m}\left(\mu_{y[1]}, + \mu_{y[m]}\right) + \frac{1}{m}\mu_{y\left[\frac{m+1}{2}\right]} \end{aligned}\right\} \quad (3)$$

With variances

$$\left.\begin{aligned} Var(\bar{X}^{ERSS_{0(m)}}) &= \frac{(m-1)}{2m^2}\left(\sigma^2_{X(1)} + \sigma^2_{X(m)}\right) + \frac{1}{m^2}\sigma^2_{x\left(\frac{m+1}{2}\right)} \\ Var(\bar{Y}^{ERSS_{0(m)}}) &= \frac{(m-1)}{2m^2}\left(\sigma^2_{Y[1]} + \sigma^2_{Y[m]}\right) + \frac{1}{m^2}\sigma^2_{y\left[\frac{m+1}{2}\right]} \end{aligned}\right\} \quad (4)$$

SRS Description

In SRS, m unit out of M units of a population are chosen in such a way that every individual unit has equiprobable chance of being selected. According to our description,

(X_{11}, Y_{11}), (X_{21}, Y_{21}), …, (X_{m1}, Y_{m1}) is the SRS.

Definition 1

Let Y be the study variable and X be the accompanying variable which is correlated with Y. Let again $(y_1, y_2, \dots, y_m)$ and $(x_1, x_2, \dots, x_m)$ be m sample values, then under Simple Random Sampling without Replacement (SRSWOR) the sample means and variances of the study and accompanying variables are given as:

$$\left.\begin{aligned} \bar{X}^{SRS} &= \frac{1}{m}\left(\sum_{i=1}^{m} X_i\right) \\ \bar{Y}^{SRS} &= \frac{1}{m}\left(\sum_{i=1}^{m} Y_i\right) \end{aligned}\right\} \quad (5)$$

$$\left.\begin{aligned} Var(\bar{X}^{SRS}) &= \left(\frac{\sigma_x^2}{m}\right) \\ Var(\bar{Y}^{SRS}) &= \left(\frac{\sigma_y^2}{m}\right) \end{aligned}\right\} \quad (6)$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}, \sigma_{XY} = Cov(\bar{Y}^{SRS}, \bar{X}^{SRS}) = \rho_{XY}\sigma_X\sigma_Y \quad (7)$$

if the finite population correction $f \to 0$

Table 1: Some existing ratio estimators with their MSE

S. No	Estimators	MSE
1.	$\bar{y}$, Sample Mean	$\left(\frac{1-f}{m}\right)\bar{Y}^2 C_y^2$
2.	$\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$, Sukhatme (1974) [48]	$\frac{\bar{Y}^2}{m}[C_y^2 + C_x^2 - 2\rho C_x C_y]$
3.	$\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$, Sukhatme (1974) [48]	$\frac{\bar{Y}^2}{m}[C_y^2 + C_x^2 + 2\rho C_x C_y]$
4.	$\bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha}$, Srivastava (1970) [46]	$\left(\frac{1}{m}-\frac{1}{M}\right)\bar{Y}^2\{C_y^2 + \alpha C_x^2(\alpha - 2\frac{\rho C_y}{C_x})\}$
5.	$\bar{y}\exp[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}]$, Bahl and Tuteja (1991) [26]	$\frac{\bar{Y}^2}{m}[C_y^2 + \frac{C_x^2}{4} - \rho C_x C_y]$
6	$\bar{y}\exp[\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}]$, Bahl and Tuteja (1991) [26]	$\frac{\bar{Y}^2}{m}[C_y^2 + \frac{C_x^2}{4} + \rho C_x C_y]$
7	$\bar{y}[\alpha(\frac{\bar{x}_2}{\bar{X}}) + (1-\alpha)(\frac{\bar{X}}{\bar{x}_1})]$, Singh & Choudhury (2012) [42]	$\bar{Y}^2\left(\frac{1}{m}-\frac{1}{M}\right)C_y^2(1-\rho^2)$
8.	$\mu_{y-A}^{SRS1} = \bar{y}_{SRS}\cdot\frac{(\mu_x+q_1)}{(\bar{x}_{SRS}+q_1)}, \bar{y}_{SRS}\cdot\frac{(\mu_x+q_3)}{(\bar{x}_{SRS}+q_3)}$ Al-Omari *et al.* (2009) [2]	$\frac{1}{m}\left(\frac{\mu_y}{\mu_x+q_j}\right)\left[\left(\frac{\mu_y}{\mu_x+q_j}\right)\sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y\rho\right]$; j=1,3
9	$\mu_{y-ST}^{SRS} = \bar{y}_{SRS}\cdot\frac{(\mu_x+\rho)}{(\bar{x}_{SRS}+\rho)}$ Singh and Tailor (2003) [45]	$\left(\frac{1-f}{m}\right)\mu_y^2\left[C_y^2 + \left(\frac{\mu_x}{\mu_x+\rho}\right)C_x^2\left(\frac{\mu_x}{\mu_x+\rho} - 2\frac{\rho C_y}{C_x}\right)\right]$
10	$\hat{\mu}_{y-KC}^{SRS} = \bar{y}_{SRS} + b\cdot\frac{(\mu_x - \bar{X}^{SRS})}{(\bar{X}_{SRS}+\rho)}(\mu_x+\rho)$ Kadilar and Cingi (2004) [32]	$\left(\frac{1-f}{m}\right)[R^2\sigma_x^2 + \sigma_y^2(1-\rho^2)]$
11	$\hat{\mu}_{y-SE}^{SRS} = \bar{y}_{SRS}\left(w\frac{\bar{X}^{SRS}}{\mu_x} + (1-w)\frac{\mu_x}{\bar{X}^{SRS}}\right)$ Singh and Espejo (2003) [44]	$\left(\frac{1-f}{m}\right)\mu_y^2\left\{C_y^2 + C_x^2(1-2w)[1-2w+2\frac{\rho C_y}{C_x}\right\}$

Notations and some useful equations

Let Y be the study variable and X be the auxiliary variable which is correlated with Y. Then following notations and expressions shall be useful in the course of this work. For all $i = 1, 2, \ldots, m$.

$$\left.\begin{aligned} \mu_x &= E(X_i) \\ \sigma_x^2 &= var(X_i) \\ \sigma_x^2 &= var(X_i) \\ \mu_{x1} &= E\big(X_{i(1)}\big) \\ \mu_{x\left(\frac{m+1}{2}\right)} &= E\left(X_{i\left(\frac{m+1}{2}\right)}\right) \\ \sigma_{x1}^2 &= var\big(X_{i(1)}\big) \\ \sigma^2_{x\left(\frac{m+1}{2}\right)} &= var(X_{i\left(\frac{m+1}{2}\right)}) \\ \sigma_{xm}^2 &= var\big(X_{i(m)}\big) \\ \sigma_{x(1,m)} &= \mathrm{cov}\big(X_{m(1)}, X_{m(m)}\big) \end{aligned}\right\} \quad (8)$$

$$\left.\begin{aligned} \mu_y &= E(Y_i) \\ \sigma_y^2 &= var(Y_i) \\ \sigma_x^2 &= var(X_i) \\ \mu_{y[1]} &= E(Y_{i[1]}) \\ \mu_{y\left(\frac{m+1}{2}\right)} &= E\left(Y_{i\left[\frac{m+1}{2}\right]}\right) \\ \sigma_{y1}^2 &= var\left(Y_{i\left[\frac{m+1}{2}\right]}\right) \\ \sigma^2_{y\left(\frac{m+1}{2}\right)} &= var\left(Y_{i\left[\frac{m+1}{2}\right]}\right) \\ \sigma_{ym}^2 &= var\big(Y_{i[m]}\big) \\ \sigma_{y(1,m)} &= \mathrm{cov}\big(Y_{m[m]}, Y_{m[m]}\big) \end{aligned}\right\} \quad (9)$$

Then proposed class of estimator based on ERSS

Motivated by Singh and Espejo (2003) [44] ratio-cum-product estimator under SRS, we advocated a class of ratio-cum-product estimators of the population mean $\bar{Y}$ of the study variable Y with a single accompanying variable using SRS and ERSS schemes as follows:

$$T_{1Ej} = \bar{y}^{ERSS_e}\left[w\left(\frac{a\bar{X}^{ERSS_e}+\rho}{a\bar{x}^{ERSS_e}+\rho}\right)^{\alpha_1} + (1-w)\left(\frac{a\bar{x}^{*ERSS_e}+\rho}{a\bar{X}^{RSS_e}+\rho}\right)^{\alpha_2}\right] \quad (10)$$

$$T_{1(o(m))j} = \bar{y}^{ERSS_{0(m)}}\left[w\left(\frac{a\bar{X}^{ERSS_{0(m)}}+\rho}{a\bar{x}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_1} + (1-w)\left(\frac{a\bar{x}^{*ERSS_{0(m)}}+\rho}{a\bar{X}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_2}\right] \quad (11)$$

Where $(a \neq 0, \rho \neq 0)$ are real numbers and may take the values of parameters associated with either the study variable y or the concomitant variable x or both(x, y); in this case, the coefficient of variation and the correlation coefficient respectively. $(\alpha_1, \alpha_2,)$ are scalars or real constants which helps in designing the estimator and can be determined suitably. We may fix α_1, α_2, and w will be selected in an optimum manner by minimizing the (MSEs) of T_{1Ei}**,** $i = 1$ to m with respect to w.

Where $\bar{x}^{*ERSS_e} = \{(1+g)\bar{X}^{ERSS_e} - g\bar{x}^{ERSS_e}\}$ is unbiased estimator of population mean $\bar{X}^{ERSS_e}$, $g = \frac{m}{(M-m)} = \frac{f}{(1-f)}$ and $f = \frac{m}{M}$

$|(1 - g\lambda_i e_i)| < 1, or$ $\left|\left(1 - g\lambda_i\left(\frac{\bar{x}^{ERSS_e} - \bar{X}^{ERSS_e}}{\bar{X}^{ERSS_e}}\right)\right)\right| < 1$ for all the ${}^{M}C_m$ Samples

$i = 0, 1, 2.$ Where, $\lambda_i = \frac{\bar{X}^{ERSS_e}}{\bar{X}^{ERSS_e}+\rho}$, $e_y = \frac{\bar{y}^{ERSS_e} - \bar{Y}^{ERSS_e}}{\bar{Y}^{ERSS_e}}$, $e_x = \frac{\bar{x}^{ERSS_e} - \bar{X}^{ERSS_e}}{\bar{X}^{ERSS_e}}$

The proposed class of estimator based on SRS

$$T_{1Sj} = \bar{y}^{SRS}\left[w\left(\frac{a\bar{X}^{SRS}+\rho}{a\bar{x}^{SRS}+\rho}\right)^{\alpha_1} + (1-w)\left(\frac{a\bar{x}^{*SRS}+\rho}{a\bar{X}^{SRS}+\rho}\right)^{\alpha_2}\right] \quad (12)$$

Where, $\bar{x}^{*SRS} = \{(1+g)\bar{X}^{SRS} - g\bar{x}^{SRS}\}$ is unbiased estimator of population mean $\bar{X}^{SRS}$, $g = \frac{m}{(M-m)} = \frac{f}{(1-f)}$ and $f = \frac{m}{M}$, $|(1 - g\lambda_i e_i)| < 1,$

or $\left|\left(1 - g\lambda_i\left(\frac{\bar{x}^{SRS} - \bar{X}^{SRS}}{\bar{X}^{SRS}}\right)\right)\right| < 1$ $i = 0, 1, 2.$ for all the ${}^{M}C_m$ samples.

