The supremacy of GARCH modelling over ARIMA modelling of Nigerian export commodity price index series

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Abstract
This study looked into the supremacy of GARCH modelling over ARIMA modelling of Nigerian export commodity price index series. Data was collected from the Central Bank database from January 2000 to December 2020 and was analyzed using EVIEWS statistical software. The time plot of Nigerian export commodity price index (figure 1) shows that the series changes over time indicating that there is clearly a secular trend in it. By the Augmented Dickey-Fuller Test in Table 1, the series is non-stationary with p = 0.8989 > 0.05. Therefore there was need for differencing (see figure 2). After the series was differenced, a stationarity test was ran for the differenced series. The Augmented Dickey Fuller test of Table 2 shows that the differenced series is stationary with p = 0.0000 < 0.05, indicating that the trend has been removed through differencing. The EVIEWS statistical software was used in estimating the model. The correlogram of Figure 3 has spikes at lag 1 for the Autocorrelation function as well as for the partial autocorrelation function suggesting an ARMA (1,1). Then ARMA (1,1) was estimated for the differenced series (table 3) which showed that the series does not follow an ARMA (1,1) as suspected, the coefficients of the parameters are both non-significant statistically. After the failure of the ARIMA (1,1,1) we fitted the ARIMA (1,1,0) and ARIMA (0,1,1) to the original data. ARIMA (1, 1, 0), ARIMA (0,1, 1) and GARCH (1,1) having met the required assumptions as the appropriate model was used to estimate its parameters. From the result obtained, it was reveal that by the AIC, SC and HQ terms, the GARCH (1,1) model when compared to other models has proven to be adequate model for the modelling of differenced Nigerian Export Commodity Price Index series.

Keywords: Modeling, ARIMA, GARCH, export commodity, price index

1. Introduction
Supremacy is the state of being superior to all others in authority, power, or status. A price index is a scale that is used to measure changes in price levels. This is the most useful gadget for determining price changes. In so many countries, price indexes are employed to monitor inflation, with a particular index focused on the valuation of a set regarding commodities and businesses significant to a specific economic sector. It is a tool to evaluate inflation by measuring the prices that vary with time. Inflation can be calculated in many ways (or deflation). The consumer price index spotlight goods and services bought by consumers; the producer price index center on items bought by businesses, and a Gross Domestic Product string-type index monitors price movements across the standard of living. A percentage-based measure of price changes is known as a price index, which can be based on the prices of either a single item or on the prices of a chosen group of items known as a market basket. The consumer price index, for instance, includes certain hundred goods and services such as automobiles, electricity, and rent. A market basket provides an extra thorough means of inflationary force than a single article because it includes a diversity of goods and services. It provides an extra comprehensive gauge of inflationary force than a single article. Export price indexes track changes in the prices of goods and services traded on the global market. They are used to deflate import and export values. The import price index is also used to forecast domestic inflation in the future.
The AR model foretells upcoming behavior established on former behavior, it is employed for predicting some relationship among worth in a time series and the worth that lead and replace them. Moving average model is a common approach to measuring univariate time series It stipulates that the dependent variable confide linearly on the ongoing and several former conditions of a presumed term. If a data Xi is given, the ARMA model is an instrument for perception and, maybe, foretelling projected values in the series. The AR component comprises reverting the variable on its own lagged (ie, former) values. The MA parts require modeling the error term as a linear organization of error terms according to contemporaneously and add at different times in the past. The model is normally regarded as the ARMA (P, q) model where P denotes the order of the AR component and q is the order of the MA component. Autoregressive moving average ARMA or autoregressive integrated moving average ARIMA models are used to find, the best-fitted model to series. The past values of the series are used to make forecasts of present values. Researchers have proposed different models to describe and foretell these patterns in volatility. A vector of stochastic variables is heteroscedastic (from ancient Greek hetero “different” and skedasis “dispersion”) if the variability of the stochastic disturbance is varied across components of the vector. Variability could be measured using the variance or other statistical dispersion tools.

The ARCH “model was developed in 1982 by Robert F Engle to describe mainly an approach to estimate volatility in financial markets. ARCH models are mostly used in situations in which there may be short periods of increasing variances, though could also describe gradually increasing variations (Engle, 2001). Engle (1982) [8] modeled the heteroscedasticity by relating the conditional (variance of the disturbance term) to the linear combination of the squared disturbances in the recent past”. The ARCH model was indiscriminate by Bollerslev (1986) [3] via modeling the conditional-change to hinge on its lagged values as well as squared-lagged values of disturbance, which is called generalized-autoregressive conditional heteroscedasticity (GARCH). GARCH is a non-linear model used for predicting future variance using past variances and predictions of variances as predictors. Expanded literature has strongly established GARCH methods in the macro econometrics literature (Bollerslev et al, 1992) [3] finding that past knowledge indeed explains conditional heteroskedastic designs in high frequency (e.g., daily, weekly, or even monthly) data. After the work of Engle (1982) [8] and Bollerslev (1986) [3], various modifications of the GARCH model have been disclosed to model volatility.

