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Optimization of CASP-CUSUM schemes based on truncated Frechet distribution using gauss-legendre integration method

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Abstract
Acceptance sampling plans are among several techniques to control quality, primarily to accept or reject lots of finished products. In destructive products like bullets, batteries, bulbs, etc., these techniques are used so that 100% inspection is impossible. The paper considers CASP-CUSUM schemes under the assumption that the continuous variable under consideration follows a truncated Frechet Distribution. By considering its distribution, the Frechet Distribution plays an important role in Statistical Quality Control, specifically in estimating reliability. Based on numerical results, optimal CASP-CUSUM schemes are suggested.

Keywords: CASP-CUSUM schemes, Type-C OC curves, ARL values, truncated Frechet distribution

1. Introduction
It is imperative for an organization manufacturing products to meet a wide range of customer demands such as availability, safety, warranty, and the full cooperation of all staff members in order to achieve a high level of quality and reliability. There have been several ways in the past to define quality, especially with regard to consumer point of view, system designer point of view, etc. Durability, Safety, low-cost and the degree of satisfaction are the main characteristics which determine the quality of a product from the Consumer's point of view. However, from the perspective of a system designer, the degree of profitability and the degree of low-cost productions determine the quality.

Acceptance Sampling Plans are among the most commonly used elements in control tools. Acceptance Sampling Plans set the ‘Sample Size’ and Criteria for accepting or rejecting a product based on its quality, using statistical principles. In industries that manufacture Bullets, Crackers, Bulbs, and Batteries, among others, where 100% inspection is impossible, a variety of techniques are used. The process of sampling enables a lot of production to be verified to meet the technical requirements. In addition, 100% inspection is too time-consuming and costly to ensure 100% compliance. We inspect or test a sample rather than the entire production lot, and make a decision about accepting or rejecting the whole lot based on that sample. A product's reliability will increase as its quality increases, because from a producer's perspective, a product or machine is more likely to last for a longer time. Better quality is a key factor in determining a product's long-term reliability a longer lifespan. It is evident that the quality of the product depends on how the system works. Usually, the quality of the product varies from time to time and also from one product to another in the same manufacturing process. For instance, the durability of electric bulbs varies from bulb to bulb. Therefore, quality is a random phenomenon.

Hawkins, D.M. 1992 [6] proposed an approximation for ARLs of CUSUM Control Charts that was fast and accurate. Using this approximation, ARLs can be computed for specific parameter values and the out-of-control ARLs for location and scale CUSUM charts. Lonnie [4]. C. Vance and colleagues determined the parameters of a CUSUM Chart based on the average run length of cumulative sum control charts. CUSUM Chart parameters include acceptable and reject able criteria along with desired respective ARL's.

Vardeman. S, Di-ou Ray 1985 [3] introduced CUSUM control charts with the restriction that the quality values were exponentially distributed. Moreover, the phenomenon under study is the occurrence of rare events and the arrival times for a homogenous poison process, which are identically distributed exponential random variables.

Kakoty. S. Chakraborthy A.B. 1990 [5] presented CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal distribution. Truncated distributions are commonly employed in many practical phenomena where there is an upper and lower limit to the variable under study. The sorting procedure, for example, eliminates items that are above or below specified tolerance limits in the production engineering items. Any continuous variable should be first approximated as an exponential variable.

Muhammad Riaz, Nasir Abbas, and Ronald J.M.M Does propose two Runs rule schemes for the CUSUM Charts 2011 [9]. A comparison of CUSUM and EWMA charts is made with the usual CUSUM and weighted CUSUM, with the first initial CUSUM as compared with the usual EWMA schemes. Using this comparison, it was concluded that the proposed schemes perform better for small and moderate shifts.

Akhtar P.Md and Sarma K.L.A.P 2004 [8] presented an optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distributions and evaluated L_0, L'_0 and Probability of Acceptance, as well as optimized CASP-CUSUM Schemes based on numerical results.

It is proposed in this paper that CASP-CUSUM Charts be used when the variable under study follows a Truncated Frechet Distribution. Therefore, it is more important to investigate some of the interesting characteristics of the distribution.

1.1 Frechet distribution

Extreme value theory plays a crucial role in statistical analysis. Generally, extreme data are described by generalized extreme value (GEV) distributions Gumbel, Weibull, and Fréchet distributions are all special cases of GEV distributions.

