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A comparative study of wild bootstrap and robust wild bootstrap estimates in linear regression

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Abstract
The Wild Bootstrap techniques are widely used today in many other fields like economics, Engineering, and medicine. Imperial evidences indicate that the use of wild bootstrap techniques may produce efficient estimate in the presence of heteroscedasticity error variance. However, in the presence of outliers the wild bootstrap is no longer efficient. It is now evidence that the robust wild Bootstrap technique based on MM-estimator was introduced to handle the outlier's problems. However, presence of high leverage outliers will introduce wrong parameter estimate and the robust wild Bootstrap is not resistance to high leverage outliers. Hence this research investigates the use of modified robust wild bootstrap techniques on regression model as an estimator in a situation where heteroscedasticity, outliers and high leverage outliers are presence. This paper proposed a modified robust wild bootstrap GM-estimator of MR Boot Wu and MR Boot Liu algorithm based on the weighted residuals which incorporate the Huber weighted function, GM-estimators, robust location and scale, and the wild bootstrap sampling procedure of Wu and Liu. The GM-estimator, was obtain using the MM-estimator as the initial and scale estimator. However, the MR Boot Wu and MR Boot Liu were obtained through a robust wild bootstrap MM-estimator (R Boot). Finally, the real data obtain from twenty-two countries based on their cross country variation on the level of income per capital in the Organization for European Economic Co-operation and Development (OECD) and simulation study. The performances of the MR Boot Wu and MR Boot Wu together with the existing, Boot Wu, Boot Liu, R Boot Wu and R Boot Liu estimators were compared using the biased, standard error and RMSE. The numerical examples indicated that the MR Boot Wu and MR Boot Liu estimator has proven to be a good alternative estimator for economic use.

Keywords: Wild bootstrap, robust estimation, bias estimation, standard error, RMSE

Introduction
One of the main problems in regression estimation methods is heteroscedasticity. Heteroscedasticity is the term used to describe cases in which when variance about the error term are not the same. The wild bootstrap method has been advocated in the literature as an alternative to LS estimation for the heteroscedasticity problem in this method, which was proposed by [1, 2] wild bootstrap estimators are used instead of classical bootstrap estimators. Wild Bootstrap is a re-sampling method that is commonly used to estimate bias, standard error and to construct the confidence interval of an estimator. However, wild bootstrap estimators usually are based on the OLS estimate, which are not robust to the presence of outliers [3, 4]. Proposed the robust wild bootstrap based on MM-estimator was introduce to handle the problems of outliers. The robust wild bootstrap method is able to produce better estimation, as the problems of outliers arise. This is due to the fact that the robust MM-estimator is resistant to presence of outlying observations. They found that robust wild bootstrap based on MM-estimator is the best estimator as it is consistently provides good estimates and shortest confidence interval. Another common problem in regression estimation methods is that of high leverage outliers. The high leverage outliers have more unattractive consequences for regression analysis such as causing biasness problem for the coefficient estimations of the data set [5]. It is now evident that high leverage outliers or outliers in the X-direction may affect these robust diagnostic methods and mislead the conclusion of the regression analysis. The main focus of this paper is to investigate the diagnostics measure which are resistant to the

heteroscedasticity, outliers and high leverage outliers in the data. The robust wild bootstrap based on MM-estimator is one of the practical methods among different heteroscedasticity, outliers and high leverage outliers diagnostics [3]. In the presence of high leverage outliers the robust wild bootstrap diagnostics measure such as robust wild bootstrap based on MM-estimator may not diagnose correctly the existence of high leverage outliers in the data set. In this respect it is imperative to utilize resistance diagnostics methods to high leverage outliers when they are present in the data set. Since robust MM-estimator can be obtained from the high break down estimator of S-estimator and robust M- estimator to provide the efficiency of fitted regression line when each of the explanatory variables is regressed to the other explanatory variables. It is required to develop a robust wild bootstrap estimator which bounded influence estimator that are resistant to heteroscedasticity, outliers and high leverage outliers. Several robust bounded influence estimators exist in the literature of robust methods [5].

