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On transmuted skewed Laplace distribution and its application

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Abstract
The aim of this study is to add a shape parameter into the existing 3-parameter skewed Laplace distribution using the method of transmutation and apply the proposed distribution on two life simulated data generated using R-statistical software. The parameters of the proposed distribution are obtained using the method of maximum likelihood estimation. The fitness of the proposed distribution was compared with that of existing skewed Laplace distribution and four parameter Weibull distribution using measures of fitness criteria, and the results obtained showed that the proposed model can be used effectively to model real life situations than the Skewed Laplace Distribution. The results also showed that the Four Parameter Weibull Distribution which is a competing model performed better than the Skewed Laplace Distribution and Transmuted Skewed Laplace Distribution.

Keywords: Transmutation, Weibull distribution, Skewed Laplace distribution, fitness criteria, cumulative distribution function

1. Introduction

The Laplace distribution also called the double exponential distribution, is the distribution of differences between two independent variants with identical exponential distribution [1]. Noted that Laplace distribution arises as tractable "lifetime" models in many areas, including life testing and telecommunications. The Laplace distribution has been found to be very useful in different aspects of human endeavors. The distribution lacks skew parameter. The inability of the Laplace distribution to account for skewness led to the development of the skewed Laplace distribution. Therefore, the Laplace distribution was extended by adding skewness parameter which is used to model the skewness of the data [2] noted that in practical situations in which some skewness are presents, the Skew Laplace distribution is more flexible to model real data. The Skewed Laplace distribution has been used in Economics, Engineering, Finance and Biology. Also, it has been used to describe the logarithm of particles, analyze bacterial sizes [3]. Skewed Laplace distribution was developed to accommodate both skewness and leptokurtic properties of data set.

2. Methods

2.1 The Existing Skewed Laplace Distribution

The following are the probability density function, f(x) and the cumulative density function, F(x) of Skewed Laplace distribution by [2].

$$f(x,\theta,\varepsilon,\mu)=\begin{cases}\left(\frac{1}{\theta+\varepsilon}\right)exp\left(\frac{x-\mu}{\theta}\right), x\le\mu\\ \left(\frac{1}{\theta+\varepsilon}\right)exp\left(\frac{-x+\mu}{\varepsilon}\right), x>\mu\end{cases} \tag{1}$$

$$G(x,\theta,\varepsilon,\mu)=\begin{cases}\left(\frac{\theta}{\theta+\varepsilon}\right)exp\left(\frac{x-\mu}{\theta}\right), x\le\mu\\ \left(1-\left(\frac{\varepsilon}{\theta+\varepsilon}\right)exp\left(\frac{-x-\mu}{\varepsilon}\right)\right), x>\mu\end{cases} \tag{2}$$

$$-\infty<x<\infty, \theta>0, -\infty<x<\infty$$

Where θ is the scale parameter, ε is the skewness parameter, and μ is the location parameter.

2.2 The transmutation method

The transmutation method was proposed by [4] as follows:

$$G(x) = (1+a)F(x) - a[F(x)]^2, |a| \leq 1 \tag{3}$$

Where $G(x)$ is the Cumulative Distribution Function (CDF) of the proposed distribution and $F(x)$ is Cumulative Distribution Function (CDF) of the baseline distribution, a is the shape parameter; when a=0, the Cumulative Distribution Function (CDF) of the proposed distribution will be the same as that of the baseline distribution. The probability density function of the proposed distribution,(x) , can be obtained from (3) by differentiating (3) with respect to the random variable X .

Then;

$$g(x) = \left(\frac{dG(x)}{dx}\right) = G^1(x)(1+a)[F^1(x)] - 2a[F(x)][F^1(x)] \tag{4}$$

But $F^1(x) = f(x), g(x)\ now\ becomes$

$$g(x) = f(x)[(1+a) - 2aF(x)] \tag{5}$$

By putting (1) and (2) in (5), we have:

$$g(x) = \begin{cases} \left[\left(\frac{1}{\theta+\varepsilon}\right) exp\left(\frac{x-\mu}{\theta}\right)\right]\left[(1+a) - \left(\frac{2a\theta}{\theta+\varepsilon}\right) exp\left(\frac{x-\mu}{\theta}\right), x \leq \mu\right] \\ \left[\left(\frac{1}{\theta+\varepsilon}\right) exp\left(\frac{-x+\mu}{\varepsilon}\right)\right]\left[(1-a) + \left(\frac{2a\varepsilon}{\theta+\varepsilon}\right) exp\left(\frac{-x+\mu}{\varepsilon}\right), x > \mu\right] \end{cases} \tag{6}$$

(6) is the Transmuted Skewed Laplace Distribution (TSLD).

2.3 Presentation of Data Set

Two real life datasets are used to demonstrate the numerical applications of the proposed distribution. The data sets are presented with the summary of their statistics.

Data set I: This dataset represent the survival times of 121 patients with breast cancer obtained from a large hospital in a period from 1929 to 1938 [5]. This data set has recently been studied by [6] on survival analysis. Table 1 presents the data set.

