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Comparison study on fuzzy two stage chain sampling plan

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Abstract
Acceptance sampling is effective tool in quality assurance. This paper compares the operating characteristics curve of Fuzzy two stage chain sampling plan with two stage chain sampling plan. OC curve of Fuzzy Two stage chain sampling plan is provided by calculating fuzzy acceptance probability of Fuzzy Two stage chain sampling plan using fuzzy Poisson distributions.

Keywords: Acceptance sampling plan, fuzzy set theory, two stage chain sampling plan

Introduction
A Category of Statistical Quality Control is Acceptance sampling plan which deals with the trust on product quality. Acceptance sampling plan are used widely in industries, to make a decision about acceptance or rejection of lots based on the samples. Two stage chain sampling plan is one of the acceptance sampling plan, uses cumulative results of current sample and results of one or more previous samples. Acceptance sampling plan the parameters are not always consider as crisp values in these cases the parameters are considered as linguistic variables.

Application of fuzzy in acceptance sampling plan has done by, Ohta. H, H. Ichihashi (1998) [2] in acceptance SSP using fuzzy membership, Kanagawa. A, Ohta. H (1990) [3] in SSP using fuzzy set theory. Tamaki. F, Kanagawa. A, Ohta. H (1991) [4] in attribute sampling inspection plans using fuzzy. Grzegorzewski. P (2001) [5] in acceptance sampling plans with fuzzy risks and quality levels. Ezzatallah Baloui Jamkhaneh, Bahram Sadeghpour Gildeh (2011) [10] designed OC curve, ASN, ATI, AOQ curves of double sampling plan when proportion defectives are fuzzy number. In 2010 they have discussed SSP and DSP where parameters are fuzzy numbers using fuzzy binomial and fuzzy Poisson distribution, in 2012 they have designed fuzzy acceptance sampling plan in which characteristic curves of single and double sampling plans are derived. Dodge and Stephens introduces a two stage chain sampling plan as an extension of chain sampling plan later Raju. C introduces a construction and selection procedure for two stage sampling plan.

Two stage chain sampling plan
Two stage chain sampling plan are extensions of chain sampling plan proposed by dodge and Stephens in 1966. The procedure make use of the each stages which requires sample size, number of samples over which the cumulative results are taken and the allowable number of nonconforming units applicable to that stage. The operating procedure for two stage chain sampling plan is given below:

- Select a sample of size n randomly from a succeeding lot, and inspect for the defectives in the samples and record the number of defectives also the cumulative defectives in the sample.
- Accept the respective lot associated with the new samples, until nonconformities obtained.
- When k1 succeeding samples are resulted in acceptance, then go for cumulation of defectives in k1 samples up to and not more than k2 samples

- Accept the respective lot associated with the new samples, during the cumulation of defectives in samples k1<k2.
- The second stage of restart period is completed when k2 samples are resulted in acceptance, then go for the cumulation of defectives in k2 samples by adding the results of current samples.

Acceptance Probability of Two stage chain sampling plan is given below:

$$Pa(p) = \left\{ \frac{p_0 + (p_1+p_2){p_0}^{k_1}\left[\frac{1-{p_0}^{k_2-k_1}}{(1-p_0)}\right] + {p_1}^2\, {p_0}^{k_1}\left[\frac{1-{p_0}^{k_2-k_1}}{(1-p_0)^2} - \frac{(k_2-k_1)\,{p_0}^{k_2-k_1-1}}{(1-p_0)}\right]}{1+(p_1+p_2){p_0}^{k_1}\left[\frac{1-{p_0}^{k_2-k_1-1}}{(1-p_0)}\right] + {p_1}^2\, {p_0}^{k_1}\frac{1-{p_0}^{k_2-k_1-1}}{(1-p_0)^2} - \frac{(k_2-k_1)\,{p_0}^{k_2-k_1-1}}{(1-p_0)^2}} \right\} \quad (1)$$