Definition: A random variable with a non-negative Frechet distribution is said to have the P.D.F is given by

$$f(x;\alpha,\beta) = \alpha\beta[x-\gamma]^{-(\alpha+1)}e^{\{-\alpha(x-\gamma)\}^{\wedge}-\beta}$$

Where $\alpha, \beta, x \geq \gamma$ …… (1.1.1)

1.2 Truncated frechet distribution

It is the ratio of the probability density function of the Frechet distribution to the corresponding cumulative distribution function at B.

$$f_B(x) = \frac{\alpha\beta[x-\gamma]^{-(\alpha+1)}e^{\{-\alpha(x-\gamma)\}^{\wedge}-\beta}}{(1-[1+e^{\{-\alpha(B-\gamma)\}^{\wedge}-\beta}])} \quad \alpha>0, \beta>0,\ x \geq \gamma \quad \text{.... (1.2.1)}$$

Where 'B' is the upper truncated point of the Frechet Distribution.

2. A Description of the Plan and the O.C Curve of Type-C

Beattie [2] proposed a method for the construction of continuous acceptance sampling plans. It consists of a chosen decision interval, "Return interval" with the length h', above which a decision line is taken.

Sum

$$S_m = \sum (X_i - k_1) X_i's(i = 1,2,3.......)$$

is distributed independently, and k_1 is the reference value, plotted on the chart. If the sum lies in the area of the normal chart, the product is accepted and if it lies in the return chart, the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point is to be plotted at maximum, i.e., h+h'.
2. Once the decision line is reached or crossed from above, the next point on the chart should be plotted from the baseline.

A network or a change of specification may be used instead of outright rejection when the CUSUM falls in the return chart. The procedure is summarized below.

1. Begin plotting the CUSUM at Zero.
2. It is accepted when

$$S_m = \sum (X_i - k) < h;$$

When

$S_m < 0$, return cumulative to zero.

3. When $h < S_m < h+h'$ the product is rejected: when S_m crosses h, i.e., when $S_m > h+h'$ and Continues rejecting the product until $S_m > h+h'$ return cumulative to h+h'.

The Type - C, OC function is determined by incoming item quality and the sampling rate in acceptance and rejection regions is equal. Therefore, the probability of acceptance P_A can be calculated as follows:

$$P_A = \frac{L_0}{L_0 + L'_0} \quad \text{… (2.1)}$$

Where,

L_0= Average Run Length in acceptance zone and

L'_0= Average Run Length in rejection zone.

Page E.S [1] has introduced the formulae for L_0 and L'_0 as

$$L_0 = \frac{N_0}{1-P_0} \quad \text{… (2.2)}$$

$$L'_0 = \frac{N'_0}{1-P'_0} \quad \text{… (2.3)}$$

Where

P_0 = Probability for the test starting from zero on the normal chart,

N_0 = ASN for the test starting from zero on the normal chart,

P'_0 = Probability for the test on the return chart and

N'_0= ASN for the test on the return chart

He further obtained integral equations for the quantities P_0, N_0, and N'_0, P'_0 as follows:

$$P_z = F(k_1 - z) + \int_0^h P(y) f(y + k_1 - z)dy, \quad \text{…… (2.4)}$$

$$N_z = 1 + \int_0^h N(y) f(y + k_1 - z)dy, \quad \text{……(2.5)}$$

$$P'_z = \int_{k1+z}^{B} f(y) + \int_0^h P'(y) f(-y + k_1 + z)dy \quad \text{……(2.6)}$$

$$N'_z = 1 + \int_0^h N'(y) f(-y + k_1 + z)dy, \quad \text{……(2.7)}$$

$$F_x = 1 + \int_A^h f(x)dx$$

$$F(k_1 - z) = 1 + \int_A^{k1-z} f(y)dy$$

And z is the distance of the starting of the test in the normal chart from zero.

3. Determination of ARLs and P_A

The Gauss-Legendre integration method helps us determine the integral equations that are related to the probability of acceptance and the average sample number values, which help estimate the value of the probability of acceptance and the average run length of the distribution. We developed computer programs to solve equations (2.4), (2.5), (2.6) and (2.7) and we obtained the following results in Tables (3.1) to (3.28).

Table 3.1 to Table 3.28 represents the values of Average Run Length and Type-C OC Cueves.

