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Some techniques of estimation and projection of a few basic demographic indicators

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Abstract
In this paper, IMR, U3MR and U5MR childhood mortality rates for sex and rural-urban differentials are estimated by constructing a procedure in which indirect techniques of child mortality estimation by Trussell's variant of Brass method proceeded to logit smoothing of the estimates and hence fitted to Weibull survival function to obtain final estimates is employed. The estimation is done based on children ever born (CEB) and children surviving (CS) data of census years of 1991, 2001 and 2011. Moreover, Life expectancies at birth are estimated for both sexes of rural and urban residents separately, by using methods of estimation of life expectancy at birth (LEB) as a function of child survivorship probabilities, developed by Phukon and Ahamed (2019). Finally, life expectancy at birth, infant mortality rate, under-three and under-five age mortality rates for both sexes and rural-urban differentials in five major districts of Bihar are estimated for reference years using Hill's (2013) method. Using the estimates for reference years combined with observed data of the indicators, LEB, IMR, U3MR, U5MR of the districts are projected till 2030.

Keywords: CEB, LEB, IMR, U3MR, U5MR

1. Introduction
Demographic indicators such as, infant mortality rate (IMR), under-three mortality rate (U3MR) and under-five mortality rates (U5MR)for sex and rural-urban differentials, state and district level are useful in the planning and implementing government policies, budget and resource allocation, decision-making processes in diverse socio-economic areas, etc. In India national level estimates ignoring spatial and socio-economic heterogeneity would not provide adequate information for representing state and district levels with its sheer population size of 1.2 billion in 2011 (Samir, *et al*., 2018) [3].
LEB (e_0), i.e., the longevity of a new born based on age-specific death rates, is an important indicator to determine the mortality level of a population from which demographers can draw an idea about the socio-economic condition and health status of that particular population (Najafi, *et al*., 2018, Motlagh, *et al*, 2012) [4, 5]. Phukon and Ahamed (2019) [1] estimated $e_0$by developing two polynomial regression models comprising child survivorship probabilities at ages one, two, three and four, i.e., l_1, l_2, l_3 and l_4 using 54 set of SRS based abridged life tables of Assam.
In this paper, IMR, U3MR and U5MR for sex and rural-urban differentials are estimated by using indirect techniques of estimation comprising Trussell's variant of Brass method along with logit smoothing and use of Weibull survival function in five major districts of Bihar. The estimation is done based on CEB and children surviving CS data of census years of 1991, 2001 and 2011. The IMR, U3MRand U5MR for the reference years are estimated with the help Hill's (2013) method. Moreover, Life expectancy at birth are also estimated for both sexes of rural and urban residents separately, by using methods of estimation of $e_0$as a function of child survivorship probabilities, developed by Phukon and Ahamed (2019) [1].
Finally, life expectancy at birth, infant mortality rate, under-three and under-five age mortality rates for five major districts of Bihar are projected till 2030.

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2. Data and Methods
2.1 Data
For estimating the demographic parameters such as, e_0, IMR and U3MR, U5MR for districts of Bihar, CEB, CS and total numbers of women in child bearing ages are taken from Indian Census data of 1991, 2001 and 2011 of Bihar. Moreover, in selecting suitable standard model life table for Bihar, each of five United Nations model life tables for developing countries (1983) and four Princeton models-Coale & Demeny model life tables (1966) [7] is examined with the help of some techniques to find out which standard model is most appropriate for Bihar. Moreover, for estimating life expectancy at birth for Bihar, 23 set of SRS-based abridged life tables of Bihar are taken into consideration to use in the second degree polynomial model.

2.2 Methods
2.2a Selection of a Model Life Table (MLT) for Bihar
The appropriate MLT family has been selected by using the following procedure.

At first, in graphical method, the estimates of $_4q_1$ are plotted against those of $_1q_0$ (Hill, 2013) on a graph showing the corresponding relationships in MLTs, shown in Figure 1. Although observed data of Bihar reflects trivial nature in this graphical representation, somehow the graph of MLT which is closely fitted to that of observed data is selected. Secondly, we calculate $\sum_{L=1}^{n}\left(1-\frac{m_x}{M_x}\right)^2$, where m_x is the Age Specific Mortality Rate (ASMR) of observed data, M_x is the ASMR of standard life table, L is the level that corresponds to that of observed data. The model set that minimizes the sum is taken as appropriate model life, shown in Table 1. In both methods, Bihar's SRS data is found to be most compatible with United Nations General Patterns of model life tables.