Table 2: Some members of the class of estimator T_{1Ej}

S/N	Estimator	w	a	ρ	α_1	α_2
1	$T_{1E1} = \bar{y}^{ERSS_e}$,	1	1	0	0	0
2	$T_{1E2} = \bar{y}^{ERSS_e}\left(\frac{\bar{X}^{ERSS_e}}{\bar{x}^{ERSS_e}}\right)$	1	1	0	1	0
3	$T_{1E3} = \bar{y}^{ERSS_e}\left(\frac{\bar{x}^{*ERSS_e}}{\bar{X}^{ERSS_e}}\right)$	0	1	0	1	1
4	$T_{1E4} = \bar{y}^{ERSS_e}\left(\frac{a\bar{X}^{ERSS_e}+\rho}{a\bar{x}^{ERSS_e}+\rho}\right)$	1	a	ρ	1	0
5	$T_{1E5} = \bar{y}^{ERSS_e}\left(\frac{a\bar{x}^{*ERSS_e}+\rho}{a\bar{X}^{ERSS_e}+\rho}\right)$	0	a	ρ	0	1
6	$T_{1E6} = \bar{y}^{ERSS_e}\left(\frac{\bar{X}^{ERSS_e}+\rho}{\bar{x}^{ERSS_e}+\rho}\right)$	1	1	ρ	1	0
7	$T_{1E7} = \bar{y}^{ERSS_e}\left(\frac{\bar{x}^{*ERSS_e}+\rho}{\bar{X}^{ERSS_e}+\rho}\right)$	0	1	ρ	0	1
8	$T_{1E8} = \bar{y}^{ERSS_e}\left(\frac{a\bar{X}^{ERSS_e}+\rho}{a\bar{x}^{ERSS_e}+\rho}\right)^{\alpha_1}$	1	a	ρ	α_1	0
9	$T_{1E9} = \bar{y}^{ERSS_e}\left(\frac{\bar{x}^{*ERSS_e}+\rho}{\bar{X}^{ERSS_e}+\rho}\right)^{\alpha_2}$	0	1	ρ	0	α_2
10	$T_{1E10} = \bar{y}^{ERSS_e}\left[w\left(\frac{\bar{X}^{ERSS_e}+\rho}{\bar{x}^{ERSS_e}+\rho}\right) + (1-w)\left(\frac{\bar{x}^{*ERSS_e}+\rho}{\bar{X}^{ERSS_e}+\rho}\right)\right]$	w	1	ρ	1	1
11	$T_{1E11} = \bar{y}^{ERSS_e}\left[w\left(\frac{\bar{X}^{ERSS_e}}{\bar{x}^{ERSS_e}}\right) + (1-w)\left(\frac{\bar{x}^{*ERSS_e}}{\bar{X}^{ERSS_e}}\right)\right]$	w	1	0	1	1
12	$T_{1E12} = \bar{y}^{ERSS_e}\left[w\left(\frac{\bar{X}^{ERSS_e}}{\bar{x}^{ERSS_e}}\right)^{\alpha_1} + (1-w)\left(\frac{\bar{x}^{*ERSS_e}}{\bar{X}^{ERSS_e}}\right)^{\alpha_2}\right]$	w	1	0	α_1	α_2
13	$T_{1E13} = \bar{y}^{ERSS_e}\left[w\left(\frac{\bar{X}^{ERSS_e}+\rho}{\bar{x}^{ERSS_e}+\rho}\right)^{\alpha_1} + (1-w)\left(\frac{\bar{x}^{*ERSS_e}+\rho}{\bar{X}^{ERSS_e}+\rho}\right)^{\alpha_2}\right]$	w	1	ρ	α_1	α_2
14	$T_{1E14} = \bar{y}^{ERSS_e}\left[w\left(\frac{a\bar{X}^{ERSS_e}+\rho}{a\bar{x}^{ERSS_e}+\rho}\right)^{\alpha_1} + (1-w)\left(\frac{a\bar{x}^{*ERSS_e}+\rho}{a\bar{X}^{ERSS_e}+\rho}\right)^{\alpha_2}\right]$	w	a	ρ	α_1	α_2

Table 3: Some members of the class of estimator $T_{1(0(m))j}$

S/N	Estimator	w	a	ρ	α_1	α_2
1	$T_{1(0(m))1} = \bar{y}^{ERSS_{0(m)}}$	1	1	0	0	0
2	$T_{1(0(m))2} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{X}^{ERSS_{0(m)}}}{\bar{x}^{ERSS_{0(m)}}}\right)$	1	1	0	1	0
3	$T_{1(0(m))3} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{x}^{*ERSS_{0(m)}}}{\bar{X}^{ERSS_{0(m)}}}\right)$	0	1	0	1	1
4	$T_{1(0(m))4} = \bar{y}^{ERSS_{0(m)}}\left(\frac{a\bar{X}^{ERSS_{0(m)}}+\rho}{a\bar{x}^{ERSS_{0(m)}}+\rho}\right)$	1	a	ρ	1	0
5	$T_{1(0(m))5} = \bar{y}^{ERSS_{0(m)}}\left(\frac{a\bar{x}^{*ERSS_{0(m)}}+\rho}{a\bar{X}^{ERSS_{0(m)}}+\rho}\right)$	0	a	ρ	0	1
6	$T_{1(0(m))6} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{X}^{ERSS_{0(m)}}+\rho}{\bar{x}^{ERSS_{0(m)}}+\rho}\right)$	1	1	ρ	1	0
7	$T_{1(0(m))7} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{x}^{*ERSS_{0(m)}}+\rho}{\bar{X}^{ERSS_{0(m)}}+\rho}\right)$	0	1	ρ	0	1
8	$T_{1(0(m))8} = \bar{y}^{ERSS_{0(m)}}\left(\frac{a\bar{X}^{ERSS_{0(m)}}+\rho}{a\bar{x}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_1}$	1	a	ρ	α_1	0
9	$T_{1(0(m))9} = \bar{y}^{ERSS_{0(m)}}\left(\frac{\bar{x}^{*ERSS_{0(m)}}+\rho}{\bar{X}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_2}$	0	1	ρ	0	α_2
10	$T_{1(0(m))10} = \bar{y}^{ERSS_{0(m)}}\left[w\left(\frac{\bar{X}^{ERSS_{0(m)}}+\rho}{\bar{x}^{ERSS_{0(m)}}+\rho}\right)+(1-w)\left(\frac{\bar{x}^{*ERSS_{0(m)}}+\rho}{\bar{X}^{ERSS_{0(m)}}+\rho}\right)\right]$	w	1	ρ	1	1
11	$T_{1(0(m))11} = \bar{y}^{ERSS_{0(m)}}\left[w\left(\frac{\bar{X}^{ERSS_{0(m)}}}{\bar{x}^{ERSS_{0(m)}}}\right)+(1-w)\left(\frac{\bar{x}^{*ERSS_{0(m)}}}{\bar{X}^{ERSS_{0(m)}}}\right)\right]$	w	1	0	1	1
12	$T_{1(0(m))12} = \bar{y}^{ERSS_{0(m)}}\left[w\left(\frac{\bar{X}^{ERSS_{0(m)}}}{\bar{x}^{ERSS_{0(m)}}}\right)^{\alpha_1}+(1-w)\left(\frac{\bar{x}^{*ERSS_{0(m)}}}{\bar{X}^{ERSS_{0(m)}}}\right)^{\alpha_2}\right]$	w	1	0	α_1	α_2
13	$T_{1(0(m))13} = \bar{y}^{ERSS_{0(m)}}\left[w\left(\frac{\bar{X}^{ERSS_{0(m)}}+\rho}{\bar{x}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_1}+(1-w)\left(\frac{\bar{x}^{*ERSS_{0(m)}}}{\bar{X}^{ERSS_{0(m)}}}\right)^{\alpha_2}\right]$	w	1	ρ	α_1	α_2
14	$T_{1(0(m))14} = \bar{y}^{ERSS_{0(m)}}\left[w\left(\frac{a\bar{X}^{ERSS_{0(m)}}+\rho}{a\bar{x}^{ERSS_{0(m)}}+\rho}\right)^{\alpha_1}+(1-w)\left(\frac{a\bar{x}^{*ERSS_{0(m)}}}{a\bar{X}^{ERSS_{0(m)}}}\right)^{\alpha_2}\right]$	w	a	ρ	α_1	α_2

Table 4: Some members of the class of estimator T_{1Sj}

S/N	Estimator	w	a	ρ	α_1	α_2
1	$T_{1S1} = \bar{y}^{SRS}$,	1	1	0	0	0
2	$T_{1S2} = \bar{y}^{SRS}\left(\frac{\bar{X}^{SRS}}{\bar{x}^{SRS}}\right)$	1	1	0	1	0
3	$T_{1S3} = \bar{y}^{SRS}\left(\frac{\bar{x}^{*SRS}}{\bar{X}^{SRS}}\right)$	0	1	0	1	1
4	$T_{1S4} = \bar{y}^{SRS}\left(\frac{a\bar{X}^{SRS}+\rho}{a\bar{x}^{SRS}+\rho}\right)$	1	a	ρ	1	0
5	$T_{1S5} = \bar{y}^{SRS}\left(\frac{a\bar{x}^{*SRS}+\rho}{a\bar{X}^{SRS}+\rho}\right)$	0	a	ρ	0	1
6	$T_{1S6} = \bar{y}^{SRS}\left(\frac{\bar{X}^{SRS}+\rho}{\bar{x}^{SRS}+\rho}\right)$	1	1	ρ	1	0
7	$T_{1S7} = \bar{y}^{SRS}\left(\frac{\bar{x}^{*SRS}+\rho}{\bar{X}^{SRS}+\rho}\right)$	0	1	ρ	0	1
8	$T_{1S8} = \bar{y}^{SRS}\left(\frac{a\bar{X}^{SRS}+\rho}{a\bar{x}^{SRS}+\rho}\right)^{\alpha_1}$	1	a	ρ	α_1	0
9	$T_{1S9} = \bar{y}^{SRS}\left(\frac{\bar{x}^{*SRS}+\rho}{\bar{X}^{SRS}+\rho}\right)^{\alpha_2}$	0	1	ρ	0	α_2
10	$T_{1S10} = \bar{y}^{SRS}\left[w\left(\frac{\bar{X}^{SRS}+\rho}{\bar{x}^{SRS}+\rho}\right)+(1-w)\left(\frac{\bar{x}^{*SRS}+\rho}{\bar{X}^{SRS}+\rho}\right)\right]$	w	1	ρ	1	1
11	$T_{1S11} = \bar{y}^{SRS}\left[w\left(\frac{\bar{X}^{SRS}}{\bar{x}^{SRS}}\right)+(1-w)\left(\frac{\bar{x}^{*SRS}}{\bar{X}^{SRS}}\right)\right]$	w	1	0	1	1
12	$T_{1S12} = \bar{y}^{SRS}\left[w\left(\frac{\bar{X}^{SRS}}{\bar{x}^{SRS}}\right)^{\alpha_1}+(1-w)\left(\frac{\bar{x}^{*SRS}}{\bar{X}^{SRS}}\right)^{\alpha_2}\right]$	w	1	0	α_1	α_2
13	$T_{1S13} = \bar{y}^{SRS}\left[w\left(\frac{\bar{X}^{SRS}+\rho}{\bar{x}^{SRS}+\rho}\right)^{\alpha_1}+(1-w)\left(\frac{\bar{x}^{*SRS}+\rho}{\bar{X}^{SRS}+\rho}\right)^{\alpha_2}\right]$	w	1	ρ	α_1	α_2
14	$T_{1S14} = \bar{y}^{SRS}\left[w\left(\frac{a\bar{X}^{SRS}+\rho}{a\bar{x}^{SRS}+\rho}\right)^{\alpha_1}+(1-w)\left(\frac{a\bar{x}^{*SRS}+\rho}{a\bar{X}^{SRS}+\rho}\right)^{\alpha_2}\right]$	w	a	ρ	α_1	α_2