Table 3.1: The values of Average Run Length and Type-C OC Cueves

α=1, β= 2,γ = 1,k = 4,h=0.01, h'= 0.01			
B	L_0	L_0'	P_A
4.4	1.815961	1.070827	0.62905937
4.3	2.216361	1.087774	0.67078405
4.2	3.033229	1.109094	0.73225301
4.1	5.516599	1.136063	0.82923167
4.0	524286.43	1.170387	0.99999779

Table 3.2: The values of Average Run Length and Type-C OC Cueves

α=1, β= 2, γ = 1, k = 4, h=0.02, h'= 0.02			
B	L_0	L_0'	P_A
4.4	1.815959	1.151011	0.61205846
4.3	2.216358	1.189504	0.65074807
4.2	3.033223	1.239005	0.70998603
4.1	5.516585	1.303175	0.80891191
4.0	524284.84	1.387081	0.99999737

Table 3.3: The values of Average Run Length and Type-C OC Cueves

α=1, β= 2, γ = 1, k = 4, h=0.03, h'= 0.03			
B	L_0	L_0'	P_A
4.4	1.815957	1.240545	0.59412914
4.3	2.216354	1.305178	0.62937200
4.2	3.033216	1.389713	0.68579357
4.1	5.516571	1.501297	0.78607499
4.0	508395.87	1.650014	0.99999672

Table 3.4: The values of Average Run Length and Type-C OC Cueves

α=1, β= 2,γ = 1,k = 4, h=0.04, h'= 0.04			
B	L_0	L_0'	P_A
4.4	1.815954	1.339423	0.57551097
4.3	2.216351	1.434788	0.60702997
4.2	3.033210	1.561197	0.66019618
4.1	5.516558	1.730397	0.76122426
4.0	508394.34	1.959124	0.99999612

Table 3.5: The values of Average Run Length and Type-C OC Cueves

α=1, β= 2, γ = 1, k = 4, h=0.05, h'= 0.05			
B	L_0	L_0'	P_A
4.4	1.815952	1.447639	0.55642741
4.3	2.216347	1.578320	0.58406877
4.2	3.033204	1.753438	0.63368093
4.1	5.516542	1.990436	0.73485517
4.0	508392.78	2.314343	0.99999547

1. By observing the above tables from (3.1) to (3.5), we see that h, h' increases and the related value of L_0 decreases. Therefore, the size of accepted and rejected zones is inversely related.

Table 3.6: The values of Average Run Length and Type-C OC Cueves

α=2, β= 1, γ = 1, k = 4, h=0.01, h'= 0.01			
B	L_0	L_0'	P_A
4.5	4.529415	1.187613	0.79226738
4.4	5.529062	1.198663	0.82183229
4.3	7.199255	1.210433	0.85606682
4.2	10.548864	1.222974	0.89611011
4.1	20.646657	1.236345	0.94350200

Table 3.7: The values of Average Run Length and Type-C OC Cueves

α=2, β= 1, γ = 1, k = 4, h=0.02, h'= 0.02			
B	L_0	L_0'	P_A
4.5	4.538419	1.427729	0.76069504
4.4	5.542046	1.455435	0.79200577
4.3	7.220275	1.485184	0.82939612
4.2	10.591212	1.517148	0.87470245
4.1	20.795610	1.551515	0.93057203

Table 3.8: The values of Average Run Length and Type-C OC Cueves

α=2, β= 1, γ = 1, k = 4, h=0.03, h'= 0.03			
B	L_0	L_0'	P_A
4.5	4.547882	1.721640	0.72539532
4.4	5.555698	1.771829	0.75819540
4.3	7.242373	1.826026	0.79863852
4.2	10.635736	1.884595	0.84947723
4.1	20.952487	1.947937	0.91493880

Table 3.9: The values of Average Run Length and Type-C OC Cueves

α=2, β= 1, γ = 1, k = 4, h=0.04, h'= 0.04			
B	L_0	L_0'	P_A
4.5	4.557870	2.070705	0.68760931
4.4	5.570128	2.149438	0.72155964
4.3	7.265759	2.234827	0.76476949
4.2	10.682889	2.327510	0.82110381
4.1	21.119188	2.428190	0.89688062

Table 3.10: The values of Average Run Length and Type-C OC Cueves

α=2, β= 1,γ = 1, k = 4, h=0.05, h'= 0.05			
B	L_0	L_0'	P_A
4.5	4.568462	2.476353	0.64848577
4.4	5.585463	2.589942	0.68320316
4.3	7.290659	2.713566	0.72875797
4.2	10.7331924	2.848224	0.79028517
4.1	21.297883	2.995023	0.87671202

2. By observing the above tables from (3.1) to (3.10), we see that h, h' increases and the related value of P_A decreases. Therefore, the parameter values and probability of acceptance is inversely related.