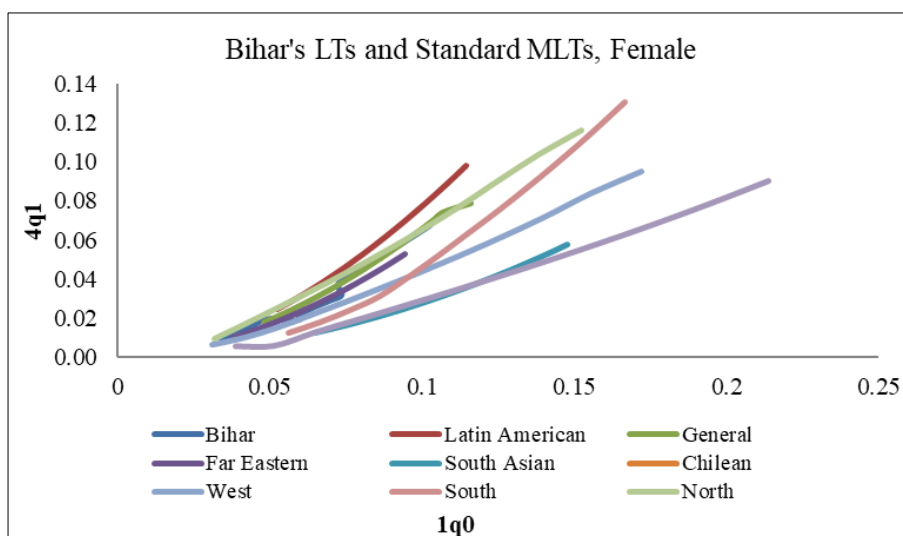**(i) Bihar, Male**
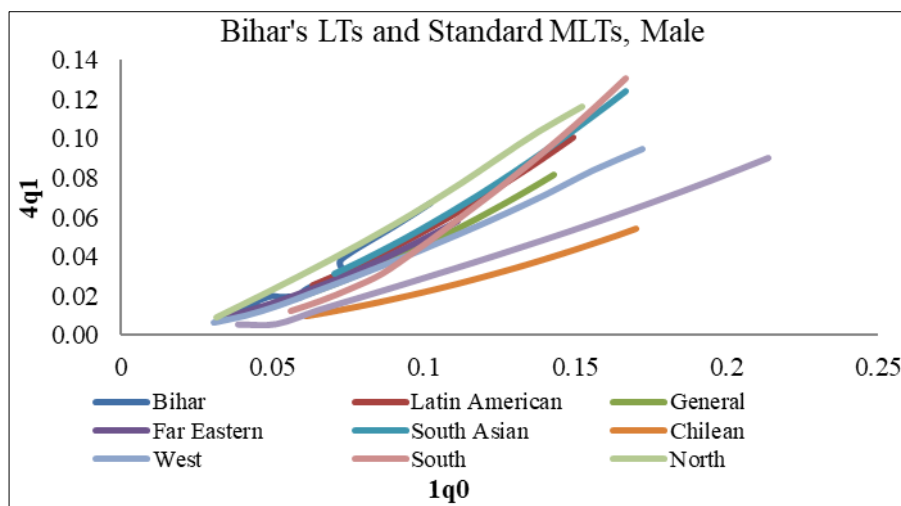
(ii) Bihar, Female

Fig 1: Comparison: Bihar's LTs and Standards MLTs, (i) Male and (ii) Female

Table 1: for male and female of Bihar

States/ Models	United Nations MLTs					Coale and Demeny MLTs			
	Chilean	Far Eastern	General	Latin American	South Asian	West	North	South	East
Bihar- Male	5.26	12.09	1.92	2.37	3.66	12.29	10.14	11.14	13.26
Bihar- Female	1.58	2.99	1.33	3.6	4.32	9.61	8.51	9.71	9.36

After obtaining the suitable MLT for Bihar, we followed the similar steps used by Phukon and Ahamed (2019) [1] in their paper, i.e., Trussell's Variant of Brass method and Weibull survival function along with logit smoothing are used to compute the l_x values.