2. Review of related literatures

According to Yang and Brossen (1992, 1993) [13], the generalized ARCH (GARCH) models surpass different advanced substitutes that represent stochastic products price series. Muhammed, Bolarinwa, and Aja (2019) [9] used exponential smoothing procedures for predicting the Nigeria consumer price index. And from the result of their study, it was observed that the Nigeria consumer price index will grow gradually over time. Thabani (2018) studied modelings and forecasting inflation in Kenya: Recent insights from ARIMA and GARCH analysis, the result of their study show that annual inflation in Kenya is anticipated to rise continually.

Box and Jenkins's approach invented a systematic class of models called Autoregressive integrated moving Average models to check time-corresponding forecasting and modeling (Shumway and Stoffer, 2010). Etuk and Toyo (2017) [15] investigated SARIMA modeling of Nigerian food consumer price indices from January 2003 to Dec 2014. From their observation seasonality existed in the trend of the variable which led to testing for serial correlogram using correlogram of the residuals later the variables were estimated and a Multiplicative Seasonal Autoregressive Integrated Moving Average (SARIMA) (0, 1, 0) x (1, 1, 1)12 was hence applied to the time series variable. They concluded that (SARIMA) (0, 1, 0) x (1, 1, 1)12 model of NFCPI is adequate because virtually all correlations are zero.

Amadi, Nwagor, and Gideon (2013) [11] studied SARIMA Modeling of Nigerian Consumer Price Index (CPI) Series between 2003-2011 and they concluded that since the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the differenced series showed a seasonal nature then the model proposes a SARIMA (0,1,1) x (0,1,1)12 model. Nyoni and Nathaniel (2018) [10] modeled rates of Inflation in Nigeria: Using an ARMA, ARIMA, and GARCH models approaches. They came to conclusion that ARMA (1, 0, 2) model is the optimum model that fits the series grounded on the minimum Theil’s U forecast estimation statistics. From the revelation of the research, it is taken into cognizance that the inflation in Nigeria is likely to increase to about 17% yearly by end of 2021 which in turn is expected to exceed that extent in 2027. Deebom and Essi (2017) Modeled the Volatility of Price of Nigerian Crude Oil Markets with GARCH Model: 1987-2017. Their statistical analysis showed that the businesses were assured of their investment and other trade-related exercises. Concerning that, the probabilities of gains were higher than losses. It was also revealed that the first-order symmetric GARCH model (GARCH, (1,1)) is more suitable modeled than the first order Asymmetric GARCH model (EGARCH (1,1)), they also advised investors or marketers to be mindful in trading in a highly volatile period. And they recommended that the Government should look for new approaches to change the economy from crude oil dependence to non-crude oil sectors like manufacturing, mining, and agriculture. Deebom et al. (2021) [7] used Symmetric and Asymmetric GARCH Models to model Price Volatility in Nigerian Crude Oil Markets. The results of the estimated models showed that using the EGARCH model to estimate conditional volatility has a strong asymmetric characteristic to news that sensitivity Asymmetric GARCH model (EGARCH (1,1)) in student’s-t error postulate gave a more considerable fit than the first order Symmetric GARCH models. Also, Using EGARCH (1,1) models with their similar error distribution in predicting crude oil price was found that the bigger the size of the approximated news elements of the model, the higher the negative news related with the high influence of volatility.

Christopher B. Barrett (1995) [5] introduced the GARCH modeling techniques to agricultural price analysis because commodity prices often exhibit volatility clustering. His work reveals the value of adding constructional regressors to the subject variance equation of GARCH models of commodity prices. Aviral et al. (2014) [2] studied the Causality between consumer price and producer price: Evidence from Mexico from January 1981 to March 2009. They made use of Squared Wavelet Coherence (WTC) or the cross-wavelet coherence system which showed a strong duplex relationship between the variables. Another relevant finding is that through cyclical movements each variable is affected by the other, on the other hand, anti-cyclical movements do not affect the variables.