Definition 2

Bias, Relative Bias and Mean Square Errors of $T_{1Ej}, T_{1(0(m))j}, T_{1Sj}$

If $T_{1Ej}, T_{1(0(m))j}, T_{1Sj}, j = 1, 2, .. m$ are classes estimator of the population mean $\bar{Y}$ under ERSS and SRS, then the biases, relative biases, and Means Square Errors (MSEs) is defined as:

(a) Biases

(1) $B\left(T_{1Ej}\right) = \left[E\left(T_{1Ej}\right) - \bar{Y}^{ERSSe}\right], j = 1,2, \dots m,$ (13)

(2) $B\left(T_{1(0(m))j}\right) = \left[E\left(T_{1(0(m))j}\right) - \bar{Y}^{ERSS_{0(m)}}\right], j = 1,2, \dots m$ (14)

(3) $B(T_{1Si}) = \left[E\left(T_{1Sj}\right) - \bar{Y}^{SRS}\right], i = 1,2, \dots m,$ (15)

(b) Relative Biases

(1). $RB\left(T_{1Ej}\right) = \frac{\left[E\left(T_{1Ej}\right) - \bar{Y}^{ERSSe}\right]}{\bar{Y}^{ERSSe}}, j = 1,2, \dots m,$ (16)

(2). $RB\left(T_{1(0(m))j}\right) = \frac{\left[E\left(T_{1(0(m))j}\right) - \bar{Y}^{ERSS_{0(m)}}\right]}{\bar{Y}^{ERSS_{0(m)}}}, j = 1,2, \dots m$ (17)

(3). $RB(T_{1Si}) = \frac{\left[E\left(T_{1Sj}\right) - \bar{Y}^{SRS}\right]}{\bar{Y}^{SRS}}, i = 1,2, \dots m,$ (18)

$T_{1Sj}, j = 1, 2, .. m$ is a class of estimator of the Population mean $\bar{Y}^{SRS}$ under SRS

(c) MSEs

(1) $MSE\left(T_{1Ej}\right) = E\left[T_{1Ej} - \bar{Y}^{ERSSe}\right]^2, j = 1,2, \dots m,$ (19)

(2) $MSE\left(T_{1[0(m)]j}\right) = E\left[T_{1(0(m))j} - \bar{Y}^{ERSS_{0(m)}}\right]^2, j = 1,2, \dots m$ (20)

(3) $MSE(T_{1Si}) = E\left[T_{1Sj} - \bar{Y}^{SRS}\right]^2, v = 1,2, \dots m,$ (21)

$T_{1Sj}, j = 1, 2, .. m$ is a class of estimator of the Population mean $\bar{Y}^{SRS}$ under SRS, for $ERSSe$ case, e: is even and $ERSS_{0(m)}$ case, $0(m)$: is median odd respectively, m the total number of members of the proposed class of estimator.

Biases T_{1Ej}, $T_{1(0(m))j}$, T_{1Sj}

To obtain the bias and Mean Square Error of the class of estimators T_{1Ej} we write

$$\left.\begin{aligned} \bar{y}^{ERSS_e} &= \bar{Y}^{ERSS_e}(1 + e_y) \\ \bar{x}^{ERSS_e} &= \bar{X}^{ERSS_e}(1 + e_x) \\ \Delta_x \Delta_y &= \left(\mu_{x(i)} - \mu_x\right)\left(\mu_{y[i]} - \mu_y\right) \\ E\left(e_y\right) &= E\left(e_x\right) = 0 \\ E\left(e_y^2\right) &= \left(\frac{1}{2m}\right)\frac{Var(\bar{y}^{ERSS_e})}{\left(\mu_y\right)^2} = C_y^2 \\ E\left(e_x^2\right) &= \left(\frac{1}{2m}\right)\frac{Var(\bar{x}^{ERSS_e})}{(\mu_x)^2} = C_x^2 \\ E\left(e_y e_x\right) = \left(\sigma_{xy} - \frac{1}{m}\sum_{i=1}^{m}\Delta_x\,\Delta_y\right) = C_{xy} &= \left(\frac{1}{2m}\right)\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_e})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_e})}}{\mu_x} = \rho_{xy}\, C_y\, C_x \end{aligned}\right\} \quad (22)$$

T_{1Ej}, $T_{1(0(m))j}$, T_{1Sj} in equations 10, 11 and 12 can be expressed in terms of $e's$ as:

$T_{1Ej} = \bar{Y}^{ERSS_e}(1 + e_y)[w(1 + \lambda_a e_x)^{-\alpha_1} + (1 - w)(1 - g\lambda_a e_x)^{\alpha_2}]$ (23)

$T_{1(0(m))j} = \bar{Y}^{ERSS_{0(m)}}(1 + e_y)[w(1 + \lambda_a e_x)^{-\alpha_1} + (1 - w)(1 - g\lambda_a e_x)^{\alpha_2}]$ (24)

$T_{1Sj} = \bar{Y}^{SRS}(1 + e_y)[w(1 + \lambda_a e_x)^{-\alpha_1} + (1 - w)(1 - g\lambda_a e_x)^{\alpha_2}]$ (25)

Expanding the right and side of (23), neglecting terms of $e's$ having power greater than two, we have and then taking the mathematical expectations of the emergent expression, yields the bias of T_{1Ej} as:

$$B(T_{1Ei}) = [E(T_{1Ej}) - \bar{Y}^{ERSS_e}] = \bar{Y}^{ERSS_e}\begin{bmatrix} w\left(1-\alpha_1\lambda_a\left(\frac{1}{2m}\right)\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_e})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_e})}}{\mu_x}+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2\left(\frac{1}{2m}\right)\frac{Var(\bar{x}^{ERSS_e})}{(\mu_x)^2}\right) \\ +(1-w)\left(1-\alpha_2 g\lambda_a\left(\frac{1}{2m}\right)\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_e})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_e})}}{\mu_x}+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2\left(\frac{1}{2m}\right)\frac{Var(\bar{x}^{ERSS_e})}{(\mu_x)^2}\right)-1 \end{bmatrix}$$

$$B(T_{1Ei}) = [E(T_{1Ej}) - \bar{Y}^{ERSS_e}] = \frac{\bar{Y}^{ERSS_e}}{2m}\begin{bmatrix} w\left((2m-\alpha_1\lambda_a C_{xy}+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_x^2\right) \\ +(1-w)\left((2m-\alpha_2 g\lambda_a C_{xy}+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2 C_x^2\right)-2m \end{bmatrix} \quad (26)$$

In like manners, we obtained the biases of $T_{1(0(m))j}$, T_{1Sj}

$$B(T_{1(0(m))j}) = [E(T_{1(0(m))j}) - \bar{Y}^{ERSS_{0(m)}}] = \bar{Y}^{ERSS_{0(m)}}\begin{bmatrix} w\left(1-\alpha_1\lambda_a\theta\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_{0(m)}})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_{0(m)}})}}{\mu_x}+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2\theta\frac{Var(\bar{x}^{ERSS_{0(m)}})}{(\mu_x)^2}\right) \\ +(1-w)\left(1-\alpha_2 g\lambda_a\theta\rho_{xy}\cdot\frac{\sqrt{Var(\bar{y}^{ERSS_{0(m)}})}}{\mu_y}\cdot\frac{\sqrt{Var(\bar{x}^{ERSS_{0(m)}})}}{\mu_x}+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2\theta\frac{Var(\bar{x}^{ERSS_{0(m)}})}{(\mu_x)^2}\right)-1 \end{bmatrix}$$

$$B(T_{1(0(m))j}) = [E(T_{1(0(m))j}) - \bar{Y}^{ERSS_{0(m)}}] =$$

$$\theta\bar{Y}^{ERSS_{0(m)}}\begin{bmatrix} w\left((\frac{1}{\theta}-\alpha_1\lambda_a C_{xy}+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_x^2\right) \\ +(1-w)\left((\frac{1}{\theta}-\alpha_2 g\lambda_a C_{xy})+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2 C_x^2\right)-\frac{1}{\theta} \end{bmatrix} \quad (27)$$

Where $\theta = \left(\frac{m-1}{2m^2}\right)$

$$B(T_{1Sj}) = [E(T_{1Sj}) - \bar{Y}^{SRS}] =$$

$$\left(\frac{1-f}{m}\right)\bar{Y}^{SRS}\begin{bmatrix} w\left(\left(\frac{m}{1-f}\right)-\alpha_1\lambda_a C_{10}+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 C_1^2\right) \\ +(1-w)\left(\left(\left(\frac{m}{1-f}\right)-\alpha_2 g\lambda_a C_{10}\right)+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2 C_1^2\right)-\left(\frac{m}{1-f}\right) \end{bmatrix} \quad (28)$$

MSEs of T_{1Ej}, $T_{1(0(m))j}$, T_{1Sj}

Recall that from equation (23)

$$(T_{1Ej} - \bar{Y}^{ERSS_e}) =$$

$$\bar{Y}^{ERSS_e}\begin{bmatrix} w\left((1+e_y-\alpha_1\lambda_a e_y e_x-\alpha_1\lambda_a e_x+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2-\cdots\right) \\ +(1-w)\left((1+e_y-\alpha_2 g\lambda_a e_y e_x-\alpha_2 g\lambda_a e_x+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2 e_x^2+\cdots\right)-1 \end{bmatrix} \quad (29)$$