2.2b Polynomial Regression Model
Phukon and Ahamed (2019) [1] developed a polynomial regression model from the perspective of the changes in mortality in the first year of life strongly affect e_0 and, at the same time, taking into consideration that child survivorship functions at early ages, i.e., (0-4) are inextricably related to e_0 and hence they proposed the following polynomial model as,

$$e_0(t) = \alpha_2 + \beta_2 * l_1 + \varphi_2 * l_2 + \eta_2 * l_3 + \kappa * l_4 + \xi * l_4^2$$

Where $\alpha_2, \beta_2, \varphi_2, \eta_2, \kappa, \xi$ are the six parameters of the second degree polynomial equation? By using the SRS-based abridged life tables data for Bihar from 1995-99 to 2012-16, the polynomial regression models are formulated for male and female separately as,

(a) $e_{0,m}(t) =$ 867.3+826.7 l_1-5307.9 l_2+6982.2 l_3-4251.27 l_4+960.78 l_4^2 and

(b) $e_{0,f}(t) =$ -588.79-1386.83 l_1+8453.90 l_2-14739.7 l_3+9031.8 l_4-700.2 l_4^2

$e_{0,m}(t), e_{0,f}(t)$ are LEB estimates at time t for male and female.

2.2c Logit Polynomial model for projection
A second degree logit polynomial model is fitted as, $Y = ax^2 + bx + c$, where, $Y = logit\ IMR$ for estimated IMR is fitted, x is the year starting from the 1977 taking as a base year i.e., (1977=0, 1980=3, till 2009=32), a, b, c are coefficients of the model. From the fitted curves, the values of Y are projected. And from the projected Y, projected IMR is sequentially calculated. The second-degree logit polynomial models are fitted to the data provided by estimates of all the four demographic indicators IMR, U3MR, U5MR by using Hill's method (2013) for reference years.

LEBs are estimated as follows
Each of the IMR obtained from reference years (1977 to 2009) is given as input in the BESTFT of MORTPAK SOFTWARE with the simultaneous selection of suitable standard model (i.e., UN-General model) and estimates of survivorship probabilities at ages one, two, three, four l_1, l_2, l_3, l_4, corresponding to input IMR is obtained. Thus for all the reference years obtained from Hill's method (2013), estimates of l_1, l_2, l_3, l_4, are obtained; these estimates are fed to the fitted polynomial regression equation of Bihar (Phukon and Ahamed, 2019) [1] to obtain LEB for each reference year.
The initial values of IMR, U3MR and U5MR are estimated from the census year 1991, 2001 and 2011, CEB and CS of five major districts of Bihar *viz*., Gaya, Madhubani, Muzaffarpur, Patna and Purba Champaran, by using the Trussell's variant of Brass method. Hill's (2013) method is also used to calculate IMR, U3MR and U5MR for the reference years 1977-2009. After estimating the IMR, U3MR, U5MR and LEB from the reference period for the years 1977-2009, projection is done with the help of fitted logit second-degree polynomial equations.

3. Results and Discussions
In Tables 2 to 3 reference years data which are estimated using Hill's (2013) method from the three consecutive census years 1991, 2001 and 2011, for the five major districts of Bihar are shown. Based on the reference years data from 1977-2009 for the five districts of Bihar, (Gaya, Madhubani, Muzaffarpur, Patna and Purba Champaran), a projection is done for all the four demographic indicators, such as, IMR, U3MR, U5MR and LEB for sex and rural-urban differentials.
In Table 2 and 3, estimated/projected IMR and U3MR for Gaya district have shown a decreasing trend for future in both the case. While inspecting the future trend of IMR in between rural and urban it has been indicated that the gender gap is higher in rural areas than the urban areas of Gaya district. But while inspecting U3MR and U5MR the gender gap is more or less similar in between rural and urban. The gap of gender is becoming lesser and lesser with the progressing years. In case of IMR, the gender gap, for rural, are shown as 0.0102 in 2010, 0.0011 in 2015 and will be 0.007 in 2027. In case of U3MR for 2010 the gender gap, for rural, is 0.004, in 2015 the gap is 0.0026 and in 2027 the gap will be 0.0003. In case of U5MR, the gap for rural, is shown as 0.0012 in 2010 and the gap will be diminished till 0.00021 in 2027. While inspecting life expectancy at birth (LEB), male LEB of Gaya district is higher than the female for both rural and urban.