One of the robust bounded influence estimators which have good properties is the Generalized M-estimators (GM-estimators). This GM-estimator attempts to down weight high leverage points as well as large residuals [6]. In this paper, a new modified robust wild bootstrap estimator which is called the modified robust wild bootstrap GM-estimator (MRBootWu and MRBootLiu) is proposed. We propose a slight modification from what have been found in the literatures, which is to combine the GM-estimator with initial and scale estimator of S-estimators defined by [7] and tukey bisquares weighted function to reduce the effect of outliers. The MRBootWu and MRBootLiu also consider wild bootstrap sampling procedures of Wu and Liu together with the GM-estimator which was develop by [8]. We would like to examine the superiority of the proposed method as an enhancement to the existing methods when handling problems of heteroscedasticity, outliers, high leverage outliers and multicollinearity. This paper is organized as follows We introduce the classical wild bootstrap in Section 2; in Section 3, the numerical example OF newly proposed technique and the existing methods together with their performances are presented. A simulation study are presented in Sections 4, and the final conclusion of the study is made in Section 5.

Wild bootstrap estimation

This bootstrap procedure has been suggested by [1, 9] for the situation when the additional assumption of $E(\varepsilon_i \mid X_i) = 0$ is appropriate. The wild bootstrap procedure for [1] estimates is discussed as follows:

Step 1. For each i, draw a random sample of t*, with replacement from an auxiliary distribution that has mean zero and variance one.

Step 2. Form a bootstrap sample of

$$y_i^* = f(X_i, \hat{\beta}_{OLS}) + t^* \hat{\varepsilon}_i^{OLS} (1 - u_{ii})^{-1}$$

Where

$u_{ii} = \mathbf{x}_i^{'}(\mathbf{x}^{'}\mathbf{x})^{-1}\mathbf{x_i}$ are the ith leverage points. y_t^{*b} is the bootstrap response and $\hat{\varepsilon}_i^{OLS}$ is the OLS residuals.

Step 3**.** Compute the OLS bootstrapped estimate to obtain the

$$\hat{\boldsymbol{\beta}}^* = (\mathbf{X'X})^{-1}\mathbf{X'y}^*$$

Step 4**.** Repeat Step 1 through 3 for *k* times to obtain $\hat{\boldsymbol{\beta}}^{*1}, ..., \hat{\boldsymbol{\beta}}^{*k}$ where k is the number of bootstrap replicates. The bootstrap procedure of this type takes in to consideration the non-constancy of the error variance. Another alternative for choice of t* based on Wu's bootstrap procedure which is different from step 1, is to sample t* from a finite population of $c_1, c_2, ..., c_n$

Where

$$c_i = (\hat{\varepsilon}_i^{OLS} - \bar{\hat{\varepsilon}}_i^{OLS})(n^{-1}\sum_{i=1}^{n}(\hat{\varepsilon}_i^{OLS} - \bar{\hat{\varepsilon}}_i^{OLS})^2)^{-1}$$

and $\bar{\hat{\varepsilon}}_i = n^{-1}\sum_{i=1}^{n}\hat{\varepsilon}_i$ where Step 2 through 5 remain the same.

When a regression model has intercept term, the $\bar{\hat{\varepsilon}}_i$ is approximately equals to zero. This procedure is a nonparametric implementation of wu's bootstrap sampling scheme because the resampling procedure is done from the empirical distribution function of the normalized residuals [1]. The bootstrap procedure is called Wu's bootstrap (BootWu).

A more refined version of classical wild bootstrap was introduced by [2], following the idea of classical wild bootstrap by [1, 9, 2]. Suggested to modify the procedure of generating the random sample of t* values. In this situation the value of t* is randomly selected from auxiliary distribution that has a third central moment equals to one, in addition to the zero mean and unit variance. He described that if this is case, Wu's shares the usual second order asymptotic properties of the ordinary least squares bootstrap. Hence, the addition for the restriction of the third central moment equals to one by [2] is used to correct the skewness term in the edge worth expansion of the sampling distribution of $I'\hat{\beta}$, where I is an n-vector of ones. Liu's wild bootstrap provided a new procedure to generate the random sample of t* value different from above. The sample bootstrap can be implemented by drawing a random sample of $t_i^* = N_i M_i - E(N_i)E(M_i),\ i = 1, 2, ..., n$ where $N_1, N_2, ..., N_n$ are independently and identically distributed normal distribution with mean $(1/2)(\sqrt{17/6}) + \sqrt{17/6}$ and variance $1/2$. $M_1, M_2, ..., M_n$ are also *i.i.d.* normally distribution with mean $(1/2)(\sqrt{(\sqrt{17/6}) - \sqrt{17/6}})$ and have variance $1/2$. N_i's and M_i's are independent. Steps 2 through 5 of these procedures remain the same. This bootstrap procedure will generate random sample of t_i^*. It is interesting to mention that when this procedure of [2] is used for the selection of t*, it produced the third central moment equal to one. The bootstrap procedure of this type is called Liu's bootstrap (BootLiu). This algorithm was then modified using other existing estimators namely, the RWMM-estimator and RWGM-estimator as a replacement to the OLS estimator.