Two stage chain sampling plan using fuzzy environment

The procedure of two stage chain sampling plan under fuzzy environment is as same as the two stage chain sampling plan. Two stage chain sampling plan are defined by the parameters n, p, k_1, k_2, are considered as linguistic variables with the use of fuzzy. Fuzzy Two stage chain sampling plan when the parameters are triangular fuzzy using the Poisson distribution is calculated as below:

Fuzzy fraction defective

Fraction defective p is defined as triangular fuzzy number and α-cut of fraction defective is derived as follows:

$$\tilde{p} = (p_a, p_b, p_c)$$

$$p(\alpha) = \{ p_a + (p_b - p_a)\alpha, p_c + (p_b - p_c)\alpha\} \quad (2)$$

Fuzzy number of events

Number of events n is defined as triangular fuzzy number and α-cut of number of events is derived as follows:

$$\tilde{n} = (n_a, n_b, n_c)$$

$$n(\alpha) = \{ n_a + (n_b - n_a)\alpha, n_c + (n_b - n_c)\alpha\} \quad (3)$$

The parameters k_1,k_2 also be defined as triangular fuzzy number $k_1(\alpha) = (k_{1a}, k_{1b}, k_{1c})$, $k_2(\alpha) = (k_{2a}, k_{2b}, k_{2c})$

$$k_1(\alpha) = \{k_{1a} + (k_{1b} - k_{1a})\alpha, k_{1c} + (k_{1b} - k_{1c})\alpha\} \quad (4)$$

$$k_2(\alpha) = \{ k_{2a} + (k_{2b} - k_{2a})\alpha, k_{2c} + (k_{2b} - k_{2c})\alpha\} \quad (5)$$

Fuzzy probability of acceptance

Acceptance Probability of two stage chain sampling plan with fuzzy parameter is given below:

$$\tilde{P}_i(\alpha) = [f_{1\,(m,y)}(\alpha), f_{r\,(m,y)}(\alpha)] \quad (6)$$

$$f_{1(m,y)}(\alpha) = min \left\{ \frac{\tilde{p}_0 + (\tilde{p}_1+\tilde{p}_2){\tilde{p}_0}^{\tilde{k}_1}\left[\frac{1-{\tilde{p}_0}^{\tilde{k}_2-\tilde{k}_1}}{(1-\tilde{p}_0)}\right] + {\tilde{p}_1}^2\, {\tilde{p}_0}^{\tilde{k}_1}\left[\frac{1-{\tilde{p}_0}^{\tilde{k}_2-\tilde{k}_1}}{(1-\tilde{p}_0)^2} - \frac{(\tilde{k}_2-\tilde{k}_1){\tilde{p}_o}^{\tilde{k}_2-\tilde{k}_1-1}}{(1-\tilde{p}_0)}\right]}{1+(\tilde{p}_1+\tilde{p}_2){\tilde{p}_0}^{\tilde{k}_1}\left[\frac{1-{\tilde{p}_0}^{\tilde{k}_2-\tilde{k}_1-1}}{(1-\tilde{p}_0)}\right] + {p_1}^2\, {p_0}^{k_1}\frac{1-{\tilde{p}_0}^{\tilde{k}_2-\tilde{k}_1-1}}{(1-\tilde{p}_0)^2} - \frac{(\tilde{k}_2-\tilde{k}_1){\tilde{p}_o}^{\tilde{k}_2-\tilde{k}_1-1}}{(1-\tilde{p}_0)^2}} \right\} \quad (7)$$

