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Stochastic analysis of two unit system

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Abstract
The paper analyzes the sensitivity of two unit system for system parameters uses Regenerative Point Graphical Technique (RPGT). Taking failure and repair rates constant. A state diagram of the system depicting the transition rates is drawn. Expressions for path probabilities mean sojourn times, mean time to system failure, availability of the system, busy period of the server, expected number of server's visits are derived using RPGT. Sensitive analysis of the system is done. Tables and graphs are prepared to compare and draw the conclusion.

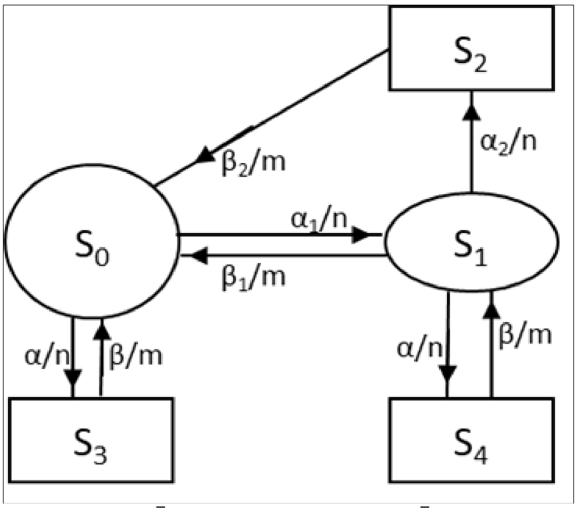
Keywords: RPGT, sensitivity analysis, system parameters

1. Introduction
Individual units play a critical role in the proper functioning of a system, which can be made up of a number of them. A stochastic analysis and sensitivity analysis of a two-unit system (one with parallel subunits and the other with series subunits) is performed in this work. Demonstrated how to solve redundancy allocation in complicated systems using a heuristic method. The major goal of this work, according to Kumar *et al*. (2019) [3], is to evaluate a washing unit in the paper sector using RPGT. Kumar *et al*. (2018, 2017) [8, 9] have calculated the behaviour analysis of a bread organization and edible oil refinery plant. Kumar *et al*. (2019) [4] analysed a cold standby framework with priority for preventive maintenance contains two identical subunits with server failure utilizing RPGT.
The 3:4: G System was described by Kumar *et al*. (2018) [9]. Kumar *et al*. (2017) [8] investigated the urea fertilizer industry's behavior analysis. For the profit analysis, we used two units, A and B, in which unit 'A' has sub units in parallel, so if one or more sub units fail, the system works at a reduced capacity, and if the number of sub unit failures is greater than a predefined number or else, the system is considered to be in a failed state. Because the sub units in Unit 'B' are connected in series, if any of them fail, the unit will fail, putting the entire system in a failed condition.
The fuzzy idea is utilized to determine a unit's failure/working condition. A transition diagram of the system is built using exponential (constant) failure rates, general & independent repair rates, and various probabilities to determine Primary circuits, Secondary circuits, Tertiary circuits, and Base state. To determine system parameters, the problem is solved using RPGT. Graphs and tables are used to illustrate system behavior. For profit analysis of the system, specific cases are taken for various repair and failure rates.

2. Assumptions and Notations
- Failures and repairs are statistically independent.
- Repair is perfect and repaired system is as good as new one.
- Nothing can fail when the system is in failed state.

α_i/n: Constant failure rate of the servers
β_i/n: Constant repair rate of the units

3. Transition Diagrams
Taking into consideration the above assumptions and notations the Transition Diagram of the system is given in Fig 1.



$S_0 = AB$, $S_1 = \bar{A}B$, $S_2 = aB$, $S_3 = Ab$, $S_4 = \bar{A}b$

Fig 1: Transition Diagrams

3.1. Model Description

Both units are initially in a good state denoted by S0, but when unit 'A' fails partially, the system enters the reduced state S1 where the server begins repairing the partially failed unit, and when it is fully repaired, the system returns to the full capacity working state S_0. If no further failures occur, the system enters the state S_2, from where it returns to the full capacity working state S_0 after repair. If in state S_0 unit 'B' fails than the system enters state S_3 where it is repaired and the system reenters the S_0 and in reduced state S_1 if the unit 'B' fails in the organization enters the S_4 where it is restored and the organization re-enters the state S_1. Disappointment and repair rates are characterized along the pathways of the transition diagram as shown in fig. 1.