Example using real data sets

This section will discuss the application of the MR Boot Wu and MR Boot Liu methods on real data by considering the numerical example that will show the advantages of the proposed method with respect to the other estimators, BWu, B

Liu, RB Wu and RB Liu estimator in the presence of outlier, high leverage outliers and heteroscedasticity error variance. The dataset described the cross-country variation in the level of income per capital in the OECD of 22 countries. We checked whether the data set contained any outliers and high leverage outliers using the LTS residuals and robust mahalobis distance (RMD) base on minimum volume ellipsoid (MVE). It was discovered that five observations were identified as outliers. If the outliers test indicate no outliers and high leverage outliers in the data set then modification become necessary. The modification followed a similar procedures proposed by various authors to generate outliers in Y and X-direction for the purpose of examining the robustness of the existing method for outliers [10]. The contamination is done by replacing the good observation of Y and X by a suitable value which is arbitrary large as outliers in Y or X observations. On the other hand, the modified robust Goldfeld-Quadl test is used for heteroscedasticity test and the null hypothesis is rejected which indicated that there is heteroscedasticity in the data. The Boot Wu, Boot Liu, RBoot $_{W}$u, R Boot Liu, MR Boot $_{W}$u and MR Boot Liu methods were then applied to the data. The results of the original data was obtained based on 1000 bootstrap replicates and are presented in Table 1 along with the standard errors, bias and RMSE of the parameter estimates from the estimators. Table 2 presents result of modified data based on their bias, RMSE and standard error of the parameter estimates. Based on the results from both tables, it is interesting to observe that both the standard error bias and RMSE of the wild bootstrap method tend to be larger followed by robust wild bootstrap. The MR Boot $_{W}$u and MR Boot Li methods have the smallest standard errors.

Table 1: Parameter estimation, bias, standard error and RMSE of original income per capital data set for the OECD of 22 countries in the world.

Par.	BootWu	BootLiu	RBootWu	RBootLiu	MRBootWu	MRBootLiu
Est.	8.263	9.107	2.738	1.111	1.819	1.921
Bias	2.336	3.456	-0.413	-0.026	-0.203	-0.108
SE	17.879	13.922	0.082	0.059	0.052	0.058
RMSE	18.03	14.34	0.421	0.064	0.210	0.122
Est.	-0.379	0.234	-0.235	0.343	0.321	0.533
Bias	-0.112	-0.287	-0.001	0.024	-0.027	-0.032
SE	0.280	0.597	0.070	0.035	0.020	0.033
RMSE	0.302	0.662	0.070	0.042	0.034	0.046
Est.	0.168	0.266	-0.205	-0.013	0.014	0.008
Bias	0.668	-0.266	0.224	-0.013	0.001	-0.041
SE	0.287	0.457	0.054	0.033	0.020	0.032
RMSE	0.727	0.529	0.231	0.035	0.020	0.052
Est.	-0.125	0.455	0.265	0.176	0.115	0.308
Bias	0.359	0.390	-0.022	-0.029	-0.022	0.018
SE	0.249	0.360	0.043	0.023	0.013	0.022
RMSE	0.436	0.530	0.049	0.037	0.026	0.028
AV.SE	4.674	3.834	0.062	0.037	0.026	0.036

Table 2: Parameter estimation, bias, standard error and RMSE of modified income per capital data set for the OECD of 22 countries in the world

Par.	BootWu	BootLiu	RBootWu	RBootLiu	MRBootWu	MRBootLiu
Est.	3.964	3.878	2.987	2.610	1.568	1.523
Bias	1.336	0.843	-0.313	-1.078	0.035	-0.019
SE	6.879	7.922	1.002	1.045	0.037	0.037
RMSE	7.008	7.966	1.050	1.502	0.051	0.041
Est.	-0.229	-0.312	-0.371	-0.288	-0.142	-0.418
Bias	0.599	0.612	-0.037	-0.040	-0.342	-0.032
SE	0.684	0.876	0.099	0.098	0.035	0.036
RMSE	0.909	1.069	0.105	0.106	0.344	0.048
Est.	0.153	0.183	0.095	0.141	0.215	0.139
Bias	0.468	0.566	0.021	0.035	0.018	0.014
SE	0.336	0.587	0.073	0.069	0.031	0.030
RMSE	0.576	0.815	0.076	0.077	0.036	0.033
Est.	-0.072	-0.657	-0.101	-0.322	-1.131	-0.957
Bias	0.296	0.490	-0.062	-0.023	0.018	-0.015
SE	0.276	0.426	0.059	0.092	0.026	0.019
RMSE	0.404	0.649	0.085	0.095	0.032	0.024
AV.	2.044	2.453	0.308	0.326	0.032	0.030