Table 3.11: The values of Average Run Length and Type-C OC Cueves

α=3, β= 1,γ = 1,k = 4, h=0.01, h'= 0.01			
B	L_0	L_0'	P_A
4.5	6.581569	1.504441	0.81394523
4.4	8.112512	1.526479	0.84163492
4.3	10.680685	1.549615	0.87329703
4.2	15.875816	1.573912	0.90980309
4.1	31.929622	1.599435	0.95229709

Table 3.12: The values of Average Run Length and Type-C OC Cueves

α=3, β= 1,γ =1, k = 4, h=0.02, h'= 0.02			
B	L_0	L'_0	P_A
4.5	6.657964	2.290787	0.74401038
4.4	8.227326	2.355027	0.77745711
4.3	10.876840	2.422938	0.81782114
4.2	16.302118	2.494756	0.86727809
4.1	33.643127	2.570737	0.92901229

Table 3.13: The values of Average Run Length and Type-C OC Cueves

α=3, β= 1, γ = 1, k = 4, h=0.03, h'= 0.03			
B	L_0	L'_0	P_A
4.5	6.747345	3.376721	0.66646587
4.4	8.3626 15	3.505384	0.70463562
4.3	11.110318	3.642014	0.75312280
4.2	16.818922	3.787166	0.81621128
4.1	35.841224	3.941438	0.90092569

Table 3.14: The values of Average Run Length and Type-C OC Cueves

α=3, β= 1,, γ = 1, k = 4, h=0.04, h'= 0.04			
B	L_0	L'_0	P_A
4.5	6.856004	4.782225	0.58909338
4.4	8.529043	4.999968	0.63042622
4.3	11.402162	5.232009	0.68546617
4.2	17.482728	5.479410	0.76137197
4.1	38.896331	5.743317	0.87134045

Table 3.15: The values of Average Run Length and Type-C OC Cueves

α=3, β= 1, γ = 1, k = 4, h=0.05, h'= 0.05			
B	L_0	L'_0	P_A
4.5	6.993865	6.530022	0.51714909
4.4	8.743942	6.864408	0.56020921
4.3	11.788059	7.221860	0.62010037
4.2	18.395919	7.604186	0.70753246
4.1	43.615077	8.013356	0.84478795

3. From the tables (3.10) to (3.15), it was observed that the value of L_0 and P_A increase as the value of truncated point 'B' decreases.

Table 3.16: The values of Average Run Length and Type-C OC Cueves

Values of ARL's AND TYPE-C OC CURVES when α=1, β= 2, γ = 1, k = 4, h=0.01, h'= 0.01			
B	L_0	L'_0	P_A
4.4	1.815961	1.070827	0.62905937
4.3	2.216361	1.087774	0.67078405
4.2	3.033229	1.109094	0.73225301
4.1	5.516599	1.136063	0.82923167
4.0	524286.43	1.170387	0.99999779

Table 3.17: The values of Average Run Length and Type-C OC Cueves

α=1, β= 2, γ = 1, k = 4, h=0.02, h'= 0.02			
B	L_0	L'_0	P_A
4.4	1.815959	1.151011	0.61205846
4.3	2.216358	1.189504	0.65074807
4.2	3.033223	1.239005	0.70998603
4.1	5.516585	1.303175	0.80891191
4.0	524284.84	1.387081	0.99999737

Table 3.18: The values of Average Run Length and Type-C OC Cueves

α=1,β= 2, γ = 1, k = 4, h=0.03, h'= 0.03			
B	L_0	L'_0	P_A
4.4	1.815957	1.240545	0.59412914
4.3	2.216354	1.305178	0.62937200
4.2	3.033216	1.389713	0.68579357
4.1	5.516571	1.501297	0.78607499
4.0	508395.87	1.650014	0.99999672

Table 3.19: The values of Average Run Length and Type-C OC Cueves

α=1,β= 2,γ = 1,k = 4, h=0.04, h'= 0.04			
B	L_0	L'_0	P_A
4.4	1.815954	1.339423	0.57551097
4.3	2.216351	1.434788	0.60702997
4.2	3.033210	1.561197	0.66019618
4.1	5.516558	1.730397	0.76122426
4.0	508394.34	1.959124	0.99999612