~12~
3. Method of Analysis

The data employed in this research was drawn and extracted from the CBN database from January 2000 to December 2020 that comprises the monthly Nigerian Export Commodity Price Index Series and the Computer Software: eviews-10 was used to estimate the parameter of the model.

3.1 Mathematical Formulation

The Box-Jenkins ARIMA model and GARCH model were used in this study. Autoregressive moving average ARMA or autoregressive integrated moving series is used to make forecasts of present values. It is a straight statistical approach and is most mighty for designing univariate data. The AR (p) bases its forecasts of the past observation from the series $X_t$ of the order (p) of past observation of series and a random error $W_t$. The moving average models MA (q) create forecasts of the past error $W_t$ of order (q) of past disturbances terms of variable predicted errors $W_1, W_2, ..., W_q$. A composition of AR of order (p) and MA of order (q) generates more flexible models which include ARMA (p, q) models, ARIMA, and the seasonal ARIMA.

The models are discussed below:

3.1.1 AR (p) Model

An AR (p) model is a model where precise lagged values of $X_t$ are employed as predictor variables. Theoretical AR (p) is written as follows:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \epsilon_t$$

(3.1)

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \epsilon_t$$

(3.2)

Where $X_{t-1}$, $X_{t-2}$, ..., $X_{t}$ represent past series observations (lags), $\varphi_1$, $\varphi_2$, ..., $\varphi_p$ represent parameters, $c$ represent a constant, and the random variable $\epsilon_t$ is error term.

Using backshift operator as

$$(1- \varphi_1 B - \varphi_2 B^2 - \ldots - \varphi_p B^p) X_t = \epsilon_t$$

(3.3)

3.1.2 Moving Average model

An MA model of with q as the order, represented as MA (q) is a process of form:

$$X_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_q \epsilon_{t-q}$$

(3.4)

$$X_t = \mu + \epsilon_t + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$

(3.5)

This can be equivalently written in terms of back shift operator B, given as:

$$X_t = (1 + \theta_1 B + \ldots + \theta_q B^q) \epsilon_t$$

(3.6)

Where $\theta_1$, $\theta_2$, $\ldots$, $\theta_q$ are estimated parameters of the model, $\mu$ is the expectation of $X_t$ (often assumed to equal to zero) and the $\epsilon_t, \epsilon_{t-1}, \ldots$, $\epsilon_{t-q}$ are white noise (error) terms.

3.1.3 Autoregressive Moving Average (ARMA) model

ARMA (p,d) is a mixture of AR process of orders p, AR(p), and MA of order q, MA(q). ARMA (p,q) is of the pattern:

$$X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \ldots + \Phi_p X_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_q \epsilon_{t-q}$$

(3.7)

$$X_t = \Phi_1 X_{t-1} - \Phi_2 X_{t-2} - \ldots - \Phi_p X_{t-p} = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_q \epsilon_{t-q}$$

(3.8)

$$X_t = c + \epsilon_t \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$

(3.9)

Using backshift operator we have:

$$(1- \Phi_1 B - \Phi_2 B^2 - \ldots - \Phi_p B^p) X_t = (1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q) \epsilon_t$$

(3.10)

$$\Phi(B) X_t = \theta(B) \epsilon_t$$

(3.11)

Where $\Phi(B) = (1- \Phi_1 B - \Phi_2 B^2 - \ldots - \Phi_p B^p)$ and $\theta(B) = (1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q)$

Generally ARMA model is,

$$X_t = c + \epsilon_t \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$

~13~
3.14 Autoregressive Integrated Moving Average (ARIMA) Model
Suppose Y_t is non-stationary in mean, the idea is to build an ARMA model on the series X_t definable as the result of the operation of differencing the series of d times (in general d= 1): X_t= ∇^dY_t.

The ARIMA model called ARIMA (p,d,q) model is written as

$$\Phi(B) \nabla^d X_t = \theta(B) \varepsilon_t$$  \hspace{1cm} (3.12)

Whereas $$\Phi(B) = (1 - \Phi_1 B - \Phi_2 B^2 - \ldots - \Phi_p B^p)$$ and $$\theta(B) = (1 + \theta_1 B + \theta_2 B^2 + \ldots + \theta_q B^q)$$.

In terms of y, the ARIMA (p d q) is stated below:

$$X_t = \mu + \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p} - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q}.$$  \hspace{1cm} (3.13)

3.1.6 ARCH Model
For the variance of a time series a well known model called ARCH (Autoregressive Conditionally Heteroscedastic) model is employed. ARCH model describes a changing, possible volatile variance.