We cannot make a final conclusion yet, just by observing the results of the original and modified real data, but a reasonable interpretation up to this stage is that the wild bootstrap is affected by outliers

Simulation study on bootstrap sample

A simulation study was designed to assess the performance of BootWu, BootLiu, RBootWu, RBootLiu and the proposed methods of MRBootWu and MRBootLiu estimator in linear regression model. This simulation study, follow a similar procedure proposed by [4, 5]. The considered design for this experiment involves a regression model with intercept and two covariates.

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \sigma_i \varepsilon_i \tag{1}$$

The covariate values x_{1i} and x_{2i} were generated from N(0; 1) and residual ε_i is drawn from normal distribution with mean zero and variance equal to 1, when no outlier was considered and for all i under homoscedasticity $\sigma = 1$. The considered sample size is when n = 20 observations. These observations are then replicated seven times to generate samples of n = 140, respectively. To replicate observations to generate large sample of observations it followed a similar procedure proposed by various authors who utilized the replication of covariate values to create large samples [11, 12]. The data is generated using $\beta_0 = \beta_1 = \beta_2 = 1$. Next, we start contamination of the data. Randomly we replace some good observations of *i.i.d.* normal errors ε_i's by N (10,9). Now our main interest is to obtain a regression design that includes heteroscedasticity, outliers and leverage outliers in the model. We form the heteroscedasticity generating procedure following [3, 12] effort, where $\sigma_i^2 = \exp(1.4x_{1i} + 1.4x_{2i})$ is used in the simulation to generate the level of heteroscedasticity in the simulation. It should be noted that the degree of heteroscedasticity should remain constant for different sample sizes, and hence, the results of our simulation are affected only by increased number of sample size. The measurement of degree for severe heteroscedasticity is set as $\phi = \max(\sigma_i^2)/\min(\sigma_i^2) = 4$, where $i = 1, 2, ..., n$. Now the heteroscedastic model is given as

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \sigma_i \varepsilon_{i(cont.)} \quad (2)$$

and $\varepsilon_{i(cont.)} = \alpha N(0,1) + (1-\alpha) N(10,9)$. However, the covariance of regression model or contaminated regressor variables is given as $x_{i(cont.)} = \alpha N(0,1) + (1-\alpha) N(10,9)$. The value of α is based on the percentage levels of outliers and leverage outliers. In this study the considered percentage of outliers for all the simulations are typically 0%, 10%, and 20% with α levels of 0.90 and 0.80 respectively. The effect of outliers were exposed to both x and y-direction. For every sample size, we fit the BootWu, BootLiu, RBootwu RBootLiu, MRBootwu and MRBootLiu linear regression model. In each bootstrap method we generated k bootstrap random sample $(k = 1, 2, \ldots, 1000)$ and construct the bootstrap estimate. We estimate the bias, root mean squares errors and standard errors of various sample size with different percentage of outliers. The bootstrap bias of BootWu and BootLiu can be obtained by subtracting the different between the true model and the estimated model as:

$$\text{Bias} = \hat{\beta}_{bOLS} - \hat{\beta}_{OLS} \quad (3)$$

Where $\hat{\beta}_{bOLS} = (k)^{-1} \sum_{b=1}^{k} \hat{\beta}_{OLS}^{*b}$

and the corresponding estimate of bootstrap standard error of BootWu and BootLiu can be acquired from the square root of the main diagonal of the covariance matrix and is given as

$$SE(\hat{\beta}_{bOLS}) = (k-1)^{-1}((\sum_{b=1}^{k}(\beta_{(OLS)}^{*b} - \hat{\beta}_{(bOLS)})(\beta_{(OLS)}^{*b} - \hat{\beta}_{(bOLS} \quad (4)$$