By Squaring both sides of (29) and neglecting terms of $e's$ having powers greater than two we have:

$$(T_{1Ej} - \bar{Y}^{ERSS_e})^2 == \begin{Bmatrix} \bar{Y}^{2ERSS_e}w^2\begin{pmatrix}(1+e_y-\alpha_1\lambda_a e_y e_x+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2+e_y+e_y^2-\alpha_1 g\lambda_a e_y e_x-\alpha_1 g\lambda_a e_y e_x \\ -\alpha_1{}^2\lambda_a^2 e_x{}^2-\alpha_1 g\lambda_1 e_y e_x+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2\end{pmatrix} \\ +\bar{Y}^{2ERSS_e}(1-w)^2\begin{pmatrix}1+e_y-\alpha_2 g\lambda_a e_y e_x+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2 e_x^2+e_y+e_y^2-\alpha_2 g\lambda_a e_y e_x \\ -\alpha_2 g\lambda_a e_y e_x-\alpha_2 g\lambda_a e_y e_x+\alpha_2{}^2\lambda_a^2 e_x{}^2+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2 e_x^2\end{pmatrix} \\ +2w(1-w)\bar{Y}^{2ERSS_e}\begin{pmatrix}1+e_y-\alpha_2 g\lambda_a e_y e_x+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2 e_x^2+e_y+e_y^2-\alpha_2 g\lambda_a e_y e_x-\alpha_1 g\lambda_a e_y e_x \\ -\alpha_1 g\lambda_a e_y e_x-\alpha_1\alpha_2 g\lambda_a^2 e_x^2+\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2\end{pmatrix} \\ -2w\bar{Y}^{2ERSS_e}\left(1+e_y-\alpha_1 g\lambda_a e_y e_x-\frac{\alpha_1(\alpha_1+1)}{2}\lambda_a^2 e_x^2\right) \\ -2(1-w)\bar{Y}^{2ERSS_e}\,1+e_y-\alpha_2 g\lambda_a e_y e_x+\frac{\alpha_2(\alpha_2-1)}{2}g^2\lambda_a^2 e_x^2\end{Bmatrix}+\bar{Y}^{2ERSS_e} \quad (30)$$

By taking the mathematical expectation of (30), gives the MSE of T_{1Ej} as:

$$MSE(T_{1Ej}) = \bar{Y}^{2^{ERSS_e}}\left[1 + w^2F_1 + (1-w)^2F_2 + 2w(1-w)F_3 - 2wF_4 - 2(1-w)F_5\right] \quad (31)$$

Where

$$F_1 = \left[1 + \frac{1}{2m}\left(1 + C_y^2 + \alpha_1\left(\alpha_1 + (\alpha_1+1)\right)\lambda_a^2C_x^2 - 4\alpha_1\lambda_aC_{xy}\right)\right] \quad (32)$$

$$F_2 = \left[1 + \frac{1}{2m}\left(1 + C_y^2 - \alpha_2\left(\alpha_2 + (\alpha_2-1)\right)g^2\lambda_a^2C_x^2 - 4\alpha_2g\lambda_aC_{xy}\right)\right] \quad (33)$$

$$F_3 = \left[1 + \frac{1}{2m}\left(C_y^2 + \left(\frac{\alpha_1(\alpha_1+1)}{2} + \frac{\alpha_2(\alpha_2-1)}{2}g^2 + \alpha_2g^2\right)\lambda_a^2C_x^2 - 2(\alpha_1 + \alpha_2g)\lambda_aC_{xy}\right)\right] \quad (34)$$

$$F_4 = \left[1 + \frac{1}{2m}\left(-\left(\frac{\alpha_1(\alpha_1+1)}{2}\right)\lambda_a^2C_x^2 - \alpha_1g\lambda_aC_{xy}\right)\right] \quad (35)$$

$$F_5 = \left[1 + \frac{1}{2m}\left(\left(\frac{\alpha_2(\alpha_2-1)}{2}\right)g^2\lambda_a^2C_x^2 - \alpha_2g\lambda_aC_{xy}\right)\right] \quad (36)$$

Similarly, the MSEs of $T_{1(0(m))j}$, T_{1Sj} were obtained to be:

$$MSE(T_{1(0(m))j} = \bar{Y}^{2^{ERSS_{0(m)}}}\left[1 + w^2H_1 + (1-w)^2H_2 + 2w(1-w)H_3 - 2wH_4 - 2(1-w)H_5\right] \quad (37)$$

Where

$$H_1 = \left[1 + \theta\left(C_y^2 + \alpha_1\left(\alpha_1 + (\alpha_1+1)\right)\lambda_a^2C_x^2 - 4\alpha_1\lambda_aC_{xy}\right)\right] \quad (38)$$

$$H_2 = \left[1 + \theta\left(C_y^2 - \alpha_2\left(\alpha_2 + (\alpha_2-1)\right)g^2\lambda_a^2C_x^2 - 4\alpha_2g\lambda_aC_{xy}\right)\right] \quad (39)$$

$$H_3 = \left[1 + \theta\left(C_y^2 + \left(\frac{\alpha_1(\alpha_1+1)}{2} + \frac{\alpha_2(\alpha_2-1)}{2}g^2 + \alpha_2g^2\right)\lambda_a^2C_x^2 - 2(\alpha_1 + \alpha_2g)\lambda_aC_{xy}\right)\right] \quad (40)$$

$$H_4 = \left[1 + \theta\left(\left(\frac{\alpha_1(\alpha_1+1)}{2}\right)\lambda_a^2C_x^2 - \alpha_1g\lambda_aC_{xy}\right)\right] \quad (41)$$

$$H_5 = \left[1 + \theta\left(\left(\frac{\alpha_2(\alpha_2-1)}{2}\right)g^2\lambda_a^2C_x^2 - \alpha_2g\lambda_aC_{xy}\right)\right] \quad (42)$$

$$MSE(T_{1Sj}) = (\bar{Y}^{SRS})^2\left[1 + w^2G_1 + (1-w)^2G_2 + 2w(1-w)G_3 - 2wG_4 - 2(1-w)G_5\right] \quad (43)$$

Where

$$G_1 = \left[1 + \left(\frac{1-f}{m}\right)\left(C_0^2 + \alpha_1\left(\alpha_1 + (\alpha_1+1)\right)\lambda_a^2C_1^2 - 4\alpha_1\lambda_aC_{10}\right)\right] \quad (44)$$

$$G_2 = \left[1 + \left(\frac{1-f}{m}\right)\left(1 + C_0^2 - \alpha_2\left(\alpha_2 + (\alpha_2-1)\right)g^2\lambda_a^2C_1^2 - 4\alpha_2g\lambda_aC_{10}\right)\right] \quad (45)$$

$$G_3 = \left[1 + \left(\frac{1-f}{m}\right)\left(C_0^2 + \left(\frac{\alpha_1(\alpha_1+1)}{2} + \frac{\alpha_2(\alpha_2-1)}{2}g^2 + \alpha_2g^2\right)\lambda_a^2C_1^2 - 2(\alpha_1 + \alpha_2g)\lambda_aC_{10}\right)\right] \quad (46)$$

$$G_4 = \left[1 + \left(\frac{1-f}{m}\right)\left(\left(\frac{\alpha_1(\alpha_1+1)}{2}\right)\lambda_a^2C_1^2 - \alpha_1g\lambda_aC_{10}\right)\right] \quad (47)$$

$$G_5 = \left[1 + \left(\frac{1-f}{m}\right)\left(\left(\frac{\alpha_2(\alpha_2-1)}{2}\right)g^2\lambda_a^2C_1^2 - \alpha_2g\lambda_aC_{10}\right)\right] \quad (48)$$

Optimal MSEs of T_{1Ej}, $T_{1(0(m))j}$, T_{1Sj}

To obtain the optimum MSE of T_{1Ej}, $T_{1(0(m))j}$, T_{1Sj} we differentiate (31), (37), and (43) partially and separately with respect to w and $(1-w)$ and equate the resulting expression to zero respectively. This procedure yields the optimal MSEs of T_{1Ej}, $T_{1(0(m))j}$, T_{1Sj} as:

$$MSE(T_{1Ej})_{opt} = \bar{Y}^{2^{ERSS_e}}\left[1 + \frac{(2F_3F_4F_5 - F_2F_4^2 - F_1F_5^2)}{(F_1F_2 - F_3^2)}\right] \quad (49)$$

$$MSE(T_{1(0(m))j})_{opt} = \bar{Y}^{2^{ERSS_{0(m)}}}\left[1 + \frac{(2H_3H_4H_5 - H_2H_4^2 - H_1H_5^2)}{(H_1H_2 - H_3^2)}\right] \quad (50)$$

$$MSE(T_{1Sj})_{opt} = \bar{Y}^{2^{SRS}}\left[1 + \frac{(2G_3G_4G_5 - G_2G_4^2 - G_1G_5^2)}{(G_1G_2 - G_3^2)}\right] \quad (51)$$

Efficiency comparison

Let $MSE(T_{1Ej})_{opt}$, $MSE(T_{1[0(m)]j})_{opt}$, and $MSE(T_{1Sj})_{opt}$ be the Mean Square Errors (MSEs) of the proposed class of estimators under ERSS for $ERSSe$ case, e: is even, $ERSS_{0(m)}$ case, $0(m)$: is median odd, and that of the estimator proposed under SRS respectively.

(i) $MSE(T_{1Ej})_{opt}$ is more efficient than $MSE(T_{1Sj})_{opt}$ under optimal condition if the ratio of $MSE(T_{1Ej})_{opt}$ in relation to $MSE(T_{1Sj})_{opt}$ is less than 1 or the reciprocal of the ratio of $MSE(T_{1Ej})_{opt}$ in relation to $MSE(T_{1Sj})_{opt}$ is greater than 1. Thus:

$$\frac{(\bar{Y}^{ERSSe})^2\left[1+\frac{(2F_3F_4F_5-F_2F_4^2-F_1F_5^2)}{(F_1F_2-F_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right]} < 1 \; or \; \frac{1}{\frac{(\bar{Y}^{ERSSe})^2\left[1+\frac{(2F_3F_4F_5-F_2F_4^2-F_1F_5^2)}{(F_1F_2-F_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right]}} > 1,$$

for $j = 1 \; to \; m$ (52)

(ii) $MSE(T_{1[0(m)]j})_{opt}$ is more efficient than $MSE(T_{1Sj})_{opt}$ under optimal condition if the ratio of $MSE(T_{1[0(m)]j})_{opt}$ in relation to $MSE(T_{1Sj})_{opt}$ is less than 1 or the reciprocal of the ratio of $MSE(T_{1[0(m)]j})_{opt}$ in relation to $MSE(T_{1Sj})_{opt}$ is greater than 1.

$$\frac{\left(\bar{Y}^{ERSS_{0(m)}}\right)^2\left[1+\frac{(2H_3H_4H_5-H_2H_4^2-H_1H_5^2)}{(H_1H_2-H_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right]} < 1 \; or \; \frac{1}{\frac{\left(\bar{Y}^{ERSS_{0(m)}}\right)^2\left[1+\frac{(2H_3H_4H_5-H_2H_4^2-H_1H_5^2)}{(H_1H_2-H_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right]}} > 1,$$

for $j = 1 \; to 14$ (53)

(iii) $MSE(T_{1[0(m)]j})_{opt}$, is most efficient than $MSE(T_{1Ej})_{opt}$, and $MSE(T_{1Sj})_{opt}$, under optimal condition if

$$\left(\bar{Y}^{ERSS_{0(m)}}\right)^2\left[1+\frac{(2H_3H_4H_5-H_2H_4^2-H_1H_5^2)}{(H_1H_2-H_3^2)}\right] < (\bar{Y}^{ERSSe})^2\left[1+\frac{(2F_3F_4F_5-F_2F_4^2-F_1F_5^2)}{(F_1F_2-F_3^2)}\right] < (\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right] \quad (54)$$