Table 1: Survival Times of 121 Patients from 1929 to 1938

0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4, 10.3,11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7, 16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5,17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1, 30.0, 31.0,31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 37.0, 38.0, 38.0, 38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0,41.0, 42.0, 43.0, 43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0, 47.0, 48.0, 49.0, 51.0, 51.0, 51.0, 52.0,54.0, 55.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0,78.0, 80.0,83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0, 111.0, 115.0, 117.0, 125.0,126.0, 127.0, 129.0, 129.0, 139.0, 154.0

Source: [5]

Data Set II: This data represent the tensile strength, measured in GPA, of 69 carbon fibers tested under tension at gauge lengths of 20mm, [7] experiment. This data set has recently been studied by [8]. Table 2 presents the data set. The summary of the data sets are obtained and presented in table 3 and 4.

Table 2: Tensile Strength of Carbon Fibres

1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.140, 2.179, 2.224, 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426, 2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.570, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770, 2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.880, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585

Source: [7]

Table 3: Summary of the Survival Times of 121 Patients with Breast Cancer (Data Set I)

Min.	1st Qu. Median Mean	3rd Qu.	Max.	Skewness	Kurtosis
0.30	17.50 40.00 46.33	60.00	154.00	1.04318	3.402139

Source: [5]

Table 4: Summary of the Tensile Strength of 69 Carbon Fibres (Data Set II)

Min.	1st Qu. Median Mean	3rd Qu.	Max.	Skewness	Kurtosis
1.312	2.098 2.478 2.451	2.773	3.585	-0.0282107	2.940733

Source: [7]

3. Results and Discussion

3.1 Results

Table 5: MLE Estimates and the Criteria Values for the Survival Times of 121 Patients with Breast Cancer Data (Data Set I)

Models Estimates	LL	AIC	BIC	HQIC
TSLD $\hat{\theta} = 3.559 \times 10^{-7}$ $\hat{\varepsilon} = 4.488 \times 10^{-1}$ $\hat{\mu} = 4.873 \times 10^{-2}$ $\hat{a} = 3.742 \times 10^{-2}$	3615.205	1015.48	302.797355	5602.05
SLD $\hat{\theta} = -80.41$ $\hat{\varepsilon} = 5712.54$ $\hat{\mu} = 82573.84$	2794.754	7740.703	425.312163	7242.952
FPWD $\hat{\theta} = 3.636 \times 10^{-4}$ $\hat{\varepsilon} = 3.008 \times 10^{-2}$ $\hat{\mu} = 3.238 \times 10^{-2}$ $\hat{a} = 5.282 \times 10^{-2}$	139.3638	286.7276	295.11	5290.134

Table 6: MLE Estimates and the Criteria Values for the Tensile Strength of 69 Carbon Fibers Data

Models Estimates	LL	AIC	BIC	HQIC
TSLD $\hat{\theta} = -3648.49$ $\hat{\varepsilon} = 75.31$ $\hat{\mu} = 706.77$ $\hat{a} = 1558.99$	49541.67	137350.70	462.684713	99094.89
SLD$\hat{\theta} = -3.062 \times 10^{-2}$ $\hat{\varepsilon} = 3.266 \times 10^{-3}$ $\hat{\mu} = 7.104 \times 10^{-3}$	150029493	415970072	678.71626	300058998
FPWD$\hat{\theta} = 0.02970$ $\hat{\varepsilon} = 0.04188$ $\hat{\mu} = 119.70322$ $\hat{a} = 0.33816$	151.0353	294.0706	287.3683	291.4116

Table 7: Simulation of Sample Size Ranging from 50 to 300 for Different Parameter Values

Sample size	True value			
$a = 0.2, \theta = 0.2, \varepsilon = 0.2, \mu = 0.2$	θ	ε	μ	a
n=50				
Bias	-0.4220	0.2833	-0.2006	0.1933
MSE	0.4255	0.2846	0.2006	0.1987
n=100				
Bias	-0.4300	0.2852	-0.2006	0.1732
MSE	0.4328	0.2832	0.2006	0.1956
n=200				
Bias	-0.4415	0.2629	-0.2006	0.1556
MSE	0.4230	0.2734	0.2006	0.1934
n=300				
Bias	-0.4468	0.2454	-0.2005	0.1341
MSE	0.3416	0.2551	0.2005	0.1822

Table 8: Simulation of Sample Size Ranging from 50 to 300 for Different Parameter Values

Sample size			True value	
a=2,θ=2,ε=2, μ=2	θ	ε	μ	a
n=50				
Bias	136.1037	56.6820	13.8021	-292.0217
MSE	137.0677	81.5719	14.0742	-293.8611
n=100				
Bias	134.0219	57.4254	15.9321	-293.1703
MSE	133.0219	55.4254	13.9321	-295.1703
n=200				
Bias	134.0176	53.1097	13.9801	-295.9100
MSE	123.0076	48.3742	13.9284	-296.1151
n=300				
Bias	131.0481	50.6765	13.0121	-296.4031
MSE	138.5481	46.1106	14.0121	-298.4031