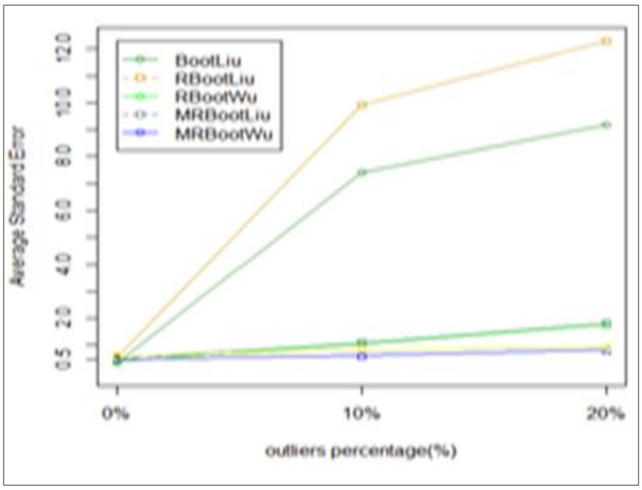

Fig 1: The average effect of outliers percentage on standard error of size n=20

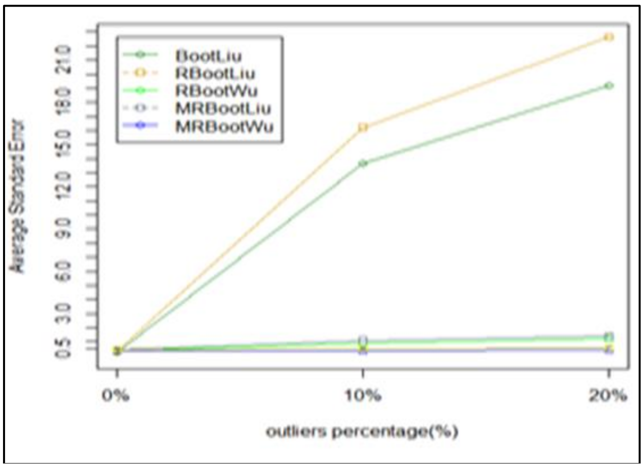

Fig 2: The average effect of outliers percentage on standard error of size n = 140

The influences of outliers on the standard errors of the estimates are visible in Figures 1 and 2. In these plots, the average standard errors of the parameters estimates are plotted OECD of 22 countries at different levels of outliers for different bootstrap methods. The results presented in Figures 1–2 show that the performances of the RBootWu, RBootLiu, MRBootWu and MRBootLiu estimates are fairly close to the Bootwu and Bootliu at the 0% level of contamination. It emerges that the average standard errors of the RBootWu, RBootLiu, MRBootWu and MRBootLiu are closer to the average standard errors of the Bootwu and Bootliu, respectively, in “clean” data, regardless of the percentage of outliers. However, at the 10%, and 20% levels of contaminations, the standard errors of the Bootwu and Bootliu, RBootWu and RBootLiu estimates become unduly large. On the contrary, it is interesting to see that not much influence is visible for the modified robust wild bootstrap techniques of MRBootWu and MRBootLiu, at the different percentage levels of outliers. It is also observed that the performance of MRBootliu is the best overall followed by MRBootwu.

Table 3: Bias, RMSE and standard error of n = 20 and n = 140 (bold) for 0% , 10% and 20% level of contaminated data based on non- robust wild bootstrap, robust wild bootstrap and modified robust wild bootstrap from normal distribution with 2 regressor variables.