Table 3.20: The values of Average Run Length and Type-C OC Cueves

α =1, β= 2, γ = 1, k = 4, h=0.05, h'= 0.05			
B	L_0	L'_0	P_A
4.4	1.815952	1.447639	0.55642741
4.3	2.216347	1.578320	0.58406877
4.2	3.033204	1.753438	0.63368093
4.1	5.516542	1.990436	0.73485517
4.0	508392.78	2.314343	0.99999547

Table 3.21: The values of Average Run Length and Type-C OC Cueves

α=1, β= 3, γ = 1, k = 4, h=0.01, h'= 0.01			
B	L_0	L'_0	P_A
4.4	1.431011	1.153281	0.55373436
4.3	1.685115	1.216317	0.58078736
4.2	2.216362	1.309142	0.62866520
4.1	3.858267	1.448540	0.72704106
4.0	349525.3750	1.662395	0.99999529

Table 3.22: The values of Average Run Length and Type-C OC Cueves

=1 β= 3γ = 1k = 4, h=0.02,h'= 0.02			
B	L_0	L'_0	P_A
4.4	1.431011	1.346411	0.51523005
4.3	1.685115	1.505245	0.52818959
4.2	2.216363	1.750597	0.55870562
4.1	3.858267	2.1381814	0.64342540
4.0	349525.375	2.764134	0.99999213

Table 3.23: The values of Average Run Length and Type-C OC Cueves

=1, β= 3,γ = 1,k = 4, h=0.03, h'= 0.03			
B	L_0	L'_0	P_A
4.4	1.431011	1.579393	0.47535523
4.3	1.685115	1.866793	0.47442528
4.2	2.216363	2.324383	0.48810544
4.1	3.858268	3.068972	0.55697047
4.0	349525.406	4.305343	0.99998766

Table 3.24: The values of Average Run Length and Type-C OC Cueves

α=1, β= 3,γ = 1, k = 4, h=0.04, h'= 0.04			
B	L_0	L'_0	P_A
4.4	1.431011	1.852230	0.43585321
4.3	1.685115	2.300969	0.42274951
4.2	2.216363	3.0305218	0.42241507
4.1	3.858268	4.240964	0.47637447
4.0	349525.468	6.2861442	0.99998205

Table 3.25: The values of Average Run Length and Type-C OC Cueves

α=1, β= 3,γ = 1, k = 4, h=0.05, h'= 0.05			
B	L_0	L_0'	P_A
4.4	1.431011	2.164927	0.39795219
4.3	1.685115	2.807781	0.37506213
4.2	2.216363	3.869033	0.36421018
4.1	3.858268	5.654208	0.40560081
4.0	349525.46	8.706662	0.99997508

Table 3.26: The values of Average Run Length and Type-C OC Cueves

α=1, β= 1,γ = 3, k = 4, h=0.01,h'= 0.01			
B	L_0	L_0'	P_A
4.4	3.033264	1.865915	0.61913716
4.3	3.858322	1.957607	0.66340595
4.2	5.516695	2.059084	0.72820162
4.1	10.508399	2.171411	0.82875055
4.0	2396770.00	2.295767	0.99999904

4. From the tables (3.20) to (3.26) it was observed that the Truncated point 'B' changes from 4.4 to 4.0 and P_A is maximum i.e. 0.99999904.

Table 3.27: The values of Average Run Length and Type-C OC Cueves

=1, β= 1,γ = 3, k = 4, h=0.02,h'= 0.02			
B	L_0	L_0'	P_A
4.5	2.541527	2.574718	0.49675628
4.4	3. 033288	2.742408	0.52518129
4.3	3.858357	2.928217	0.56852787
4.2	5. 516753	3.134166	0.63770717
4.1	10.508543	3.362516	0.75758767

Table 3.28: The values of Average Run Length and Type-C OC Cueves

α=1, β= 1, γ = 3, k = 4, h=0.03, h'= 0.03			
B	L_0	L_0'	P_A
4.5	2.541544	3.375372	0.42953863
4.4	3.033312	3.629998	0.45522600
4.3	3.858392	3.912539	0.49651607
4.2	5.516811	4.226209	0.56623214
4.1	10.50869	4.574633	0.69670939

5. By observing the tables from (3.1) to (3.28), we can see that as the value of parameters α, β, γ, k, h and h'of Frechet distribution changes, P_A changes along with it.