Table 2: Estimated/Projected IMR and U3MR for Gaya from 2010 to 2030

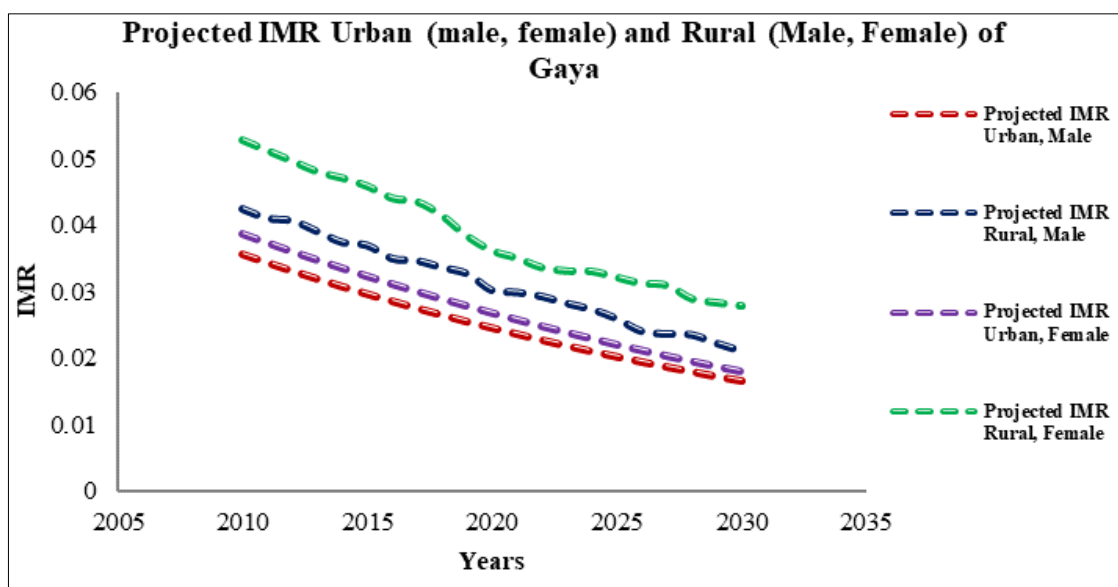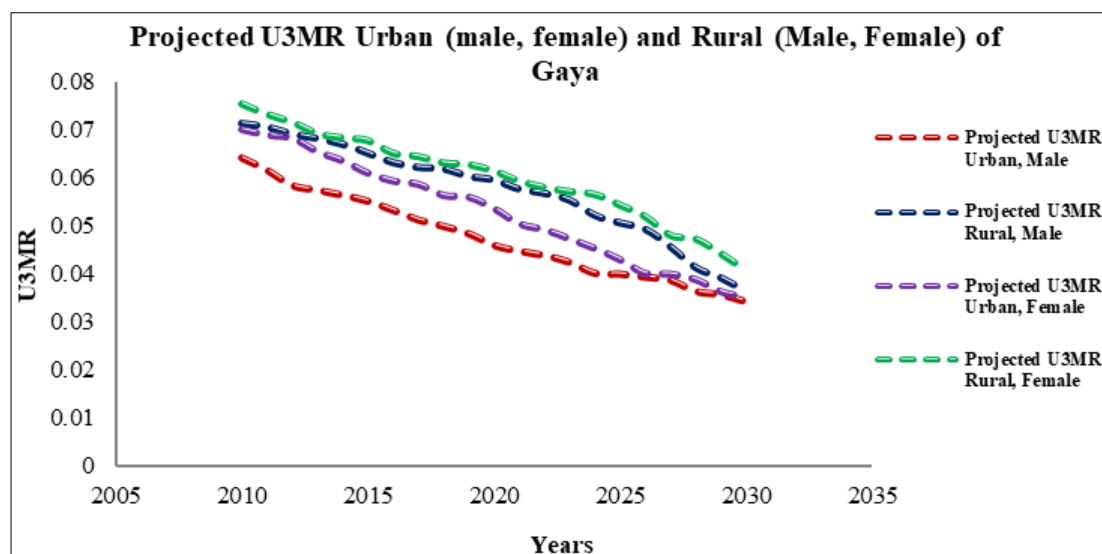| Year	Projected IMR Urban		Projected IMR Rural		Projected U3MR Urban		Projected U3MR Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.03562	0.03861	0.04255	0.05283	0.06421	0.06995	0.07142	0.0755
2011	0.03435	0.03728	0.04115	0.05126	0.06167	0.06879	0.07067	0.07336
2012	0.03312	0.03597	0.04076	0.04967	0.05842	0.06815	0.06922	0.07168
2013	0.03193	0.03471	0.03915	0.04804	0.05746	0.06534	0.06829	0.06934
2014	0.03077	0.03348	0.03752	0.04709	0.05641	0.06344	0.06706	0.06844
2015	0.02965	0.03228	0.03706	0.04586	0.05514	0.06088	0.06521	0.06788
2016	0.02857	0.03111	0.03505	0.04403	0.05325	0.05922	0.06322	0.06522
2017	0.02752	0.02998	0.03473	0.04348	0.05123	0.05856	0.06223	0.06456
2018	0.0265	0.02888	0.03369	0.04138	0.04989	0.05622	0.06189	0.06322
2019	0.02551	0.02781	0.03279	0.03828	0.04834	0.05587	0.06034	0.06287
2020	0.02456	0.02677	0.03027	0.03607	0.0459	0.05352	0.05959	0.06152
2021	0.02364	0.02576	0.02996	0.03497	0.04467	0.05034	0.05767	0.05944
2022	0.02274	0.02478	0.02935	0.03352	0.04378	0.04911	0.05678	0.05811
2023	0.02188	0.02384	0.02828	0.03302	0.04234	0.04722	0.05534	0.05722
2024	0.02105	0.02292	0.02738	0.03296	0.04002	0.04522	0.05216	0.05663
2025	0.02024	0.02203	0.02598	0.03204	0.03987	0.04289	0.0506	0.05439
2026	0.01946	0.02116	0.02406	0.03113	0.03925	0.04001	0.04925	0.05202
2027	0.01871	0.02033	0.02371	0.03085	0.03857	0.03989	0.04557	0.04809
2028	0.01799	0.01952	0.02357	0.0288	0.03623	0.03874	0.04122	0.04735
2029	0.01728	0.01874	0.02226	0.02822	0.03556	0.03623	0.03906	0.04423
2030	0.01661	0.01799	0.02108	0.02773	0.03405	0.035048	0.03625	0.040812