3.1.1. Transition Probability and the Mean sojourn times

Table 1: Transition Probabilities

$q_{i,j}(t)$	$P_{ij} = q^*_{i,j}(0)$
$q_{0,1}(t) = (\alpha_1/n)e^{-\left[\frac{(\alpha_1+\alpha)}{n}\right]t}$ $q_{0,3}(t) = (\alpha/n)e^{-\left[\frac{(\alpha_1+\alpha)}{n}\right]t}$	$p_{0,1} = \alpha_1/(\alpha_1+\alpha)$ $p_{0,3} = \alpha/(\alpha_1+\alpha)$
$q_{1,0}(t) = (\beta_1/m)e^{-\left[\frac{(\beta_1)}{m}+\frac{(\alpha+\alpha_2)}{n}\right]t}$ $q_{1,2}(t) = (\alpha_2/n)e^{-\left[\frac{(\beta_1)}{m}+\frac{(\alpha+\alpha_2)}{n}\right]t}$ $q_{1,4}(t) = (\alpha/n)e^{-\left[\frac{(\beta_1)}{m}+\frac{(\alpha+\alpha_2)}{n}\right]t}$	$p_{1,0} = n\beta_1/(n\beta_1+m\alpha+m\alpha_2)$ $p_{1,2} = m\alpha_2/(n\beta_1+m\alpha+m\alpha_2)$ $p_{1,4} = m\alpha/(n\beta_1+m\alpha+m\alpha_2)$
$q_{2,0} = \beta_2 e^{-\beta_2 t}$	$p_{2,0} = 1$
$q_{3,0} = \beta e^{-\beta t}$	$p_{3,0} = 1$
$q_{4,1} = \beta e^{-\beta t}$	$p_{4,1} = 1$

Table 2: Mean Sojourn Times

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_0(t) = e^{-\left[\frac{(\alpha_1+\alpha)}{n}\right]t}$	$\mu_0 = n/(\alpha_1+\alpha)$
$R_1(t) = e^{-\left[\frac{(\beta_1)}{m}+\frac{(\alpha+\alpha_2)}{n}\right]t}$	$\mu_1 = mn/(n\beta_1+m\alpha+m\alpha_2)$
$R_2(t) = e^{-(\beta_2/m)t}$	$\mu_2 = m/\beta_2$
$R_3(t) = e^{-\left(\frac{\beta}{m}\right)t}$	$\mu_3 = m/\beta$
$R_4(t) = e^{-\left(\frac{\beta}{m}\right)t}$	$\mu_4 = m/\beta$

4. Evaluation of Parameters

The Mean time to system failure and all the key parameters of the system under steady state conditions are evaluated, applying Regenerative Point Graphical Technique (RPGT) and using '0' as the base-state of the system as under:

$V_{0,0} = 1$ (Verified)
$V_{0,1} = (0,1)/\{1-(1,4,1)\}$
$= p_{0,1}/(1-p_{1,4}p_{4,1})$
$V_{0,2} = (0,1,2)/\{1-(1,4,1)\}$
$= p_{0,1}p_{1,2}/(1-p_{1,4}p_{4,1})$
$V_{0,3} = (0,3)$
$= p_{0,3}$
$V_{0,4} = (0,1,4)/\{1-(1,4,1)\}$
$= p_{0,1}p_{1,4}/(1-p_{1,4}p_{4,1})$

MTSF (T₀): The regenerative un-failed states to which the system can transit (initial state '0'), before entering any failed state are: 'i' = 1 taking 'ξ' = '0'.

$$\text{MTSF}(T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}i\right)\right\}\mu i}{\prod_{m_1\neq\xi}\{1-V_{\overline{m_1m_1}}\}}\right\}\right] \div \left[1-\sum_{sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}\xi\right)\right\}}{\prod_{m_2\neq\xi}\{1-V_{\overline{m_2m_2}}\}}\right\}\right]$$

$T_0 = (V_{0,0}\mu_0+V_{0,1}\mu_1)/\{1-(0,1,0)\}$

Availability of the System (A_0): The regenerative states at which the system is available are 'j' = 0,1 and the regenerative states are 'i' = 0 & 1 taking 'ξ' = '0'