(iv) $MSE\;(T_{1Ej})_{opt}$ is more efficient than $MSE(T_{1Sj})_{opt}$ in terms of PRE, if

$$\left.\begin{array}{l} \frac{(\bar{Y}^{ERSSe})^2\left[1+\frac{(2F_3F_4F_5-F_2F_4^2-F_1F_5^2)}{(F_1F_2-F_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right]} \times 100 < 100 \; or \\ \frac{1}{\frac{(\bar{Y}^{ERSSe})^2\left[1+\frac{(2F_3F_4F_5-F_2F_4^2-F_1F_5^2)}{(F_1F_2-F_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right]}} \times 100 > 100 \\ \text{for } j = 1 \; to \; 14 \end{array}\right\} \quad (55)$$

(v) $MSE\;(T_{1[0(m)]j})_{opt}$ is more efficient than $MSE(T_{1Sj})_{opt}$ in terms of PRE, if

$$\left.\begin{array}{l} \frac{\left(\bar{Y}^{ERSS_{0(m)}}\right)^2\left[1+\frac{(2H_3H_4H_5-H_2H_4^2-H_1H_5^2)}{(H_1H_2-H_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right]} \times 100 < 100 \; or \\ \frac{1}{\frac{\left(\bar{Y}^{ERSS_{0(m)}}\right)^2\left[1+\frac{(2H_3H_4H_5-H_2H_4^2-H_1H_5^2)}{(H_1H_2-H_3^2)}\right]}{(\bar{Y}^{SRS})^2\left[1+\frac{(2G_3G_4G_5-G_2G_4^2-G_1G_5^2)}{(G_1G_2-G_3^2)}\right]}} \times 100 > 100 \\ \text{for } j = 1 \; to \; 14 \end{array}\right\} \quad (56)$$

Table 5: Members of $T_{1Ej}, j = 1, 2, \dots 14$ with their Bias

S/N	T_{1Ei}	Bias
1	T_{1E1}	0
2	T_{1E2}	$\frac{\bar{Y}^{ERSS_e}}{2m}(C_x^2 - C_{xy})$
3	T_{1E3}	$\frac{\bar{Y}^{ERSS_e}}{2m}(-gC_{xy})$
4	T_{1E4}	$\frac{\bar{Y}^{ERSS_e}}{2m}\lambda_a(\lambda_a C_x^2 - C_{xy})$
5	T_{1E5}	$\frac{\bar{Y}^{ERSS_e}}{2m}\lambda_a(-gC_{xy})$
6	T_{1E6}	$\frac{\bar{Y}^{ERSS_e}}{2m}\lambda_1(\lambda_1 C_x^2 - C_{xy})$
7	T_{1E7}	$\frac{\bar{Y}^{ERSS_e}}{2m}\lambda_a(-gC_{xy})$
8	T_{1E8}	$\frac{\bar{Y}^{ERSS_e}}{2m}\lambda_a\left(\left(\frac{\alpha_1(\alpha_1+1)}{2}\right)\lambda_a C_x^2 - \alpha_1 C_{xy}\right)$
9	T_{1E9}	$\frac{\bar{Y}^{ERSS_e}}{2m}\lambda_1\left(\left(\frac{\alpha_2(\alpha_2-1)}{2}\right)g^2\lambda_1 C_x^2 - g\alpha_2 C_{xy}\right)$
10	T_{1E10}	$\frac{\bar{Y}^{ERSS_e}}{2m}\left(w(2m + \lambda_1^2 C_x^2 - \lambda_1 C_{xy}) + (1-w)(2m - \lambda_1 C_{xy}) - 2m\right)$
11	T_{1E11}	$\frac{\bar{Y}^{ERSS_e}}{2m}\left(w(2m + C_x^2 + C_{xy}) + (1-w)(2m - gC_{xy}) - 2m\right)$
12	T_{1E12}	$\frac{\bar{Y}^{ERSS_e}}{2m}\left(w\left(2m + \left(\frac{\alpha_1(\alpha_1+1)}{2}\right)C_x^2 - \alpha_1 C_{xy}\right) + (1-w)\left(2m + \left(\frac{\alpha_2(\alpha_2-1)}{2}\right)g^2 C_x^2 - g\alpha_2 C_{xy}\right) - 2m\right)$
13	T_{1E13}	$\frac{\bar{Y}^{ERSS_e}}{2m}\left(w\left(2m + \left(\frac{\alpha_1(\alpha_1+1)}{2}\right)\lambda_1^2 C_x^2 - \alpha_1\lambda_1 C_{xy}\right) + (1-w)\left(2m + \left(\frac{\alpha_2(\alpha_2-1)}{2}\right)\lambda_1^2 g^2 C_x^2 - g\lambda_1\alpha_2 C_{xy}\right) - 2m\right)$
14	T_{1E14}	$\frac{\bar{Y}^{ERSS_e}}{2m}\left(w\left(2m + \left(\frac{\alpha_1(\alpha_1+1)}{2}\right)\lambda_a^2 C_x^2 - \alpha_1\lambda_a C_{xy}\right) + (1-w)\left(2m + \left(\frac{\alpha_2(\alpha_2-1)}{2}\right)\lambda_a^2 g^2 C_x^2 - g\lambda_a\alpha_2 C_{xy}\right) - 2m\right)$

Table 6: Members of $T_{1Ej}, j = 1, 2, \dots 14$ with their MSE

S/N	T_{1Ei}	MSE
1	T_{1E1}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2)$
2	T_{1E2}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2 + C_x^2 - 2C_{xy})$
3	T_{1E3}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2 + g^2 C_x^2 + 2gC_{xy})$
4	T_{1E4}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2 + \lambda_a^2 C_x^2 - 2\lambda_a C_{xy})$
5	T_{1E5}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2 + g^2\lambda_a^2 C_x^2 - 2\lambda_a g C_{xy})$
6	T_{1E6}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2 + \lambda_1^2 C_x^2 - 2\lambda_1 C_{xy})$
7	T_{1E7}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2 + g^2\lambda_1^2 C_x^2 + 2g\lambda_1 C_{xy})$
8	T_{1E8}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2 + \alpha_1^2\lambda_a^2 C_x^2 - 2\alpha_1\lambda_a C_{xy})$
9	T_{1E9}	$\frac{\bar{Y}^{2^{ERSS_e}}}{2m}(C_y^2 + g^2\alpha_2^2\lambda_1^2 C_x^2 + 2g\alpha_2\lambda_1 C_{xy})$
10	T_{1E10}	$\lim_{(\lambda_a\to\lambda_1, \alpha_1\to 1, \alpha_2\to 1)} \bar{Y}^{2^{ERSS_e}}[1 + w^2F_1 + (1-w)^2F_2 + 2w(1-w)F_3 - 2wF_4 - 2(1-w)F_5]$
11	T_{1E11}	$\lim_{(\lambda_a\to 1, \alpha_1\to 1, \alpha_2\to 1)} \bar{Y}^{2^{ERSS_e}}[1 + w^2F_1 + (1-w)^2F_2 + 2w(1-w)F_3 - 2wF_4 - 2(1-w)F_5]$
12	T_{1E12}	$\lim_{(\lambda_a\to 1)} \bar{Y}^{2^{ERSS_e}}[1 + w^2F_1 + (1-w)^2F_2 + 2w(1-w)F_3 - 2wF_4 - 2(1-w)F_5]$
13	T_{1E13}	$\lim_{(\lambda_a\to\lambda_1)} \bar{Y}^{2^{ERSS_e}}[1 + w^2F_1 + (1-w)^2F_2 + 2w(1-w)F_3 - 2wF_4 - 2(1-w)F_5]$
14	T_{1E14}	$\bar{Y}^{2^{ERSS_e}}[1 + w^2F_1 + (1-w)^2F_2 + 2w(1-w)F_3 - 2wF_4 - 2(1-w)F_5]$

Empirical study

In order to investigate the efficiency of the proposed class of estimators and its members under ERSS over its corresponding counterpart estimator based on SRS, and some existing ratio type estimators, we have considered three natural populations data sets. The real-life data sets were obtained from various sources and the description of the population and the values of the required parameters are specified below:

Population I: [Source: Murthy (1967)] [36]. The population consists of 80 factories in a region, the character X and Y being fixed capital and output respectively. The variables are defined as follows:
Y = Output of factory
X= Fixed capital
$M = 80, m = 8, \bar{Y} = 8.480904,\ \bar{X} = 6.750716,\ C_x = 0.7459, C_y = 0.3519,\ \rho_{xy} = 0.9640175.$

Population II: [Source: Steel and Torrie (1960)] [49].
Y: Log of leaf burn in seconds,
X: Potassium percentage
$M = 30, m = 4, \bar{Y} = 0.6860, \bar{X} = 4.6437, C_x = 0.47906, C_y = 0.693\ \rho_{xy} = 0.1794.$

Population III: [Source: Khare and Rehman (2015)] [34].
Y: Number of Agricultural labour pp
X: Area of village hectares
$M = 96, m = 24, \bar{Y} = 137.9271, \bar{X} = 144.8720, C_x = 0.8115,\ C_y = 1.3232$
$\rho_{xy} = 0.786.$

TABLE 7
Results of Biases of the proposed class of estimators
Estimators/populations

	T_{1Ej}				$T_{1(o(m))j}$				T_{1Sj}		
members	*I*	*II*	*III*	*members*	*I*	*II*	*III*	*members*	*I*	*II*	*III*
T_{1E1}	0.00	0.00	0.00	$T_{1(o(m))1}$	0.00	0.00	0.00	T_{1S1}	0.00	0.00	0.00
T_{1E2}	0.16078	0.01457	-0.4928	$T_{1(o(m))2}$	0.140684	0.010929	-0.47226	T_{1S2}	0.289407	0.025259	-0.73919
T_{1E3}	-0.01497	-0.0008	-0.79502	$T_{1(o(m))3}$	-0.01304	-0.00059	-0.761899	T_{1S3}	-0.026825	-0.00136	-0.192537
T_{1E4}	0.095173	0.012126	-0.50186	$T_{1(o(m))4}$	0.083277	0.009094	-0.48094	T_{1S4}	0.171312	0.021018	-0.75278
T_{1E5}	-0.01251	-0.00073	-0.78983	$T_{1(o(m))5}$	-0.01094	-0.00055	-0.75692	T_{1S5}	-0.02251	0.00126	-1.18475
T_{1E6}	0.108445	0.013326	-0.50017	$T_{1(o(m))6}$	0.094889	0.009994	-0.47933	T_{1S6}	0.1952	0.023098	-0.75025
T_{1E7}	-0.01304	-0.00076	-0.79081	$T_{1(o(m))7}$	-0.011404	-0.00057	-0.757855	T_{1S7}	-0.023473	-0.00131	-1.186208
T_{1E8}	0.112572	0.004726	2.369494	$T_{1(o(m))8}$	0.098501	0.003545	2.270765	T_{1S8}	0.20263	0.008192	3.334241
T_{1E9}	-0.01304	-0.000756	-0.790806	$T_{1(o(m))9}$	-0.01141	-0.00057	-0.757855	T_{1S9}	-0.023473	-0.00131	-1.186208
T_{1E10}	-0.37738	-0.0414599	-7.670937	$T_{1(o(m))10}$	-0.403384	-0.0445	-7.53934	T_{1S9}	-0.210967	-0.03112	-9.250023
T_{1E11}	0.024491	-0.03018	-7.620882	$T_{1(o(m))11}$	-0.05175	-0.03653	-7.49137	T_{1S10}	0.512403	-0.01157	-9.17494
T_{1E12}	0.024491	-0.030181	-7.620882	$T_{1(o(m))12}$	-0.051746	-0.03653	-7.49137	T_{1S11}	0.512403	-0.01157	-9.17494
T_{1E13}	-0.09703	-0.03185	-7.59978	$T_{1(o(m))13}$	-0.15808	-0.03777	-7.47115	T_{1S12}	0.293669	-0.01446	-9.149329
T_{1E14}	-0.17835	-0.033403	-7.587901	$T_{1(o(m))14}$	-0.2292346	-0.0389421	-7.45977	T_{1S13}	0.147283	-0.01716	-9.125469