Fig 2a: Projected IMR Urban (male, female) and Rural (Male, Female) of Gaya

Fig 2b: Projected U3MR Urban (male, female) and Rural (male, female) of Gaya

Table 3: Estimated/Projected U5MR and LEB for Gaya from 2010 to 2030

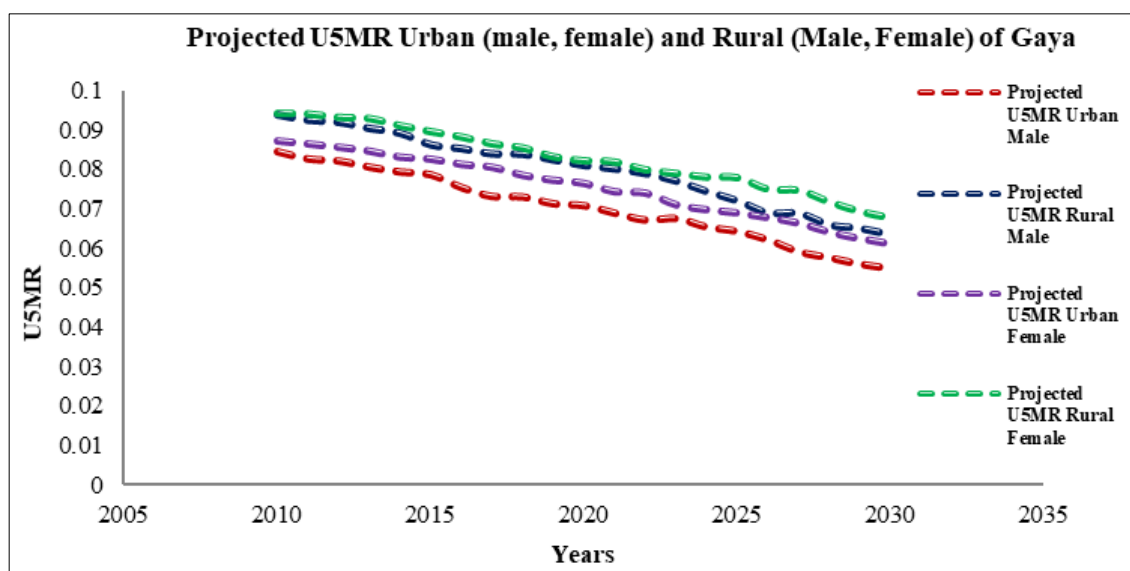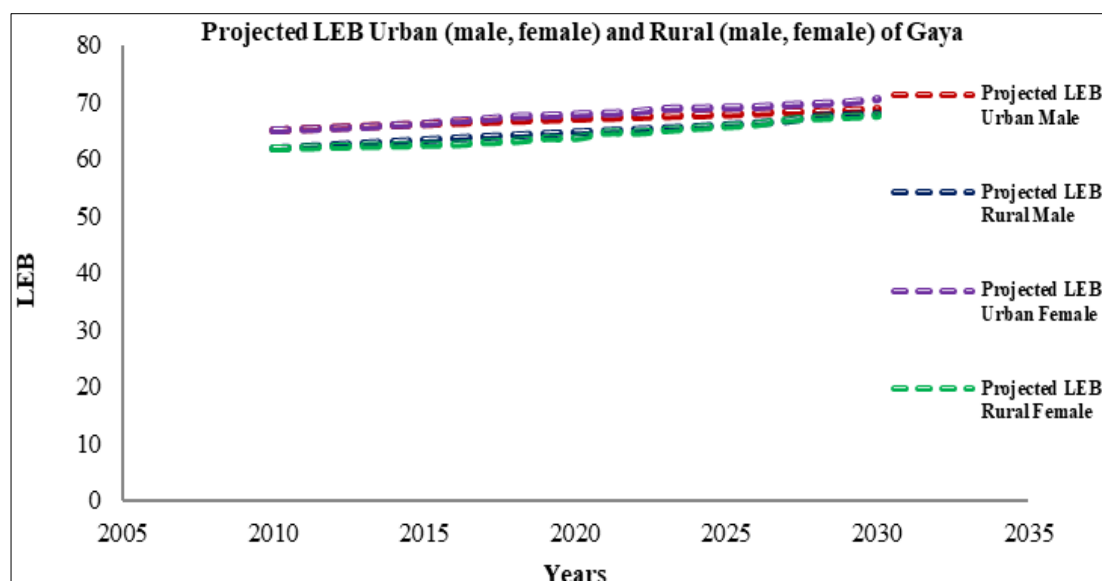| Year	Projected U5MR Urban		Projected U5MR Rural		Projected LEB Urban		Projected LEB Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.08436	0.08712	0.09389	0.0941	64.87	65.01	61.75	61.69
2011	0.08255	0.08637	0.09255	0.09399	65.12	65.23	62.06	61.86
2012	0.08201	0.08555	0.09201	0.09315	65.37	65.44	62.38	62.02
2013	0.08045	0.08456	0.09045	0.092856	65.61	65.63	62.68	62.15
2014	0.07922	0.08305	0.08922	0.09105	65.84	65.98	62.98	62.27
2015	0.07853	0.08255	0.08631	0.08955	66.07	66.19	63.27	62.37
2016	0.07529	0.08122	0.08533	0.08822	66.28	66.75	63.56	62.45