$$f_{r(m,y)}(\alpha) = max \left\{ \frac{\tilde{p}_0 + (\tilde{p}_1+\tilde{p}_2){\tilde{p}_0}^{\tilde{k}_1}\left[\frac{1-{\tilde{p}_0}^{\tilde{k}_2-\tilde{k}_1}}{(1-\tilde{p}_0)}\right] + {\tilde{p}_1}^2\, {\tilde{p}_0}^{\tilde{k}_1}\left[\frac{1-{\tilde{p}_0}^{\tilde{k}_2-\tilde{k}_1}}{(1-\tilde{p}_0)^2} - \frac{(\tilde{k}_2-\tilde{k}_1){\tilde{p}_o}^{\tilde{k}_2-\tilde{k}_1-1}}{(1-\tilde{p}_0)}\right]}{1+(\tilde{p}_1+\tilde{p}_2){\tilde{p}_0}^{\tilde{k}_1}\left[\frac{1-{\tilde{p}_0}^{\tilde{k}_2-\tilde{k}_1-1}}{(1-\tilde{p}_0)}\right] + {p_1}^2\, {p_0}^{k_1}\frac{1-{\tilde{p}_0}^{\tilde{k}_2-\tilde{k}_1-1}}{(1-\tilde{p}_0)^2} - \frac{(\tilde{k}_2-\tilde{k}_1){\tilde{p}_o}^{\tilde{k}_2-\tilde{k}_1-1}}{(1-\tilde{p}_0)^2}} \right\} \quad (8)$$

Fuzzy operating characteristics curve

Operating characteristics curve plots the values of, probability of acceptance versus proportion defectives. Fuzzy operating characteristic curve is a band with upper and lower bounds. Where these bandwidth depends on the uncertainty level on the proportion defective and the number of samples. The bandwidth and the uncertainty are directly proportional, that is the bandwidth increases when uncertainty increases and decreases when the uncertainty decreases.

Example: In chocolate manufacturing company, some products are identified as defectives after the completion of production. Hence the parameters are considered as linguistic variables these are expressed as fuzzy numbers as follows. Let the proportion defective, sample size, k_1 and k_2 are considered as precisely unknown. And represent these parameters as triangular fuzzy numbers as follows:

$\tilde{p} = (p_a, p_b, p_c)$, $\tilde{n} = (n_a, n_b, n_c)$, $k_1(\alpha) = (k_{1a}, k_{1b}, k_{1c})$, $k_2(\alpha) = (k_{2a}, k_{2b}, k_{2c})$

Let consider $\tilde{p} = (0.0005, 0.001, 0.0015)$, $\tilde{n} = (19, 20, 21)$, $k_1 = (2, 3, 4)$, $k_2 = (6, 7, 8)$.
For finding the probability of acceptance we are using the equations "2, 3, 4, 5", and α cut of the above parameters are calculated as

$\tilde{p} = (0.0005+0.0005\alpha, 0.0015-0.0005\alpha)$,
$\tilde{n} = (19+\alpha, 21-\alpha)$,
$k_1 = (2+\alpha, 4-\alpha)$, $k_2 = (6+\alpha, 8-\alpha)$ respectively.

As a result the Fuzzy probability of acceptance is calculated using Fuzzy Poisson distribution and using the equations "6" are calculated.

$\tilde{P}_i(\alpha) = [f_{1\,(m,y)}(\alpha), f_{r\,(m,y)}(\alpha)]$

$$f_{1(m,y)}(\alpha) = min\left\{\frac{\widetilde{0.0005} + (\widetilde{0.001} + \widetilde{0.0015})\widetilde{0.005}^{\tilde{3}}\left[\frac{1-\widetilde{0.005}^{\tilde{7}-\tilde{3}}}{(1-\widetilde{0.005})}\right] + \widetilde{0.001}^2\,\widetilde{0.005}^{\tilde{3}}\left[\frac{1-\widetilde{0.005}^{\tilde{7}-\tilde{3}}}{(1-\widetilde{0.005})^2} - \frac{(\tilde{7}-\tilde{3})\widetilde{0.005}^{\tilde{7}-\tilde{3}-1}}{(1-\widetilde{0.005})}\right]}{1 + (\widetilde{0.001} + \widetilde{0.0015})\widetilde{0.005}^{\tilde{3}}\left[\frac{1-\widetilde{0.005}^{\tilde{7}-\tilde{3}-1}}{(1-\widetilde{0.005})}\right] + \widetilde{0.001}^2\,\widetilde{0.005}^{\tilde{3}}\,\frac{1-\widetilde{0.005}^{\tilde{7}-\tilde{3}-1}}{(1-\widetilde{0.005})^2} - \frac{(\tilde{7}-\tilde{3})\widetilde{0.005}^{\tilde{7}-\tilde{3}-1}}{(1-\widetilde{0.005})^2}}\right\}$$