TABLE 8
Results of MSEs of the proposed class of estimators
ESTIMATORS/POPULATIONS

	T_{1Ej}				$T_{1(o(m))j}$				T_{1Sj}		
members	*I*	*II*	*III*	*members*	*I*	*II*	*III*	*members*	*I*	*II*	*III*
T_{1E1}	0.56	0.03	693.95	$T_{1(o(m))1}$	0.49	0.02	665.01	T_{1S1}	1.00	0.05	1040.88
T_{1E2}	0.78276	0.034744	296.9833	$T_{1(o(m))2}$	0.608491	0.026058	284.609	T_{1S2}	1.408962	0.060222	445.4749
T_{1E3}	0.84033	0.029648	942.2298	$T_{1(o(m))3}$	0.735289	0.022236	902.9702	T_{1S3}	1.512594	0.05139	1413.345
T_{1E4}	0.371361	0.033367	295.7841	$T_{1(o(m))4}$	0.357979	0.024995	285.4706	T_{1S4}	0.736414	0.057766	446.8236
T_{1E5}	0.362621	0.027527	504.4298	$T_{1(o(m))5}$	0.320486	0.020645	483.635	T_{1S5}	0.659285	0.047713	756.9945
T_{1E6}	0.481031	0.034019	297.7121	$T_{1(o(m))6}$	0.420902	0.025514	285.3074	T_{1S6}	0.865855	0.058965	446.5682
T_{1E7}	0.801509	0.0295845	940.7588	$T_{1(o(m))7}$	0.701321	0.022188	901.5606	T_{1S7}	1.442716	0.05128	1411.1383
T_{1E8}	3.932467	0.046335	1603.054	$T_{1(o(m))8}$	3.699479	0.034721	1538.271	T_{1S8}	7.610357	0.080245	2407.728
T_{1E9}	0.801509	0.0298845	940.7588	$T_{1(o(m))9}$	0.701321	0.0221884	901.5606	T_{1S9}	1.442717	0.05128	2407.728
T_{1E10}	5.504698	0.0664712	625.193	$T_{1(o(m))10}$	4.826006	0.050898	599.3288	T_{1S9}	9.784045	0.108657	934.28496
T_{1E11}	8.496157	0.0705618	629.6667	$T_{1(o(m))11}$	7.457911	0.054074	603.6199	T_{1S10}	14.98426	0.115089	940.92237
T_{1E12}	5.727823	0.038455	834.0606	$T_{1(o(m))12}$	5.066987	0.029463	800.1377	T_{1S11}	9.638809	0.062773	1235.6422
T_{1E13}	4.970879	0.0380057	829.1195	$T_{1(o(m))13}$	4.390756	0.02911	795.3884	T_{1S12}	8.44028	0.062856	1228.4902
T_{1E14}	4.765378	0.03755396	828.2811	$T_{1(o(m))14}$	4.207528	0.028756	794.5831	T_{1S13}	8.111063	0.061393	1227.2663

TABLE 9
Relative efficiency of the proposed class of estimators
ESTIMATORS/POPULATIONS

$RE(T_{1(o(m))j}, T_{1Ej})$ members	I	II	III	$RE(T_{1Ej}, T_{1Sj})$ members	I	II	III	$RE(T_{1(o(m))j}, T_{1Sj})$ members	I	II	III
$(T_{1(o(m))1}, T_{1E1})$	0.875	0.6666666	0.9582967	(T_{1E1}, T_{1S1})	0.56	0.60	0.67	$(T_{1(o(m))1}, T_{1S1})$	0.49	0.40	0.64
$(T_{1(o(m))2}, T_{1E2})$	0.7773692	0.75	0.9583333	(T_{1E2}, T_{1S2})	0.5555	0.576932	0.666667	$(T_{1(o(m))2}, T_{1S2})$	0.413872	0.432699	0.638889
$(T_{1(o(m))3}, T_{1E3})$	0.8750003	0.75	0.9583333	(T_{1E3}, T_{1S3})	0.555537	0.569216	0.666667	$(T_{1(o(m))3}, T_{1S3})$	0.436111	0.432691	0.638889
$(T_{1(o(m))4}, T_{1E4})$	0.963964	0.7490934	0.9651316	(T_{1E4}, T_{1S4})	0.154274	0.577623	0.661971	$(T_{1(o(m))4}, T_{1S4})$	0.436111	0.432691	0.638889
$(T_{1(o(m))5}, T_{1E5})$	0.8838043	0.7499909	0.9587756	(T_{1E5}, T_{1S5})	0.550022	0.576928	0.666358	$(T_{1(o(m))5}, T_{1S5})$	0.436111	0.432691	0.638889
$(T_{1(o(m))6}, T_{1E6})$	0.8749997	0.7499926	0.9583332	(T_{1E6}, T_{1S6})	0.555556	0.579355	0.666666	$(T_{1(o(m))6}, T_{1S6})$	0.436111	0.432697	0.638889
$(T_{1(o(m))7}, T_{1E7})$	0.8750000	0.7500008	0.9583333	(T_{1E7}, T_{1S7})	0.555556	0.569231	0.666666	$(T_{1(o(m))7}, T_{1S7})$	0.436111	0.432693	0.638889
$(T_{1(o(m))8}, T_{1E8})$	0.9407527	0.7493471	0.9595877	(T_{1E8}, T_{1S8})	0.516726	0.5774192	0.665795	$(T_{1(o(m))8}, T_{1S8})$	0.436111	0.432687	0.638889
$(T_{1(o(m))9}, T_{1E9})$	0.8750000	0.7424718	0.9583333	(T_{1E9}, T_{1S9})	0.555555	0.5827733	0.390725	$(T_{1(o(m))9}, T_{1S9})$	0.436111	4.326928	0.374445
$(T_{1(o(m)10}, T_{1E10})$	0.8767067	0.7657155	0.9586301	(T_{1E10}, T_{1S10})	0.562542	0.6117506	0.669167	$(T_{1(o(m)10}, T_{1S10})$	0.493184	0.468265	0.493184
$(T_{1(o(m))11}, T_{1E11})$	0.8777981	0.7663370	0.9586340	(T_{1E11}, T_{1S11})	0.567005	0.613107	0.6692	$(T_{1(o(m))11}, T_{1S11})$	0.497716	0.469846	0.641519
$(T_{1(o(m))12}, T_{1E12})$	0.8846270	0.7661594	0.9593280	(T_{1E12}, T_{1S12})	0.594246	0.6126045	0.675002	$(T_{1(o(m))12}, T_{1S12})$	0.525686	0.46935	0.647548
$(T_{1(o(m))13}, T_{1E13})$	0.8832957	0.7659472	0.9593169	(T_{1E13}, T_{1S13})	0.588946	0.61215244	0.674909	$(T_{1(o(m))13}, T_{1S13})$	0.520214	0.468875	0.647452
$(T_{1(o(m))14}, T_{1E14})$	0.8829368	0.7657153	0.9593157	(T_{1E14}, T_{1S14})	0.587516	0.61169838	0.674899	$(T_{1(o(m))14}, T_{1S14})$	0.51874	0.468399	0.647441

TABLE 10
Percentage relative efficiency of $T_{1Ej}, T_{1(o(m))j}, T_{1Sj}$
ESTIMATORS/POPULATIONS

$PRE(T_{1(o(m))j}, T_{1Ej})$ Members	I	II	III	$PRE(T_{1Ej}, T_{1Sj})$ members	I	II	III	$PRE(T_{1(o(m))j}, T_{1Sj})$ members	I	II	III
$(T_{1(o(m))1}, T_{1E1})$	87.5	66.666667	95.82967	(T_{1E1}, T_{1S1})	56.00	60.00	66.67	$(T_{1(o(m))1}, T_{1S1})$	49.00	40.00	63.89
$(T_{1(o(m))2}, T_{1E2})$	77.736922	75	95.83333	(T_{1E2}, T_{1S2})	55.55	57.6932	66.6667	$(T_{1(o(m))2}, T_{1S2})$	41.3872	43.2699	63.8889
$(T_{1(o(m))3}, T_{1E3})$	87.50003	75	95.83333	(T_{1E3}, T_{1S3})	55.553746	56.9216	66.66665	$(T_{1(o(m))3}, T_{1S3})$	43.6111	43.2691	63.88887
$(T_{1(o(m))4}, T_{1E4})$	96.396498	74.909342	96.51317	(T_{1E4}, T_{1S4})	15.4274	57.7623	66.19706	$(T_{1(o(m))4}, T_{1S4})$	43.6111	43.2691	63.88887
$(T_{1(o(m))5}, T_{1E5})$	88.38043	74.999092	95.87756	(T_{1E5}, T_{1S5})	55.002161	57.6928	66.6358	$(T_{1(o(m))5}, T_{1S5})$	43.6111	43.2691	63.88887
$(T_{1(o(m))6}, T_{1E6})$	87.499974	74.999265	95.83332	(T_{1E6}, T_{1S6})	55.55556	57.9355	66.6666	$(T_{1(o(m))6}, T_{1S6})$	43.6111	43.2697	63.88887
$(T_{1(o(m))7}, T_{1E7})$	87.500006	75.000085	95.83333	(T_{1E7}, T_{1S7})	55.55557	56.9231	66.6666	$(T_{1(o(m))7}, T_{1S7})$	43.6111	43.2693	63.88887
$(T_{1(o(m))8}, T_{1E8})$	94.075271	74.934715	95.95878	(T_{1E8}, T_{1S8})	51.67256	57.74192	66.57953	$(T_{1(o(m))8}, T_{1S8})$	43.6111	43.2687	63.88887
$(T_{1(o(m))9}, T_{1E9})$	87.500006	74.247185	95.83333	(T_{1E9}, T_{1S9})	55.55554	58.27733	39.07247	$(T_{1(o(m))9}, T_{1S9})$	43.6111	432.6928	37.44445
$(T_{1(o(m)10}, T_{1E10})$	87.670677	76.571551	95.86301	(T_{1E10}, T_{1S10})	56.25418	61.17506	66.91674	$(T_{1(o(m)10}, T_{1S10})$	49.31842	46.84265	64.1484
$(T_{1(o(m))11}, T_{1E11})$	87.779818	76.633704	95.86341	(T_{1E11}, T_{1S11})	56.70053	61.31074	66.92032	$(T_{1(o(m))11}, T_{1S11})$	49.77162	46.98464	64.15194
$(T_{1(o(m))12}, T_{1E12})$	88.462709	76.615941	95.9328	(T_{1E12}, T_{1S12})	59.4246	61.26045	67.50017	$(T_{1(o(m))12}, T_{1S12})$	52.56861	46.93496	64.7548
$(T_{1(o(m))13}, T_{1E13})$	88.329573	76.594724	95.93169	(T_{1E13}, T_{1S13})	58.89467	61.21524	67.49093	$(T_{1(o(m))13}, T_{1S13})$	52.0214	46.88753	64.7452
$(T_{1(o(m))14}, T_{1E14})$	88.293689	76.571532	95.93158	(T_{1E14}, T_{1S14})	58.75158	61.16984	67.48992	$(T_{1(o(m))14}, T_{1S14})$	51.87394	46.83991	64.74414