2017	0.07302	0.08048	0.08402	0.08648	66.49	67.03	63.84	62.85
2018	0.07289	0.07846	0.08389	0.08546	66.69	67.54	64.12	63.05
2019	0.07115	0.07722	0.08248	0.08322	66.88	67.66	64.38	63.58
2020	0.07067	0.07642	0.08107	0.08223	67.07	67.88	64.65	63.59
2021	0.06876	0.0743	0.0802	0.08193	67.24	68.07	64.9	64.58
2022	0.06701	0.07398	0.079	0.07998	67.41	68.33	65.16	64.56
2023	0.0674	0.07101	0.07714	0.07905	67.57	68.94	65.4	65.05
2024	0.06526	0.06978	0.07446	0.07811	67.72	68.99	65.64	65.45
2025	0.06413	0.06897	0.07213	0.07797	67.87	69.11	65.97	65.77
2026	0.062	0.06778	0.06904	0.07505	68.12	69.23	66.29	66.12
2027	0.05897	0.06638	0.06897	0.07478	68.33	69.58	66.72	66.95
2028	0.05745	0.06407	0.06601	0.07187	68.35	69.79	67.54	67.22
2029	0.05582	0.06248	0.06512	0.06944	68.66	70.06	67.74	67.46
2030	0.05472	0.06113	0.06352	0.06789	68.91	70.64	67.95	67.69