$$f_{r(m,y)}(\alpha) = max\left\{\frac{\widetilde{0.0005} + (\widetilde{0.001} + \widetilde{0.0015})\widetilde{0.005}^{\tilde{3}}\left[\frac{1-\widetilde{0.005}^{\tilde{7}-\tilde{3}}}{(1-\widetilde{0.005})}\right] + \widetilde{0.001}^2\,\widetilde{0.005}^{\tilde{3}}\left[\frac{1-\widetilde{0.005}^{\tilde{7}-\tilde{3}}}{(1-\widetilde{0.005})^2} - \frac{(\tilde{7}-\tilde{3})\widetilde{0.005}^{\tilde{7}-\tilde{3}-1}}{(1-\widetilde{0.005})}\right]}{1 + (\widetilde{0.001} + \widetilde{0.0015})\widetilde{0.005}^{\tilde{3}}\left[\frac{1-\widetilde{0.005}^{\tilde{7}-\tilde{3}-1}}{(1-\widetilde{0.005})}\right] + \widetilde{0.001}^2\,\widetilde{0.005}^{\tilde{3}}\,\frac{1-\widetilde{0.005}^{\tilde{7}-\tilde{3}-1}}{(1-\widetilde{0.005})^2} - \frac{(\tilde{7}-\tilde{3})\widetilde{0.005}^{\tilde{7}-\tilde{3}-1}}{(1-\widetilde{0.005})^2}}\right\}$$

$\tilde{P}a(f_{1(m,y)}(\alpha)) = (0.944, 0.980, 0.99)$

The membership function of $P_i(\alpha)$ is shown in the figure1 below.

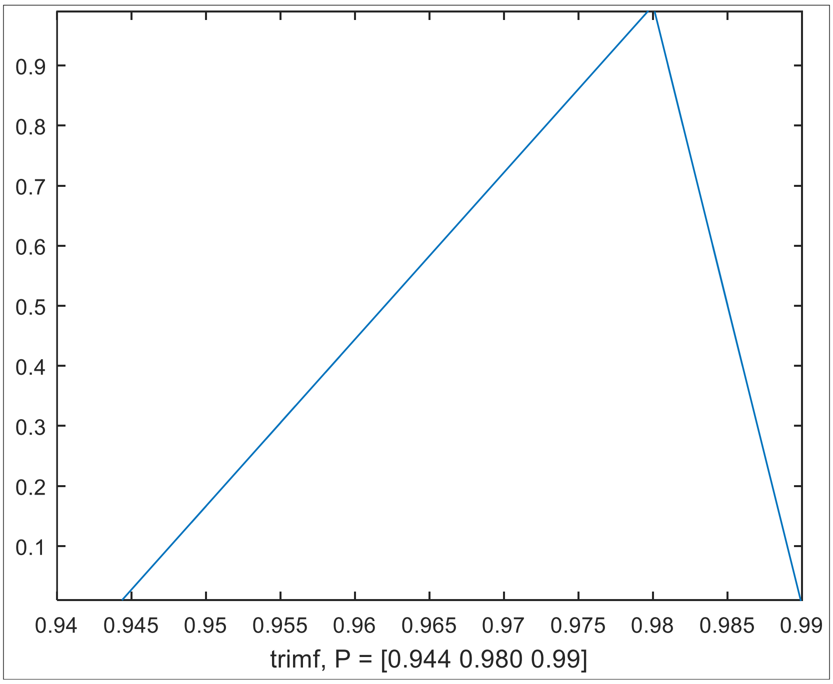

Fig 1: Membership function of $P_i(\alpha)$

The following table 2 represents the value of fuzzy acceptance probability values, based on these values the operating curve is displayed below in the figure 2.