An ARCH (1) model is given by

$$Y_t = \sigma_0 \varepsilon_t,$$  \hspace{1cm} (3.14)

where $$\sigma_t = \sqrt{\alpha_0 + \alpha_1 Y_{t-1}^2}, \alpha_0 \geq 0, \alpha_1 \geq 0.$$  \hspace{1cm} (3.15)

$$o_t$$ is white noise and distributed independently with mean zero and a unit variance.

The variance of an ARCH (p) model at time t is depends on measurement at past p number of times and it is given as:

$$\text{Var} (Y_t | Y_{t-1}, \ldots, Y_{t-p}) = \sigma_t^2 = \alpha_0 + \alpha_1 Y_{t-1}^2 + \ldots + \alpha_p Y_{t-p}^2 \text{ (3.15)}$$

With the constraints $$\alpha_0 \geq 0$$ and $$\alpha_1 \geq 0$$ to avoid negative variance.

3.1.7 GARCH Model
The model that uses observation of the past squared measurements and previous changes to estimate the change at time t is known as GARCH (Generalized Autoregressive Conditionally Heteroscedastic) model. This is a non-linear model used for predicting future change using past changes and predictions of changes as the predictors.

A GARCH (1,1) is given as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 Y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$  \hspace{1cm} (3.16)

GARCH (p, q) is written as

$$\sigma_t^2 = \alpha_0 + \alpha_1 Y_{t-1}^2 + \alpha_2 Y_{t-2}^2 + \ldots + \alpha_p Y_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_q \sigma_{t-q}^2 \text{ (3.17)}$$

This can be written as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i Y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$  \hspace{1cm} (3.18)

$$\sigma_t^2$$ is the dependent variable, $$\alpha_0, \alpha_1, \ldots, \alpha_p$$ are parameters and $$\beta_1, \ldots, \beta_q$$ are explanatory variables/independent variables.

In order to recognize the appropriate model for this series, Box and Jenkins (1976) [46] methodology will be adopted. The Box and Jenkins methodology entail the following process;

1. **Model identification:** It was accomplished using the time plot and the diagram of the sample ACF and PACF acquired from the series. These graphs of ACF and PACF (correlogram) indicate the degree of correlation within the series for lags 1, 2, 3, 4… k. Let $$X_t$$ denotes the rate of a time series at time t. The ACF of the series gives the interaction linking $$X_t$$ and $$X_{t+h}$$ for h = 1,2,3,…etc. In a time series $$X_t$$, t = 1,2,3,…, the ACF at lag k is given as

$$\rho_k = \frac{\text{Cor}(X_tX_{t+k})}{\sqrt{\text{Var}(X_t)\text{Var}(X_{t+k})}} = \gamma_k/y_0$$  \hspace{1cm} (3.19)

where $$\text{Var}(X_t) = \gamma_0$$ as a function of k

$$\gamma_k$$ is the autocovariance function

$$\rho_k$$ is the autocorrelation function

where -1$$\leq \rho_k \leq 1.$$
The PACF on the other hand measures the correlation between an observation \( k \) period in the past and the present observation. The PACF is given as:

\[
\phi_{k+1,k+1} = \frac{\rho_{k+1-\sum_{j=1}^{k} \phi_{j} \rho_{j+k-1}}}{1-\sum_{j=1}^{k} \phi_{j} \rho_{j}}
\]  

(3.20)

where:

\[
\phi_{k+1,j} = \phi_{k,j} - \phi_{k+1,1} \phi_{k+1-j,1}, j = 1, 2, ..., k.
\]

The graph of ACF shows the level of correlation in the series for the lags (k). It is defined with lag k as:

\[
\rho_{k} = \frac{\text{Cov}(X_{t+k}, X_{t})}{\sqrt{\text{Var}(X_{t}) \text{Var}(X_{t+k})}} = \frac{\gamma_{k}}{\gamma_{0}}
\]  

(3.21)

where \( \text{Var}(X_{t}) = \text{Var}(X_{t+k}) = \gamma_{0} \) as a function of k

\( \gamma_{k} \) is the autocovariance function

\( \rho_{k} \) is the autocorrelation function.

Similarly, the PACF shows the level of correlation at a given lag after considering for the correlation from the intervening lags. The PACF is denoted by:

\[
\phi_{k+1,k+1} = \frac{\rho_{k+1-\sum_{j=1}^{k} \phi_{j} \rho_{j+k-1}}}{1-\sum_{j=1}^{k} \phi_{j} \rho_{j}}
\]  

(3.22)

where:

For a time series, the partial autocorrelation is the interaction linking two variables under the presumption that we realize and take into consideration the worth of other set of variables. When the ACF and PACF are plotted, there will be spikes appearing at each lag order. Any spike that resides outside the confidence limit lines, the correlation is remarkable at that lag.