Out. (%)		0%			10%			20%		
Coef	**Method**	**Bias**	**RMSE**	**SE**	**Bias**	**RMSE**	**SE**	**Bias**	**RMSE**	**SE**
$\hat{\beta}_0$	BootWu	-0.043	0.287	0.284	-6.565	13.502	11.798	11.526	29.915	27.606
		-0.441	0.909	0.261	-0.578	12.152	6.832	-0.646	21.002	11.831
	BootLiu	-0.122	0.311	0.127	8.077	33.200	16.900	13.416	62.195	32.424
		-0.357	0.689	0.156	-1.947	12.894	7.009	-1.740	28.039	15.723
	RBootWu	-0.104	0.479	0.250	-0.353	1.550	0.800	-0.528	3.411	1.851
		-0.498	1.062	0.334	-0.353	1.548	0.799	-0.436	1.724	0.869
	RBootLiu	0.110	0.537	0.282	-0.393	1.471	0.731	-0.515	3.263	1.768
		-0.398	1.006	0.405	0.399	1.670	0.853	-0.499	1.823	0.899
	MRBootWu	-0.056	0.514	0.284	0.053	0.949	0.533	-0.366	1.317	0.647
		-0.098	0.405	0.207	-0.212	0.603	0.266	-0.323	0.811	0.324
	MRBootLiu	-0.054	0.283	0.150	0.054	1.119	0.629	-0.392	1.243	0.582
		-0.070	0.527	0.289	-0.215	0.809	0.403	-0.355	1.224	0.592
$\hat{\beta}_1$	BootWu	0.330	0.383	0.194	7.453	15.354	13.424	9.409	30.723	29.247
		-0.303	0.567	0.104	0.622	9.935	5.571	0.877	23.899	13.455
	BootLiu	-0.551	1.032	0.188	5.800	35.476	19.156	11.099	67.777	36.593
		-0.266	0.731	0.315	0.218	16.551	9.335	0.959	33.317	18.773
	RBootWu	0.113	0.416	0.206	0.411	2.702	1.468	0.623	2.809	1.457
		-0.278	0.881	0.412	0.329	1.273	0.638	0.431	1.596	0.790
	RBootLiu	-0.174	0.502	0.224	0.405	2.950	1.614	0.450	3.111	1.697
		-0.249	1.006	0.510	-0.532	1.685	0.787	0.503	1.773	0.865
	MRBootWu	-0.064	0.145	0.051	0.222	0.883	0.446	0.302	1.208	0.611
		-0.059	0.467	0.257	0.299	0.738	0.290	0.204	0.734	0.361
	MRBootLiu	-0.121	0.804	0.437	0.328	1.067	0.505	0.338	1.344	0.679
		-0.076	0.569	0.312	0.224	0.916	0.466	0.277	0.897	0.424
$\hat{\beta}_2$	BootWu	0.421	0.452	0.164	8.410	18.999	17.037	12.026	34.838	32.697
		0.322	0.785	0.304	0.326	11.485	6.471	0.711	20.071	11.302
	BootLiu	-0.312	0.838	0.355	7.652	34.258	17.749	13.005	75.986	40.850
		0.347	0.966	0.421	0.307	14.728	8.304	1.235	30.912	17.397
	RBootWu	-0.253	0.626	0.247	0.432	1.763	0.896	0.524	3.645	1.989
		0.370	0.852	0.307	0.455	1.745	0.873	0.421	1.685	0.852
	RBootLiu	0.094	0.485	0.257	0.428	1.849	0.951	0.556	3.663	1.990
		0.398	0.914	0.328	0.387	1.547	0.782	0.597	1.993	0.953
	MRBootWu	0.110	0.664	0.358	-0.322	1.058	0.503	0.480	1.515	0.707
		0.052	0.368	0.201	0.295	0.695	0.259	0.324	0.795	0.309
	MRBootLiu	0.125	0.242	0.055	-0.332	1.266	0.632	0.423	1.599	0.797
		0.055	0.729	0.408	0.214	0.681	0.319	0.387	1.014	0.421

Conclusion

This paper has described the finite sample behavior of wild bootstrap and robust wild bootstrap estimators, namely Boot Wu, Boot Liuu, R Boot Wu, R Boot Liu, MR Boot GMWu and MRW Boot GMLiu in linear regression model, in the presence of outliers, high leverage outliers and heteroscedasticity error variance using original real data, modified real data and simulation study. Our investigations have produced several important results. In our findings, the estimator proved to work very well using original real data, modified real data and simulated study when outliers, high leverage outliers and heteroscedasticity are present. By investigating the result of the standard error, it was interesting to observe that the resistance of outlier and high leverage outliers by the MRW Boot GMWu and MRW Boot GM Liu in producing small standard error of the estimate. Secondly, a weighted estimator based on Tukey biweight procedure are preferred to unweight ones when estimating the parameter of regression model since they performed better when heteroscedasticity, outliers and high leverage outliers were present. Thirdly, the R Boot Wu and R Boot Liu estimators, which are commonly used in applied research, when heteroscedasticity, outliers and high leverage outliers were present, exhibited a substantial result of small sample bias, RMSE and standard error, both in the presence of outliers and heteroscedasticity error variance. The MRW Boot GM Wu and MRW Boot GMLiu performed much better than the R Boot Wu and R Boot Liu in all the cases. Our results suggest that the MRW Boot GMWu and MRW Boot GMLiu estimators based on studies by [1, 2], the bootstrapping procedure constitutes a much more appealing alternative. In conclusion, the MRW Boot GMWu and MRW Boot GMLiu estimators of regression model prove to be robust against unfavourable regression data.

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