Table 1: Fuzzy Acceptance Probability

P	Pa(p) L	Pa(p) U	Pa(p) (without fuzzy)	P	Pa(p) L	Pa(p) U	Pa(p) (without fuzzy)
(0.0005,0.0015)	0.9774	0.9999	0.99771	(0.485,0.495)	3E-05	1E-04	5.5E-05
(0.002,0.003)	0.9664	0.9979	0.98788	(0.5,0.51)	2E-05	7E-05	4.1E-05
(0.0035,0.0045)	0.9497	0.9963	0.97314	(0.515,0.525)	2E-05	6E-05	3E-05
(0.005,0.015)	0.7576	0.8754	0.89217	(0.53,0.54)	1E-05	4E-05	2.3E-05
(0.02,0.03)	0.5534	0.7908	0.66622	(0.545,0.555)	9E-06	3E-05	1.7E-05
(0.035,0.045)	0.3958	0.6008	0.48027	0.56,0.57)	6E-06	2E-05	1.2E-05
(0.05,0.06)	0.2857	0.4437	0.34612	(0.575,0.585)	5E-06	2E-05	9.2E-06
(0.065,0.075)	0.2075	0.3239	0.25174	(0.59,0.6)	3E-06	1E-05	6.8E-06
(0.08,0.09)	0.1512	0.2365	0.18457	(0.605,0.615)	2E-06	1E-05	5E-06
(0.095,0.105)	0.1103	0.1736	0.13601	(0.62,0.63)	2E-06	8E-06	3.7E-06
(0.11,0.12)	0.0805	0.1282	0.10049	(0.635,0.645)	1E-06	6E-06	2.8E-06
(0.125,0.135)	0.0587	0.0952	0.07435	(0.65,0.66)	1E-06	4E-06	2E-06
(0.14,0.15)	0.0429	0.071	0.05505	(0.665,0.675)	7E-07	3E-06	1.5E-06
(0.155,0.165)	0.0313	0.0531	0.04077	(0.68,0.69)	5E-07	2E-06	1.1E-06
(0.17,0.18)	0.0228	0.0398	0.0302	(0.695,0.705)	4E-07	2E-06	8.3E-07
(0.185,0.195)	0.0167	0.0299	0.02237	(0.71,0.72)	3E-07	1E-06	6.2E-07
(0.2,0.21)	0.0122	0.0224	0.01657	(0.725,0.735)	2E-07	1E-06	4.6E-07
(0.215,0.225)	0.0089	0.0168	0.01228	(0.74,0.75)	1E-07	8E-07	3.4E-07
(0.23,0.24)	0.0065	0.0127	0.0091	(0.755,0.765)	1E-07	6E-07	2.5E-07
(0.245,0.255)	0.0047	0.0095	0.00674	(0.77,0.78)	8E-08	4E-07	1.9E-07
(0.26,0.27)	0.0034	0.0072	0.00499	(0.785,0.795)	6E-08	3E-07	1.4E-07
(0.275,0.285)	0.0025	0.0054	0.0037	(0.8,0.81)	4E-08	3E-07	1E-07
(0.29,0.3)	0.0018	0.004	0.00274	(0.815,0.825)	3E-08	2E-07	7.5E-08
(0.305,0.315)	0.0013	0.003	0.00203	(0.83,0.84)	2E-08	1E-07	5.6E-08
(0.32,0.33)	0.001	0.0023	0.0015	(0.845,0.855)	2E-08	1E-07	4.1E-08
(0.335,0.345)	0.0007	0.0017	0.00111	(0.86,0.87)	1E-08	8E-08	3.1E-08
(0.35,0.36)	0.0005	0.0013	0.00083	(0.875,0.885)	8E-09	6E-08	2.3E-08
(0.365,0.375)	0.0004	0.001	0.00061	(0.89,0.9)	6E-09	5E-08	1.7E-08
(0.38,0.39)	0.0003	0.0007	0.00045	(0.905,0.915)	5E-09	3E-08	1.2E-08
(0.395,0.405)	0.0002	0.0006	0.00034	(0.92,0.93)	3E-09	3E-08	9.2E-09
(0.41,0.42)	0.0001	0.0004	0.00025	(0.935,0.945)	2E-09	2E-08	6.8E-09
(0.425,0.435)	0.0001	0.0003	0.00018	(0.95,0.96)	2E-09	1E-08	5.1E-09
(0.44,0.45)	8E-05	0.0002	0.00014	(0.965,0.975)	1E-09	1E-08	3.8E-09
(0.455,0.465)	6E-05	0.0002	0.0001	(0.98,0.99)	9E-10	8E-09	2.8E-09
(0.47,0.48)	4E-05	0.0001	7.5E-05				