2. Parameter Estimation: This entails the assessment of the coefficients of the identified model parameters. A non seasonal ARIMA model is classified as an “ARIMA(p,d,q)” model, where:

\( P \) represents the number of autoregressive terms,
\( d \) represents the number of non seasonal differences needed for stationarity, and
\( q \) represents the number of lagged forecast errors in the prediction equation ie the number of moving average terms.

The construction of the predicting equation is stated below. First, let \( x \) represent the \( d^{th} \) difference of \( X \), which implies:

If \( d=0: \ x_{t} = Y_{t} \)
If \( d=1: \ x_{t} = Y_{t} - Y_{t-1} \)
If \( d=2: \ x_{t} = (Y_{t} - Y_{t-1}) - (Y_{t-1} - Y_{t-2}) = Y_{t} - 2Y_{t-1} + Y_{t-2} \)

In respect to \( y \), the predicting equation is generally written as:

\[
\hat{y}_{t} = \mu + \phi_{1} y_{t-1} + ... + \phi_{p} y_{t-p} + \theta_{1} \varepsilon_{t-1} + ... + \theta_{q} \varepsilon_{t-q}
\]

The GARCH (p,q) model

\[
\sigma_{t}^{2} = \alpha_{0} + \alpha_{1} Y_{t-1}^{2} + \alpha_{2} Y_{t-2}^{2} + ... + \alpha_{p} Y_{t-p}^{2} + \beta_{1} \sigma_{t-1}^{2} + ... + \beta_{q} \sigma_{t-q}^{2}
\]

where \( \alpha_{0}, \alpha_{1}, ..., \alpha_{p}, \beta_{1}, ... \beta_{q} \) are parameters that are to be estimated, \( Y_{t-1}^{2}, Y_{t-2}^{2} ... Y_{t-1-q}^{2} \) are past squared observations and \( \sigma_{t}^{2} \) \( \sigma_{t-1}^{2} \) \( ... \sigma_{t-q}^{2} \) are past variances.

3. Model Diagnostics: Diagnostic check is done to ensure that the residuals from the estimated model imitate a white noise process (Enders, 2010).

The identified models was used to fit the data and to be able to test the validity of each model, diagnostic check was done by employing the Akaike Information Criterion (AIC), Schwarz criterion and Hannan-Quinn criterion. Akaike Information criterion (AIC) is a mathematical approach for estimating how competent a model describes the data it was created from. In statistics, AIC is used to collate various feasible models and decide which one is the best fit for the data.

Generally, a P-value less than 5% imply you can drop the null hypothesis that there exists a unit root. You can also contrast the calculated DFT statistic with a tabulated critical value. In statistics and econometrics ADF checks the null hypothesis that exists a unit root in a time series specimen. The alternative hypothesis is distinct based on which type of the test used, but frequently it is stationary or trend-stationary. It is an augmented version of the DF Test for a bigger and complex group of time series models. In the test the ADF statistics employed is a negative number. On some extent of confidence the higher negative the ADF, the stronger the negligence of the hypothesis that a unit roots exist. The DF evaluation is checking if \( \phi = 0 \)

\[
Y_{t} = \alpha + \beta_{1} Y_{t-1} + \epsilon_{t} \text{ which can be composed as}
\]
\[ \Delta Y_t = Y_t - Y_{t-1} = \alpha + \beta_t + \gamma Y_{t-1} + \varepsilon_t, \] where \( Y_t \) is the data. It is expressed this approach so we can regress linearly \( \Delta Y_t \) of against \( t \) and \( Y_{t-1} \) and check if \( \gamma \) is separate from 0. If \( \gamma = 0 \), then there exist a random walk process. If not and \( -1 < 1 + \gamma < 1 \), then there exist a stationarity procedure. The ADF expands the Dickey-Fuller Test equation to include a higher order autoregressive procedure by adding \( \Delta Y_{t-p} \) in the model. But the test however remains if \( \gamma = 0 \). Therefore the ADF test is given as

\[ \Delta Y_t = \alpha + \beta_t + \gamma Y_{t-1} + \phi_1 \Delta Y_{t-1} + \phi_2 \Delta Y_{t-2} + \ldots + \phi_p \Delta Y_{t-p}. \]

4. Data Analysis

![Time Plot of the Nigerian Export Price Index Series (2000-2020)](image)