Table 9: Simulation of Sample Size Ranging from 50 to 300 for Different Parameter Values

Sample size	True value			
$a = 0.5, \theta = 0.5, \varepsilon = 0.5, \mu = 0.5$	θ	ε	μ	a
n=50				
Bias	0.0132	0.3269	-0.4991	-0.2166
MSE	0.0137	0.3269	0.4991	0.2155
n=100				
Bias	0.0131	0.3165	-0.6991	0.2165
MSE	0.0135	0.3242	13.9321	0.2143
n=200				
Bias	0.0120	0.3053	-0.8985	0.2143
MSE	0.0133	0.3232	0.4471	0.1163
n=300				
Bias	0.0103	0.3032	-0.9991	-0.9991
MSE	0.0112	0.2314	0.4131	0.1131

3.2 Discussion of Results

Table 5 shows the parameter estimates of the models when fitted on data set I and the measure of fitness criteria values for comparison. The mode of comparison of the proposed distribution with the existing distribution is based on the Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and the Haman Quinn Information Criteria (HQIC). The proposed four parameter transmuted skewed Laplace distribution is compared with two existing distributions, first is the Skewed Laplace Distribution (SLD) of three parameters while the second is the Four Parameter Weibull Distribution (FPWD). Each of these distributions are fitted into the data set I and the estimate of the parameters; log-likelihood (LL), AIC, BIC and HQIC values were obtained. The objective of fitting the proposed four parameter distribution and compared with other distributions is to determine it flexibility in modeling real life situations.

The results obtained show that the proposed four parameter TSLD having the least AIC, BIC and HQIC values has a better fit than the parent distribution (SLD) of less parameter which shows that the proposed model can be used effectively to model real life situations than the SLD. The competing model with the proposed model is the four parameter Weibull distribution. The four parameter Weibull distribution has the least AIC, BIC and HQIC values than the proposed four parameter TSLD which shows that the FPWD can be used to model this real life situation (the survival data used here) effectively than the TSLD. This result shows an impressive analysis about improving on an existing distribution. Once an existing distribution is improved upon by the addition of extra parameter, the new distribution becomes more flexible than the parent distribution. However, it becomes a competitive case when models of equal parameter are compared just like the FPWD has demonstrated better fitness than it counterpart (TSLD).

Table 6 shows the parameter estimates of the models when fitted on data set II and the measure of fitness criteria values for comparison. Each of the distributions is fitted into the data set II and the estimate of the parameters, log-likelihood, AIC, BIC and HQIC values were obtained. The results obtained showed that the proposed four parameter TSLD having the least AIC, BIC and HQIC values has a better fit than the parent distribution (SLD) of less parameter which shows that the proposed model can be used effectively to model real life situations than the SLD. The competing model with the proposed model is the four parameter Weibull distribution. The four parameter Weibull distribution has the least AIC, BIC and HQIC values than the proposed four parameter TSLD which shows that the FPWD can be used to model this real life situation effectively than the TSLD.

The simulation approach for this study is the Monte Carlo simulations approach. The distribution used is the proposed Transmuted Skewed Laplace Distribution (TSLD). Table 7, 8 and 9 show the results of the simulations for different sample size ranging from 50 to 300 samples for different parameter values. The parameter values (True values) are chosen randomly for each sample size. The results obtained in table 7, 8 and 9 show that as the sample size is increasing, the bias and the mean square error values of the proposed distribution are reducing respectively. This shows the flexibility nature of the proposed transmuted skewed Laplace distribution to model real life situations of different nature. This is in line with the findings of [9, 10, 11, 12, 13]. All these researchers found that by extending or improving on the existing distributions that the new distributions perform better than their respective parent distributions in terms of fitness which is one of the major finding of this study.

4. Conclusion

This study expanded the existing skewed Laplace distribution by the injection of a shape parameter into the existing 3-parameter skewed Laplace distribution to make it a 4-parameter distribution. The transmuted skewed Laplace distribution was compared with existing skewed Laplace distribution using measures of fitness criteria such as Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and Haman Quinn Information Criteria (HQIC). Application of this distribution was demonstrated empirically using survival times of 121 patients with breast cancer obtained from a large hospitals in a period from 1929 to 1938 [5] and tensile strength of 69 carbon fibres tested under tension at gauge length of 20mm of the experiment conducted by [7]. Results were generated using R statistical software. The results obtained showed that the proposed four parameter Transmuted Skewed Laplace Distribution (TSLD) having the least AIC, BIC and HQIC values has better fit than the Skewed Laplace Distribution (SLD) of less parameter, which shows that the proposed model can be used effectively to model real life situation than the SLD. Also, the Four Parameter Weibull Distribution (FPWD) which is a competing model outperformed the Transmuted Skewed Laplace Distribution (TSLD).

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