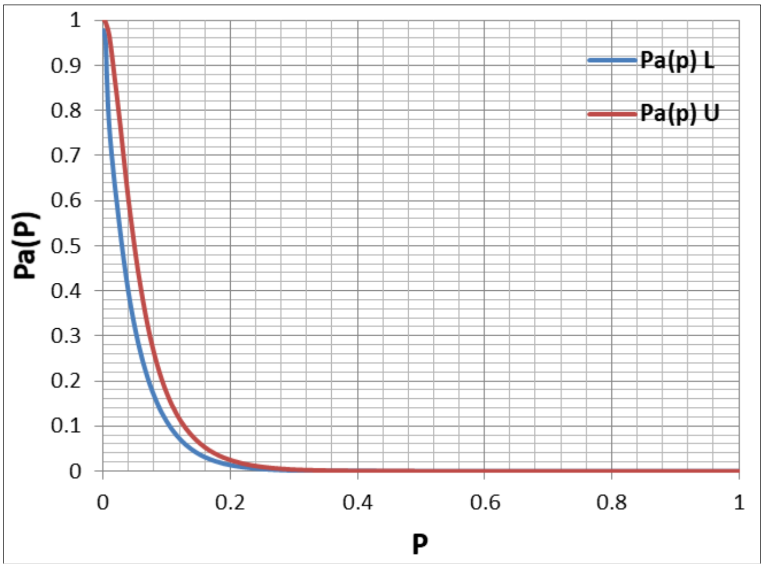

Fig 2: Fuzzy OC curve of Fuzzy two stage chain sampling plan

The probability of Acceptance of two stage chain sampling with Fuzzy parameter and the probability of acceptance two stage chain sampling without fuzzy parameter are compared in the below figure3. The use of fuzzy parameter gives higher and lower possibilities of a respective sampling plan. Bandwidth depends upon the uncertain values in the fuzzy parameters. From below figure3 we can see that the bandwidth decreases at certain point this due to decrease in the uncertainty in the values.