**Table 1: Stationarity test for the raw series.**

<table>
<thead>
<tr>
<th>Null Hypothesis: NECI has a unit root</th>
<th>Exogenous: Constant</th>
<th>Lag Length: 1 (Automatic - based on SIC, maxlag=15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>t-Statistic</td>
<td>Prob.*</td>
</tr>
<tr>
<td>Test critical values: 1% level</td>
<td>-0.439684</td>
<td>0.8980</td>
</tr>
<tr>
<td>5% level</td>
<td>-3.456408</td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-2.872904</td>
<td></td>
</tr>
<tr>
<td>25% level</td>
<td>-2.572900</td>
<td></td>
</tr>
</tbody>
</table>


Augmented Dickey-Fuller Test Equation

Dependent Variable: D(NECI)
Method: Least Squares
Date: 09/15/21 Time: 23:45
Sample (adjusted): 2000M03 2020M12
Included observations: 250 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NECI(-1)</td>
<td>-0.005351</td>
<td>0.012170</td>
<td>-0.439684</td>
<td>0.6868</td>
</tr>
<tr>
<td>D(NECI(-1))</td>
<td>-0.353648</td>
<td>0.059924</td>
<td>-5.016709</td>
<td>0.0000</td>
</tr>
<tr>
<td>C</td>
<td>2.493546</td>
<td>2.410941</td>
<td>1.030842</td>
<td>0.3036</td>
</tr>
</tbody>
</table>

R-squared: 0.128197
Adjusted R-squared: 0.121137
S.E. of regression: 19.07572
Akaike info criterion: 8.745637
S. of squared resid: 89989.13
Schwarz criterion: 8.780895
Log likelihood: -1090.330
Hannan-Quinn criterion: 8.763845
F-statistic: 18.16038
Durbin-Watson stat: 2.034826
Prob(F-statistic): 0.000000
Fig 2: Time Plot of the differenced series.

Table 2: Stationarity Test for the differenced series

<table>
<thead>
<tr>
<th>Null Hypothesis: DNECI has a unit root</th>
<th>Exogenous: Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag Length: 0 (Automatic - based on SIC, maxlag=15)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test</th>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-22.87998</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Test critical values:
- 1% level: -3.456406
- 5% level: -2.872904
- 10% level: -2.572900


Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DNECI)
Method: Least Squares
Date: 03/15/21 Time: 23:51
Sample (adjusted): 2000M03 2020M12
Included observations: 250 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNECI(-1)</td>
<td>-1.357091</td>
<td>0.059313</td>
<td>-22.87998</td>
<td>0.0000</td>
</tr>
<tr>
<td>C</td>
<td>1.572200</td>
<td>1.206451</td>
<td>1.303152</td>
<td>0.1937</td>
</tr>
</tbody>
</table>

R-squared | 0.678546 | Mean dependent var | -0.000560 |
Adjusted R-squared | 0.677249 | S.D. dependent var | 33.52277 |
S.E. of regression | 19.04467 | Akaike info criterion | 8.739420 |
Sum squared resid | 89949.47 | Schwarz criterion | 8.767591 |
Log likelihood | -1090.427 | Hannan-Quinn crier. | 8.750758 |
F-statistic | 523.4935 | Durbin-Watson stat | 2.037399 |
Prob(F-statistic) | 0.000000 |
Fig 3: Correlogram of the differenced NECI

Table 3: Estimation of the ARMA (1, 1) for the differenced NECI

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>-0.165676</td>
<td>0.120200</td>
<td>-1.360002</td>
<td>0.1588</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.209110</td>
<td>0.132302</td>
<td>-1.579830</td>
<td>0.1154</td>
</tr>
<tr>
<td>SIGMAQ</td>
<td>359.6520</td>
<td>15.03919</td>
<td>23.91432</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared: 0.124380  Mean dependent var: 1.157949
Adjusted R-squared: 0.117318  S.D. dependent var: 20.30721
S.E. of regression: 19.07885  Akaike info criterion: 8.747479
Sum squared resid: 90272.65  Schwarz criterion: 8.739615
Log likelihood: -1094.809  Hannan-Quinn criterion: 8.764435
Durbin-Watson stat: 1.987647

Inverted AR Roots: -0.17
Inverted MA Roots: 0.21
Table 4: Estimation of the AR (1) model for the differenced NECI