TABLE 11
MSEs of $T_{1(o(m))j}$, T_{1Sj} and some existing estimators
ESTIMATORS/POPULATIONS

Some existing Estimators	I	II	III	$T_{1(o(m))j}$ members	I	II	III	T_{1Sj} members	I	II	III
$\bar{y}$	1.0020176	0.0489674	1040.8789	$T_{1(o(m))1}$	0.49	0.02	665.01	T_{1S1}	1.00	0.05	1040.88
Sukhatme(1974)	1.7292328	0.080987	892.668	$T_{1(o(m))2}$	0.608491	0.026058	284.609	T_{1S2}	1.408962	0.060222	445.4749
Sukhatme(1974)	10.501746	0.0860152	2926.9959	$T_{1(o(m))3}$	0.735289	0.022236	902.9702	T_{1S3}	1.512594	0.05139	1413.345
Srivastava (1970)	1.408962	0.0602221	445.47492	$T_{1(o(m))4}$	0.357979	0.024995	285.4706	T_{1S4}	0.736414	0.057766	446.8236
Bahl & Tuteia (1991)	0.088899	0.0562439	860.40421	$T_{1(o(m))5}$	0.320486	0.020645	483.635	T_{1S5}	0.659285	0.047713	756.9945
Bahl & Tuteia (1991)	4.6388749	0.0702579	2176.2696	$T_{1(o(m))6}$	0.420902	0.025514	285.3074	T_{1S6}	0.865855	0.058965	446.5682
Singh and Choudhury(2012)	0.0708128	0.0473914	418.92358	$T_{1(o(m))7}$	0.701321	0.022188	901.5606	T_{1S7}	1.442716	0.05128	1411.1383
Singh & Tailor(2003)	0.8658551	0.0589654	446.56818	$T_{1(o(m))8}$	3.699479	0.034721	1538.271	T_{1S8}	7.610357	0.080245	2407.728
Kadilar & Cingi(2004)	4.5727355	0.0707916	810.41864	$T_{1(o(m))9}$	0.701321	0.0221884	901.5606	T_{1S9}	1.442717	0.05128	2407.728
Al-Omari et al(2009)	0.3935055	0.0675158	588.44038	$T_{1(o(m))10}$	4.826006	0.050898	599.3288	T_{1S10}	9.784045	0.108657	934.28496
Singh & Espejo (2003)	10.979546	0.0872581	6004.3547	$T_{1(o(m))11}$	7.457911	0.054074	603.6199	T_{1S11}	14.98426	0.115089	940.92237
				$T_{1(o(m))12}$	5.066987	0.029463	800.1377	T_{1S12}	9.638809	0.062773	1235.6422
				$T_{1(o(m))13}$	4.390756	0.02911	795.3884	T_{1S13}	8.44028	0.062856	1228.4902
				$T_{1(o(m))14}$	4.207528	0.028756	794.5831	T_{1S14}	8.111063	0.061393	1227.2663

Simulation study

A computer simulation study was conducted using the R-software to examine the theoretical underpinnings of the work and the performances of the proposed class of estimator of population mean based on *ERSS* (even and odd median) and on *SRS*, when ranking is done on a single accompanying variable *X*. Bivariate random observations were generated from a bivariate normal distribution having parameters $\mu_X = 25$, $\mu_Y = 15$, $\sigma_X^2 = \sigma_y^2 = 1$, and $\rho_{XY} = \pm 0.99, \pm 0.90, \pm 0.70$ and ± 0.50. Using 5000 simulations, estimates of MSE's for the estimators in question were computed. We considered sample sizes *m = 3, 4, 5, 6, 7, 8, 9, 10* and *r* = 1 respectively to study the performances of the proposed ratio-cum-product estimators under *ERSSe*, *ERSSo(m)* and on *SRS*, the results is as shown in table 12 and table 13.

Table 12: Simulation Results of MSEs, R.E, AND P.R.E of $T_{1Ej}, T_{1(o(m))j}, T_{1Sj}$

m	T_{1Ej}	$T_{1(o(m))j}$	T_{1Sj}	ρ_{xy}=0.99 RE_1	RE_2	PRE_1	PRE_2
3	2.9333865	2.06268234	5.8654474	0.500113	0.351666667	50.0113	35.1666667
4	2.7537503	2.14067801	5.50656724	0.50008475	0.38875	50.008475	38.875
5	2.648793	2.17701262	5.2968677	0.5000678	0.411	50.00678	41.1
6	2.8414645	2.41970715	5.68228685	0.5000565	0.425833333	50.00565	42.5833333
7	2.7810122	2.4271913	5.5614858	0.500048429	0.436428571	50.00484286	43.6428571
8	2.7904075	2.47976454	5.58034214	0.500042375	0.444375	50.0042375	44.4375
9	2.8344908	2.55399878	5.66855462	0.500037667	0.450555556	50.00376667	45.0555556
10	2.7223081	2.47985453	5.44424704	0.5000339	0.4555	50.00339	45.55
m				ρ_{xy}= - 0.99			
3	4.2260183	3.15466985	8.45000853	0.50012	0.373333333	50.012	37.3333333
4	4.1497192	3.36066766	8.29794484	0.50009	0.405	50.009	40.5
5	4.0674988	3.44874234	8.13382628	0.500072	0.424	50.0072	42.4
6	4.1568577	3.62988682	8.31271792	0.50006	0.436666667	50.006	43.6666667
7	4.1652753	3.71266355	8.32969385	0.500051429	0.445714286	50.00514286	44.5714286
8	4.0902068	3.70130407	8.17967751	0.500045	0.4525	50.0045	45.25
9	4.2857021	3.92348445	8.57071845	0.50004	0.457777778	50.004	45.7777778
10	4.2681986	3.94353161	8.53578271	0.500036	0.462	50.0036	46.2
m				ρ_{xy}=0.90			
3	2.1071912	1.47902634	4.21375024	0.500075	0.351	50.0075	35.1
4	2.1748577	1.68858706	4.34922616	0.50005625	0.38825	50.005625	38.825
5	2.2164454	1.8199812	4.43249197	0.500045	0.4106	50.0045	41.06
6	2.1689496	1.84563767	4.33757385	0.5000375	0.4255	50.00375	42.55
7	2.1083178	1.83893727	4.21636452	0.500032143	0.436142857	50.00321429	43.6142857
8	2.2829573	2.02772278	4.56565782	0.500028125	0.444125	50.0028125	44.4125
9	2.1680773	1.95261735	4.33593787	0.500025	0.450333333	50.0025	45.0333333
10	2.2303415	2.03085754	4.46048218	0.5000225	0.4553	50.00225	45.53
m				ρ_{xy}= - 0.90			
3	0.739528	0.51913824	1.47902634	0.50001	0.351	50.001	35.1
4	3.5715981	2.77330429	7.14308896	0.5000075	0.38825	50.00075	38.825
5	3.5821267	2.94160712	7.16416736	0.500006	0.4106	50.0006	41.06
6	3.6636968	3.11777481	7.32732035	0.500005	0.4255	50.0005	42.55
7	3.6077125	3.14692911	7.21536317	0.5000043	0.436142857	50.00042857	43.6142857
8	3.5225697	3.1288991	7.04508664	0.5000038	0.444125	50.000375	44.4125
9	3.5933394	3.23637946	7.18663093	0.5000033	0.450333333	50.00033333	45.0333333
10	3.5929281	3.27170068	7.18581304	0.500003	0.4553	50.0003	45.53

Table 13: Second simulation Results of MSEs, R.E, AND P.R.E of $T_{1Ej}, T_{1(o(m))j}, T_{1Sj}$ Advocated classes of estimators

m	T_{1Ej}	$T_{1(o(m))j}$	T_{1Sj}	ρ_{xy}=0.70 RE_1	RE_2	PRE_1	PRE_2
3	1.4019339	0.8299883	2.45092732	0.572001433	0.33864256	57.200143	33.8642559
4	1.2389521	0.92921411	2.4899649	0.497578157	0.37318362	49.757816	37.3183618
5	1.1655997	0.932479774	2.47790429	0.470397392	0.37631791	47.039739	37.6317914
6	1.2166386	1.01386546	2.33119944	0.521893808	0.43491151	52.189381	43.4911507
7	1.2393883	1.062332799	2.4332771	0.509349414	0.43658521	50.934941	43.6585212
8	1.2543398	1.097547303	2.47877653	0.50603181	0.44277783	50.603181	44.2777834
9	1.2216126	1.085877852	2.50867955	0.486954415	0.43284837	48.695442	43.2848369
10	1.0912346	0.9651243	2.44322517	0.446636935	0.39502061	44.663694	39.5020612
m				ρ_{xy}= - 0.70			
3	2.3694209	1.57961392	4.73884176	0.5	0.33333333	50	33.3333333
4	2.2138623	1.660396699	4.42772453	0.5	0.375	50	37.5
5	2.239746	1.79179679	4.47949197	0.5	0.4	50	40
6	2.3188298	1.93235819	4.63765966	0.5	0.41666667	50	41.6666667
7	2.2643066	1.940834217	4.52861317	0.5	0.42857143	50	42.8571429
8	2.3252148	2.034562913	4.65042952	0.5	0.4375	50	43.75
9	2.2950216	2.04001916	4.59004311	0.5	0.44444444	50	44.4444444
10	2.237049	2.013344079	4.47409795	0.5	0.45	50	45

m				ρ_{xy}=0.50			
3	0.5937952	0.39586345	1.18759035	0.5	0.33333333	50	33.3333333
4	0.5186365	0.388977396	1.03727306	0.5	0.375	50	37.5
5	0.5542557	0.443404557	1.10851139	0.5	0.4	50	40
6	0.5377956	0.44816302	1.07559125	0.5	0.41666667	50	41.6666667
7	0.5536828	0.474585235	1.10736555	0.5	0.42857143	50	42.8571429
8	0.5536053	0.484404623	1.10721057	0.5	0.4375	50	43.75
9	0.5373568	0.477650448	1.07471351	0.5	0.44444444	50	44.4444444
10	0.5678814	0.511093304	1.1357629	0.5	0.45	50	45
m				ρ_{xy}= - 0.50			
3	1.2824622	0.854974816	2.56492445	0.5	0.33333333	50	33.3333333
4	1.2584511	0.94383832	2.51690219	0.5	0.375	50	37.5
5	1.2824775	1.025981993	2.56495498	0.5	0.4	50	40
6	1.2816042	1.068003536	2.56320849	0.5	0.41666667	50	41.6666667
7	1.2963588	1.111164661	2.59271754	0.5	0.42857143	50	42.8571429
8	1.2984354	1.13613097	2.59687079	0.5	0.4375	50	43.75
9	1.3156134	1.169434112	2.63122675	0.5	0.44444444	50	44.4444444