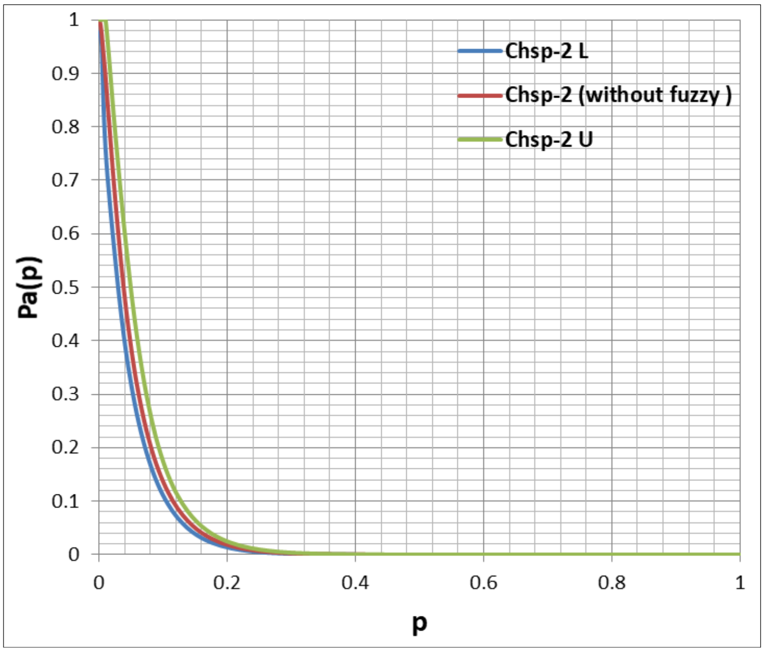

Fig 3: Comparison OC curves of two stage chain sampling plan with and without fuzzy

Conclusion

Mainly in production environments, it may not be easy to define the parameters fraction of nonconforming, acceptance number, or sample size as crisp values. In these cases, these parameters can be expressed by linguistic variables, which can be done with the use of Fuzzy set theory. In this paper the two stage chain sampling plan is discussed with fuzzy parameter and without fuzzy parameter. The fuzzy parameter is considered as triangular fuzzy numbers. Then the probability of acceptance of two stage chain sampling plan in both the conditions are compared and the results shows that the use of fuzzy parameter in sampling plan gives upper and lower bounds in probability of acceptance.

Reference

1. Dubis D, Prade H. Operations of fuzzy number, Int. J syst; c1978.
2. Ohta H, Ichihashi H. Determination of single sampling attribute plans based on membership function. Int. J of Production Research. 1998;26(9):1477-1485.
3. Kanagawa H Ohta. A design for single sampling attribute plan based on fuzzy sets theory, Fuzzy Sets and Systems. 1990;37(2):173-181.
4. Tamaki F, Kanagawa A, Ohta H. A fuzzy design of sampling inspection plans by attributes. Japanese Journal of Fuzzy Theory and Systems. 1991;3(4):315-327.
5. Grzegorzewski P. Acceptance sampling plans by attributes with fuzzy risks and quality levels, Frontiers in Statistical Quality Control, 6: eds., Wilrich P. Th. Lenz H. J Springer, Heidelberg; c2001, p 36-46.
6. Kahraman Cengiz, İhsan Kaya. Fuzzy acceptance sampling plans. Production engineering and management under fuzziness. Springer, Berlin, Heidelberg; c2010, p 457-481.
7. Jamkhaneh, Ezzatallah Baloui, Sadeghpour-Gildeh Bahram, Gholamhossein Yari. Acceptance single sampling plan with fuzzy parameter with the using of Poisson distribution. World Academy of Science, Engineering and Technology. 2009;49:1017-1021.
8. Jamkhaneh Baloui E, Sadeghpour-Gildeh Bahram, Gholamhossien Yari. Important criteria of rectifying inspection for single sampling plan with fuzzy parameter. Int. J Contemp. Math. Sci. 2009;4:1791-1801.
9. Raju C, Narasimha Murthy MN. Two-stage chain sampling plans chsp-(0, 2) and chsp-(l, 2)-part 1. Communications in Statistics-Simulation and Computation. 1996;25(2):557-572.
10. Jamkhaneh Ezzatallah Baloui, Bahram Sadeghpour Gildeh. Chain sampling plan using fuzzy probability theory. Journal of Applied Sciences. 2011;11(24):3830-3838.