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>-0.351293</td>
<td>0.030671</td>
<td>-11.45366</td>
<td>0.0000</td>
</tr>
<tr>
<td>SIGMASQ</td>
<td>380.5228</td>
<td>15.17624</td>
<td>23.77233</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared  0.121530  Mean dependent var  1.157648
Adjusted R-squared  0.110002  S.D. dependent var  20.30721
S.E. of regression  10.07146  Akaike info criterion  8.742724
Sum squared resid  90566.46  Schwarz criterion  8.770816
Log likelihood  -1965.212  Hannan-Quinn criter.  8.754029
Durbin-Watson stat  2.034548

Inverted AR Roots  -0.35

Table 5: Estimation of the MA(1) model for the DNECI series

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA(1)</td>
<td>-0.355987</td>
<td>0.035772</td>
<td>-10.23104</td>
<td>0.0000</td>
</tr>
<tr>
<td>SIGMASQ</td>
<td>350.3491</td>
<td>15.12948</td>
<td>23.81925</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared  0.122683  Mean dependent var  1.157649
Adjusted R-squared  0.118159  S.D. dependent var  20.30721
S.E. of regression  19.06895  Akaike info criterion  8.741430
Sum squared resid  90447.53  Schwarz criterion  8.769551
Log likelihood  -1965.053  Hannan-Quinn criter.  8.752784
Durbin-Watson stat  2.026239

Inverted MA Roots  0.37

Table 6: Estimation of the GARCH (1,1) model for the differenced NECI.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>10.42537</td>
<td>3.225705</td>
<td>3.231854</td>
<td>0.0012</td>
</tr>
<tr>
<td>RESID(-1)*2</td>
<td>0.169918</td>
<td>0.035272</td>
<td>4.817382</td>
<td>0.0000</td>
</tr>
<tr>
<td>GARCH(-1)</td>
<td>0.827623</td>
<td>0.026243</td>
<td>31.53658</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared  -0.003263  Mean dependent var  1.157649
Adjusted R-squared  0.000734  S.D. dependent var  20.30721
S.E. of regression  20.29975  Akaike info criterion  8.513776
Sum squared resid  103432.0  Schwarz criterion  8.555913
Log likelihood  -1065.479  Hannan-Quinn criter.  8.530733
Durbin-Watson stat  2.705354
4. Discussion of Results

4.1 Model Identification

The table in the appendix shows a complete presentation of the Nigerian Export Price Index Series from Jan. 2000 to Dec. 2020 making a sum of 250 computations for the study. The time series plot (figure 1) shows that the series changes over time indicating that there is a secular trend in it. In Table 1: Stationarity test for the raw series, the series is non-stationary with $p = 0.8989 > 0.05$. Therefore there was the need for differencing (see figure 2 Time Plot of the differenced series). After the series was differenced, a stationarity test was run for the differenced series. Table 2 shows that the differenced series is stable with $p = 0.0000 < 0.05$, indicating that the trend has been removed through differencing. The correlogram of Figure 3 has spikes at lag 1 for the Autocorrelation function as well as for the partial autocorrelation function suggesting an ARMA.

The correlogram of Figure 3 has spikes at lag 1 for the Autocorrelation function as well as for the partial autocorrelation function suggesting an ARMA (1,1). Then ARMA (1,1) was estimated for the differenced series (table 3) which showed that the series does not follow an ARMA as suspected, the coefficients of the parameters are both non-significant statistically. After the failure of the ARIMA (1,1,0) and ARIMA (0,1,1) to the NECI. ARIMA (1, 1, 0), ARIMA (0,1, 1), and GARCH (1,1) having met the required assumptions as the appropriate model was used to estimate its parameters. Therefore required models are given as

$$X_t = X_{t-1} + \phi_1 (X_{t-1} - X_{t-2}) + \epsilon_t$$

Thus the fitted model is

$$D(NECI)_t = D(NECI)_{t-1} - 0.351293 (D(NECI)_{t-1} - D(NECI)_{t-2}) + 360.8226$$

$$D(NECI)_t = D(NECI)_{t-1} - 0.351293 (D(NECI)_{t-1} - 0.351293 D(NECI)_{t-2}) + 360.8226$$

$$D(NECI)_t = (1 - 0.351293) (D(NECI)_{t-1} - 0.351293 D(NECI)_{t-2}) + 360.8226$$

$$X_t = X_{t-1} - 0.121530 + \epsilon_t$$

The fitted model is

$$D(NECI)_t = D(NECI)_{t-1} + 0.365987 \epsilon_{t-1} + 360.3491$$

The differenced series show evidence of volatility. Hence, the GARCH model was tried. GARCH (1,1) having met the required assumptions as the appropriate model was used to estimate its parameters. The required GARCH model is GARCH (1, 1) which can be formulated as