Conclusion

A class of ratio-cum-product estimators of population mean of the study variable Y have been successfully proposed following information on a single accompanying variable under ERSS as shown in equations (10) and (11) while keeping track record of the SRS version of the proposed estimators as shown in (12) for the purpose of efficiency comparison. Members of the proposed class of the estimators were obtained by varying the scalars that helps in designing the estimator and were presented in table 2, table 3, and table 4 respectively. Their properties such as biases, and MSEs were all derived as can be envisage in equations (26), (27), (28) for biases and (31), (37), (43) for MSEs. The Optimal Mean Square Errors were also calculated to the quadratic polynomial form of Taylor's series approximation and presented in (49), (50) and (51) respectively. Theoretical underpinnings and the condition for which the proposed class of estimator would provide an appreciable gain in efficiency over its counterpart estimator were established and shown in (52). (53), (54), (55), and (56). Empirical and simulation studies were conducted to ascertain the veracity of the theoretical underpinnings of the work. From where it was discovered from the results that the proposed class of estimators based on ERSS provided smaller MSEs, R.E, P.R.E, for all values of the correlation coefficients and sample sizes considered in the work and are therefore adjudged to be more efficient than the corresponding counterpart under SRS. This evidence is presented in table 7 to table 13.

The efficiency of T_{1Ej} , $T_{1(o(m))j}$ T_{1Sj} increases for smaller values of correlation coefficient ρ_{XY} =*-0.80, ±0.70,* and *±0.50* and for smaller values of sample size and decreases for the values of the correlation coefficient ρ_{XY} =*±0.99, ±0.90,* and *+0.80* and as the sample size increases in most cases in table 12 and table 13.

The proposed estimators are approximately unbiased for all cases, correlation coefficients, and sample sizes considered in the simulation study.

The estimator $T_{1(o(m))j}$ performs better than that of T_{1Ej} and T_{2Sj} for all the values of the correlated coefficient and the samples sizes considered in this work.

Therefore, the estimators $T_{1(o(m))j}$ was adjudged to be the most efficient estimators among their brethren T_{1Ej}, T_{1Sj} since it produces the smallest MSEs in all the population, correlation coefficients, and sample sizes considered in this work. The estimators in question were therefore adjudged to be efficient and provide a better alternative whenever efficiency is required.

References

1. Al-Omari AI. Improved ratio-cum-product estimators of population mean under ERSS and SRS. Academic Press; c2019. p. 25-42.
2. Al-Omari AI, Jemain AA, Ibrahim K. New ratio estimators of the mean using simple random sampling and ranked set sampling methods. Rev. Investig. Oper. 2009;30(2):97-108.
3. Ali Asad, Butt MM, Iqbal K, Hanif M, Zubair M. Estimation of population by using a generalized family estimators under classical ranked set sampling. Journals of Research in Mathematics & Statistics. 2021;8(1):1-9.
4. Abu-Dayyeh WA, Samawi HM, Bani-Hani LA. On distribution function estimation using double ranked set samples with application. Journal of Modern Applied Statistical Methods. 2002;1(2):443-451.
5. Abujiya M, Muttlak H. Quality control chart for the mean using double ranked set sampling. Journal of Applied Statistics. 2004;31(10):1185-1201.
6. Agarwal GK, Allende SM, Bouza CN. Double sampling with ranked set selection in the second phase with nonresponse: Analytical results and Monte Carlo experiences. Journal of Probability and Statistics; c2012. Article ID 214959.
7. Asad A, Muhammad MB, Kanwal I, Muhammad H, Muhammad Z. Estimation of Population Mean by using a generalized family of estimators under classical ranked set sampling. Research in Mathematics & Statistics (RSM). 2021;8(1):1-9.
8. Al-jaraha J, Ahmed MS. The class of chain estimator for finite population variance using double sampling. Information and Management Sciences. 2002;13(21):13-18.
9. Al-Nasser AD, Mustafa AB. Robust extreme ranked set sampling. Journal of Statistical Computation and Simulation. 2009;79(7):859-867.
10. Al-Omari AI, Haq A. Improved quality control charts for monitoring the process mean, using double-ranked set sampling methods. Journal of Applied Statistics. 2012;39(4):745-763.
11. Al-Omari AI, Raqab MZ. Estimation of the population mean and median using truncation based ranked set samples. Journal of Statistical Computation and Simulation. 2013;83(8):1-19.

12. Al-Omari AI, Al-Nasser AD. On the population median estimation using robust extreme ranked set sampling. Monte Carlo Methods and Applications. 2012;18(2):109-118.
13. Al-Omari AI, Al-Saleh MF. Quartile double ranked set sampling for estimating the population mean. Economic Quality Control. 2009;24(2):243-253.
14. Al-Omari AI. Estimation of mean based on modified robust extreme ranked set sampling. Journal of Statistical Computation and Simulation. 2011;81(8):1055-1066.
15. Al-Omari AI, Jemain AA, Ibrahim K. New ratio estimators of the mean using simple random sampling and ranked set sampling methods. Investigation Operational. 2009;30(2):97-108.
16. Al-Saleh MF, Al-Shrafat K. Estimation of average milk yield using ranked set sampling. Environ metrics. 2001;12(4):395-399.
17. Al-Saleh MF, Samuh MH. On multistage ranked set sampling for distribution and median estimation. Computational Statistics and Data Analysis. 2008;52(4):2066-2078.
18. Al-Saleh MF, Al-Kadiri MA. Double-ranked set sampling. Statistics and Probability Letters. 2000;48(2):205-212.
19. Al-Saleh MF, Samawi H. On estimating the odds using Moving extreme ranked set sampling. Statistical Methodology. 2010;7(2):133-140.
20. Al-Saleh MF, Ababneh A. Test for accuracy in ranking in moving extreme ranked set sampling. International Journal of Computational Theoretical Statistics. 2015;2(2):67-77.
21. Al-Saleh MF, Al-Kadiri M. Double ranked set sampling. Statistical Probability Letter. 2000;48:205-212.
22. Al-Saleh MF, Al-Omari AI. Multistage ranked set sampling. J Stat. Plan. Inference. 2002;102(2):273-286.
23. Al-Saleh MF, Darabseh M. Inference on the skew normal distribution using ranked set sampling. International Journal of Computational Theoretical Statistics. 2017;4(1):65-76.
24. Al-Saleh MF, Samawi HM. On the efficiency of Monte Carlo methods using steady state ranked simulated samples. Communication in Statistics and Simulation Computation. 2000;29(3):941-954.
25. Al-Saleh MF, Zheng G. Estimation of bivariate characteristics using ranked set sampling. Australia N. Z. Journal Statistics. 2002;44(2):221-232.
26. Bahl S, Tuteja RK. Ratio and product type exponential estimator. Information and Optimization Sciences. 1991;12(1):159-163.
27. Bowley AL. Measurements of precision attained in sampling. Bull. Inst. Internet Statist. 1926;22:1-62.
28. Cochran WG. The estimation of yield of cereal experiments by sampling for the yield of grains to total produce; c1940. p. 2075-2081.
29. Cochran WG. Sampling theory when the sampling units are unequal sizes J Amer. Stats. Assoc. 1942;37(128):199-212.
30. Dell TR, Clutter JL. Ranked set sampling theory with order statistics background, Biometrics. 1972;28(2):545-555.
31. Haq A, Shabbir J. A family of ratio estimators for population mean in extreme ranked set sampling using two auxiliary variables. Sort. 2010;34(1):45-64.
32. Kadilar C, Cingi H. Ratio estimators in simple random sampling. Appl. Math. Comput. 2004;151(3):893-902.
33. Kaur A, Patil G, Sinha AK, Taillie C. Ranked set sampling an annotated bibliography. Environ. Ecol. Stat. 1995;2(1):25-54.
34. Khare BB, Rehman HU. Improved ratio in regression type estimator for population mean using coefficient of variation of the study character in the presence of non-response. International Journals of Technology and Innovations. 2015;14:1-7.
35. McIntyre GA. A method for unbiased selective sampling using ranked sets. Aust. J Agric. Res. 1952;3(4):385-390.
36. Murthy MN. Sampling theory and methods. Statistical publishing society Calcutta India; c1967.
37. Neyman J. One the two different aspects of representative method: The method of Stratified Sampling and the method of purposive selection. J Roy Satist. Soc. 1934;97(4):558-606.
38. Neyman J. Contribution to the theory of sampling human populations. Journal of American Statistical Association. 1938;33(201):101-116.
39. O'zturk O, Wolfe DA. Optimal allocation procedure in ranked set sampling for unimodal and multi-modal distributions. Environmental and Ecological Statistics. 2000;7(4):343-356.
40. O'zturk O. Ranked set sample inference under a symmetry restriction. Journal of Statistical Planning and Inference. 2002;102(2):317-336.
41. Samawi HM, Ahmed MS, Abu-Dayyeh W. Estimating the population mean using extreme ranked set sampling. Biom. J. 1996;38(5):577-586.
42. Singh BK, Choudhury S. Double sampling product –cum- Dual to ratio estimators of finite population mean in sample surveys. International Journals of fiscientic and Engineering Research. 2012;3(4):1-7.
43. Singh HP, Vishwakarma GK. Modified Exponential Ratio and Product Estimators for finite population mean in double sampling. Austrian Journal of Statistics. 2007;36(3):217-225.
44. Singh HP, Espejo MR. On linear regression and ratio-product estimation of a finite population mean. Statistician. 2003;52(1):59-67.
45. Singh HP, Tailor R. Use of known correlation coefficient in estimating the finite population mean. Stat. Transit. 2003;6(4):555-560.
46. Srivastava SK. Two Phase sampling estimator in sample surveys. American Journals of Statistics. 1970;12(1):23-27.
47. Steel RGD, Torrie JH. Principle and procedures of Statistics. McGraw-Hill book Co; c1960.
48. Sukhatme BV. Some ratio-type estimators in two-phase sampling. Journal of the American Statistical Association. 1974;57(299):628-632.
49. Steel RGD, Torrie JH. Principle and procedures of Statistics. McGraw-Hill book Co; c1960.
50. Takahasi K, Wakimoto K. On unbiased estimates of the population mean based on the sample stratified by means of ordering. Ann. Inst. Stat. Math. 1968;20(1):1-31.
51. Watson DJ. The estimation of leaf areas. J Agricultural sci. 1937;27(3):474-483.