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\}fj,\mu j}{\prod_{m_1\neq\xi}\{1-V_{\overline{m_1m_1}}\}}\right\}\right] \div \left[\sum_{i,sr}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\prod_{m_2\neq\xi}\{1-V_{\overline{m_2m_2}}\}}\right\}\right]$$

$A_0 = \left[\sum_j V_{\xi,j}, f_j, \mu_j\right] \div \left[\sum_i V_{\xi,i}, f_j, \mu_i^1\right]$

Busy Period of the Server: The regenerative states where server is busy are j = 1, 2, 3, 4 and regenerative states are 'i' = 0 & 1, taking ξ = '0', the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\},nj}{\prod_{m_1\neq\xi}\{1-V_{\overline{m_1m_1}}\}}\right\}\right] \div \left[\sum_{i,sr}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\prod_{m_2\neq\xi}\{1-V_{\overline{m_2m_2}}\}}\right\}\right]$$

$B_0 = \left[\sum_j V_{\xi,j}, n_j\right] \div \left[\sum_i V_{\xi,i}, \mu_i^1\right]$

$B_0 = (V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,3}\mu_3+V_{0,4}\mu_4)/D$

Expected Fraction of Inspections by the repair man: The regenerative states where the repair man to do repair are j = 1, 3, 4 the regenerative states are i = 0 & 1, Taking 'ξ' = '0', the number of visit by the repair man is given by

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\}}{\prod_{k_1\neq\xi}\{1-V_{\overline{k_1k_1}}\}}\right\}\right] \div \left[\sum_{i,sr}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\prod_{k_2\neq\xi}\{1-V_{\overline{k_2k_2}}\}}\right\}\right]$$

$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}, \mu_i^1\right]$

5. Particular Cases: $\alpha = \alpha_i$; $\beta = \beta_i$

Profit Function: The system can be done by utilizing profit function

$P_0 = D_1A_0 - (D_2B_0 + D_3V_0)$
$= D_1A_0 - D_2B_0 - D_3V_0$

Where: $D_1 = 1200$;
$D_2 = 100$;
$D_3 = 200$

Table 3: Profit Function (P_0)

β α	0.5	0.6	0.7
0.1	892	934	964
0.2	645	732	877
0.3	363	384	597

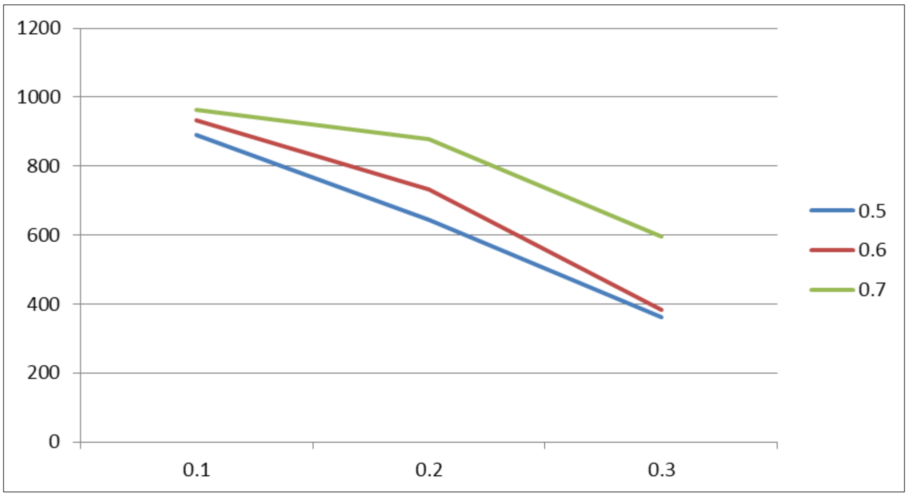


Fig 2: Profit Function

6. Conclusion

From the fig. 2 and table 3, it is seen that profit increase with the expansion in repair rates for example profit is directly proportional to the repair rates of units and profit diminishes with the expansion in the estimations of failure rates of units, henceforth profit function is conversely proportional to the disappointment/failure rates, in this way for optimum profit function esteems repairman ought to be effective concerning as could reasonably be expected and failure rates of units ought to be as small as these can be for example good quality units and over-structured units ought to be utilized for better outcomes.

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