4. Results and Conclusions

The hypothetical values of the parameters k, h, and h' are given at the top of each table, By determining optimum truncated point B, we can determine at which value P_A the probability of accepting an item is maximum, and by obtaining ARL's values, we can determine the acceptance zone and rejection zone.

For random variable 'X', we have the values for the truncated point B, and the values for the Type – C OC Curve, that is, P_A are given in columns I, II, III and IV respectively.

The following conclusions can be drawn from the above tables 3.1 to 3.28.

1. It is observed that the Table - 4.1 values of Maximum Probabilities increased as the decreased values of h & h' as shown below the Figure-4.1.

Table 4.1: Maximum Probabilities increased and the decreased values of h & h'

h& h'	P_A
0.01	0.999999
0.02	0.999998
0.03	0.999997
0.04	0.999995
0.05	0.999992

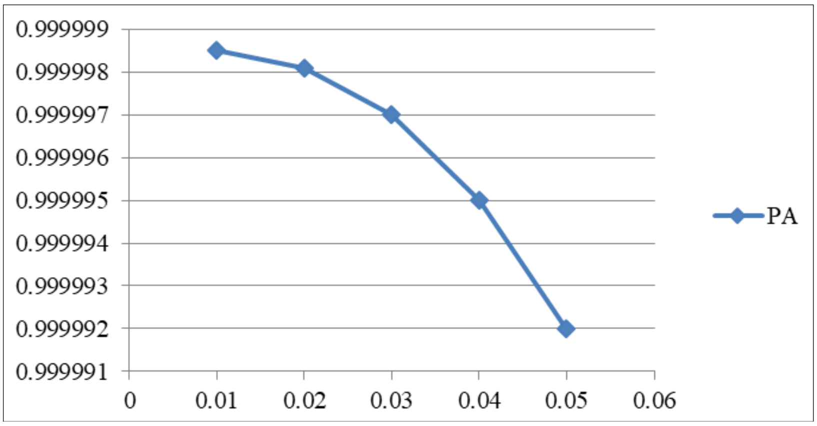

Fig 4.1: Graphical representation of the Probabilities increased and decreased values

Table 4.2: Consolidated Table

B	α	β	γ	k	h	h'	P_A
4.0	1	2	1	4	0.01	0.01	0.99999779
4.0	1	2	1	4	0.02	0.02	0.99999737
4.0	1	2	1	4	0.03	0.03	0.99999672
4.0	1	2	1	4	0.04	0.04	0.99999612
4.0	1	2	1	4	0.05	0.05	0.99999547
4.1	2	1	1	4	0.01	0.01	0.94350200
4.1	2	1	1	4	0.02	0.02	0.93057203
4.1	2	1	1	4	0.03	0.03	0.91493880
4.1	2	1	1	4	0.04	0.04	0.89688062
4.1	3	1	1	4	0.01	0.01	0.95229709
4.1	3	1	1	4	0.02	0.02	0.92901229
4.1	3	1	1	4	0.03	0.03	0.90092569
4.1	3	1	1	4	0.04	0.04	0.87134045
4.0	1	2	1	4	0.01	0.01	0.99999779
4.0	1	2	1	4	0.02	0.02	0.99999737
4.0	1	2	1	4	0.03	0.03	0.99999672
4.0	1	2	1	4	0.04	0.04	0.99999612
4.0	1	2	1	4	0.05	0.05	0.99999547
4.0	1	3	1	4	0.01	0.01	0.99999529
4.0	1	3	1	4	0.02	0.02	0.99999213
4.0	1	3	1	4	0.03	0.03	0.99998766
4.0	1	3	1	4	0.04	0.04	0.99998205
4.0	1	3	1	4	0.05	0.05	0.99997508
4.0	1	1	3	4	0.01	0.01	0.99999904
4.1	1	1	3	4	0.02	0.02	0.75758767
4.1	1	1	3	4	0.03	0.03	0.69670939

From the following Table No.4.2, we can observe the different relationships between the ARL's and Type-C OC Curves with the parameters of the CASP-CUSUM is

$$\begin{bmatrix} B = 4.0 \\ \alpha = 1 \\ \beta = 1 \\ \gamma = 3 \\ k = 4 \\ h = 0.01 \\ h' = 0.01 \end{bmatrix}$$

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