$$\delta_{t+1} = \alpha_0 + \alpha_1 Y_{t+1}^2 + \beta_1 \delta_{t+1}$$

Thus the fitted model is expressed as;

$$\delta_{t+1} = 10.43 + 0.17 Y_{t+1}^2 + 0.83 \delta_{t+1}$$

Where $\alpha_0 = 10.43$, $\alpha_1 = 0.17$, $\beta_1 = 0.83$

4.2 Parameter Estimation

The EVIEWS statistical software was used in estimating the model. The correlogram of Figure 3 has spikes at lag 1 for the Autocorrelation function as well as the partial autocorrelation function suggesting an ARMA (1,1). Then ARMA (1,1) was estimated for the differenced series (table 3) which showed that the series does not follow an ARMA (1,1) as suspected, the coefficients of the parameters are both non-significant statistically. After the failure of the ARIMA (1,1,0) and ARIMA (0,1,1) to the NECI. ARIMA (1, 1, 0), ARIMA (0,1, 1), and GARCH (1,1) having met the required assumptions as the appropriate model was used to estimate its parameters. Therefore required models are given as

$$X_t = X_{t-1} + \phi_1 (X_{t-1} - X_{t-2}) + \epsilon_t$$

Thus the fitted model is

$$D(NECI)_t = D(NECI)_{t-1} - 0.351293 (D(NECI)_{t-1} - D(NECI)_{t-2}) + 360.8226$$

$$D(NECI)_t = D(NECI)_{t-1} - 0.351293 (D(NECI)_{t-1} - 0.351293 D(NECI)_{t-2}) + 360.8226$$

$$D(NECI)_t = (1 - 0.351293) (D(NECI)_{t-1} - 0.351293 D(NECI)_{t-2}) + 360.8226$$

$$X_t = X_{t-1} - 0.121530 + \epsilon_t$$

The fitted model is

$$D(NECI)_t = D(NECI)_{t-1} + 0.365987 \epsilon_{t-1} + 360.3491$$

The differenced series show evidence of volatility. Hence, the GARCH model was tried. GARCH (1,1) having met the required assumptions as the appropriate model was used to estimate its parameters. The required GARCH model is GARCH (1, 1) which can be formulated as

$$\delta_{t+1} = \alpha_0 + \alpha_1 Y_{t+1}^2 + \beta_1 \delta_{t+1}$$

Thus the fitted model is expressed as;

$$\delta_{t+1} = 10.43 + 0.17 Y_{t+1}^2 + 0.83 \delta_{t+1}$$

Where $\alpha_0 = 10.43$, $\alpha_1 = 0.17$, $\beta_1 = 0.83$

4.3 Diagnostic Check

On the basis of R-squared, AIC, Hannan-Quinn and Schwarz criteria the ARIMA(0,1,1) model has 0.122683, 8.741460, 8.752764 and 8.769551 values while ARIMA(1,1,0) has 0.121530, 8.742724, 8.754029 and 8.770816 values. The differenced series show evidence of volatility. Hence, the GARCH model was tried. By the AIC, SC and HQ terms, the GARCH (1, 1) model has 8.513776, 8.55913, 8.530733 values.

5. Conclusion

This study investigated the supremacy of GARCH modelling over ARIMA modelling of Nigerian export commodity price index series. Data was obtained from the Central Bank database from January 2000 to December 2020 and was analyzed using EVIEWS statistical software. The time plot of Nigerian export commodity price index (figure 1) shows that the series changes over time indicating that there is clearly a secular trend in it. The correlogram of Figure 3 has spikes at lag 1 for the Autocorrelation function as well as for the PACF suggesting an ARMA (1, 1). Then ARMA (1,1) was estimated for the differenced series (table 3) which showed that the series does not follow an ARMA(1,1) as suspected, the coefficients of the parameters are both non-significant statistically. After the failure of the ARIMA (1,1,1), the ARIMA (1,1,0) and ARIMA (0,1,1) was fitted to the NECI. ARIMA (1, 1, 0), ARIMA (0,1, 1) and GARCH (1,1) having met the required assumptions as the appropriate model was used to estimate its parameters. On the basis of R-squared, AIC, Hannan-Quinn and Schwarz criteria the ARIMA(0,1,1) model is a superior model when compared to ARIMA(1,1,0) The differenced series show evidence of volatility. Hence, the GARCH model was tried. From the result obtained, it was reveal that by the AIC, SC and HQ terms, the GARCH (1,1) model when compared to other models has proven to be adequate model for the Modelling of differenced Nigerian Export Commodity Price Index series.
6. References