Fig 2c: Projected U5MR Urban (male, female) and Rural (male, female) of Gaya

Fig 2d: Projected LEB Urban (male, female) and Rural (male, female) of Gaya

Table 4: Estimated/Projected IMR and U3MR for Madhubani, from 2010-30

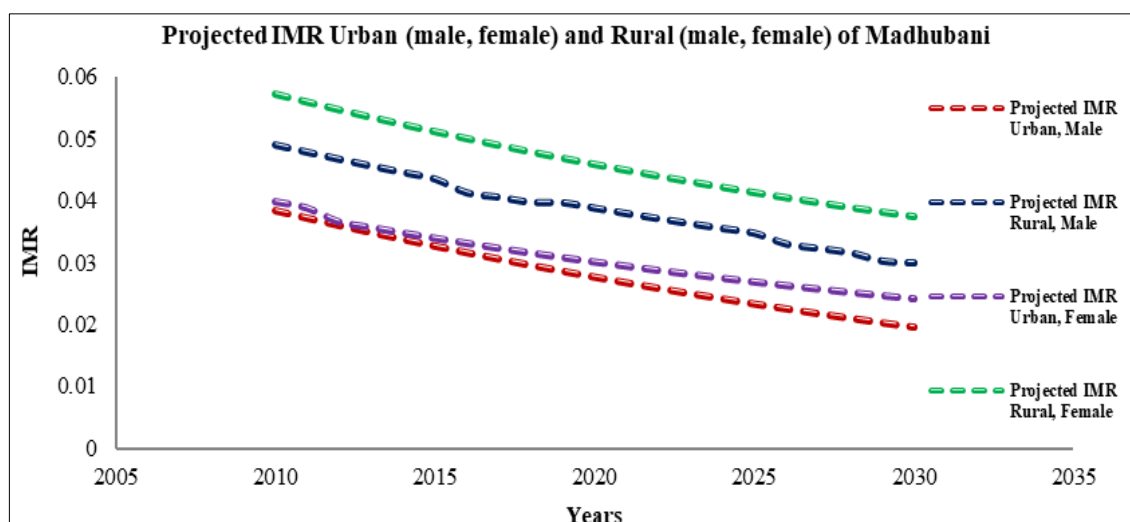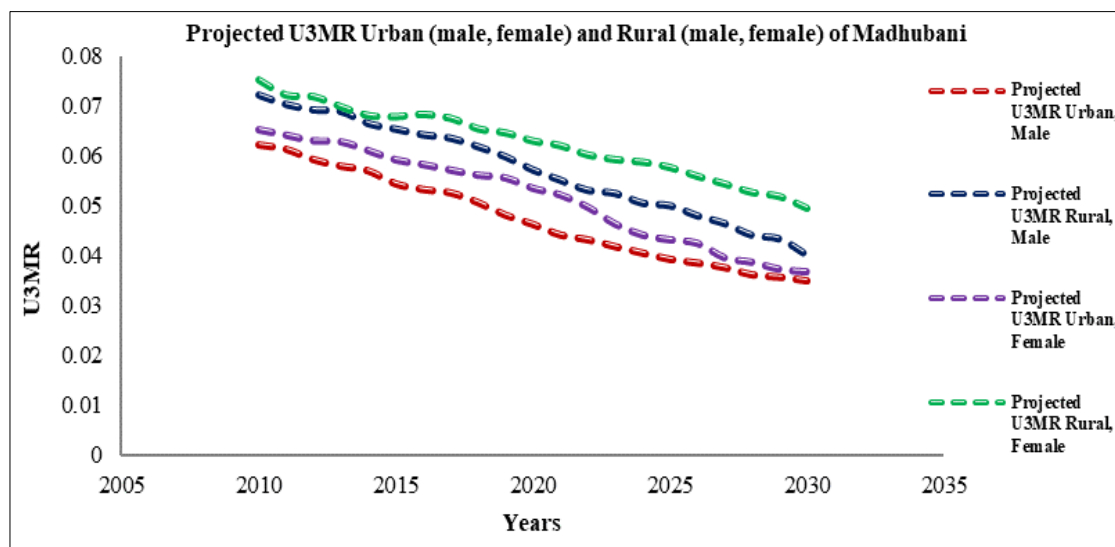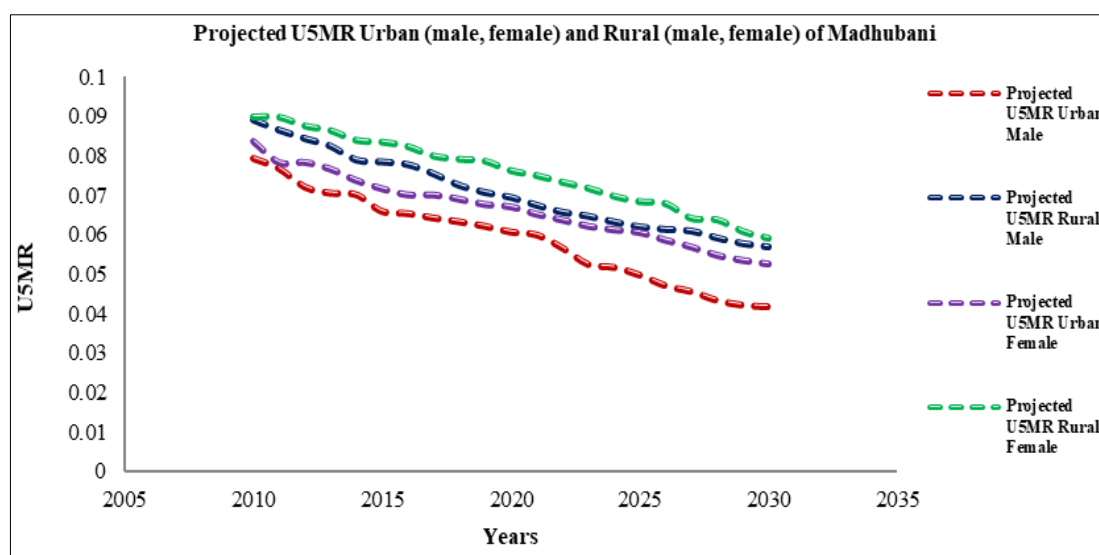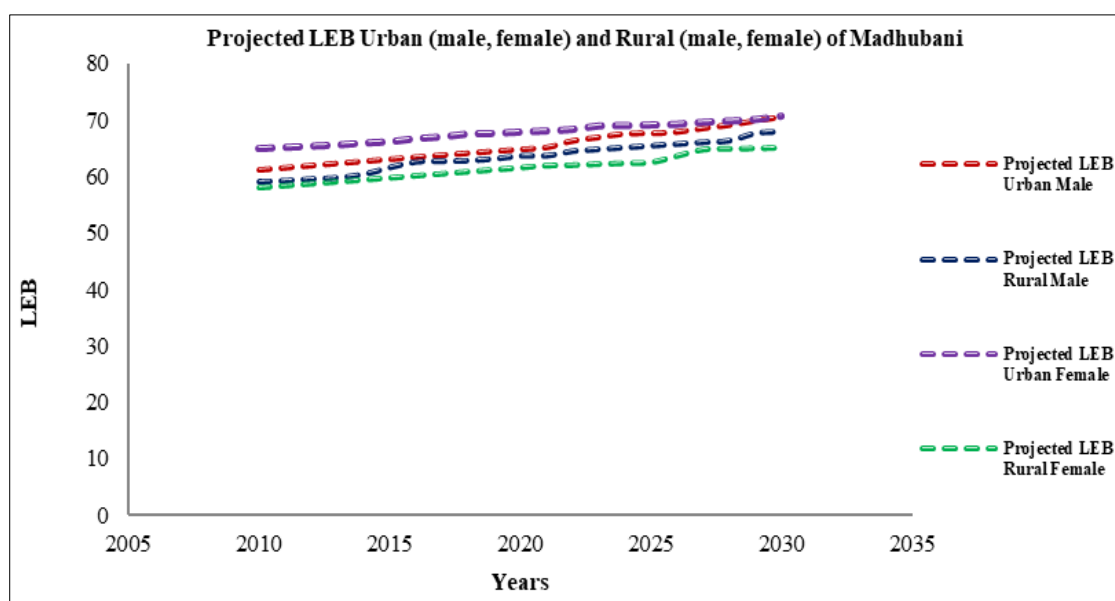| Year	Projected IMR Urban		Projected IMR Rural		Projected U3MR Urban		Projected U3MR Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.03836	0.03995	0.04905	0.05712	0.06222	0.06534	0.07222	0.07534
2011	0.03715	0.03891	0.04789	0.05585	0.06133	0.06426	0.07033	0.07226
2012	0.03597	0.03665	0.04676	0.05461	0.05923	0.06315	0.06923	0.07198
2013	0.03482	0.03573	0.04566	0.0534	0.05793	0.06293	0.06893	0.06991
2014	0.0337	0.03483	0.0446	0.05223	0.05698	0.06115	0.06648	0.06815
2015	0.03261	0.03397	0.04357	0.05109	0.05437	0.05931	0.06537	0.06791
2016	0.03155	0.03314	0.04133	0.04997	0.05322	0.05836	0.06421	0.0684
2017	0.03051	0.03233	0.04064	0.04889	0.05255	0.05725	0.06355	0.0676
2018	0.02951	0.03155	0.03981	0.04784	0.05059	0.05624	0.06182	0.06547
2019	0.02853	0.0308	0.03975	0.04682	0.04805	0.05569	0.059805	0.06464
2020	0.02758	0.03008	0.03886	0.04583	0.04622	0.05359	0.05716	0.06301
2021	0.02666	0.02938	0.038	0.04487	0.04406	0.05226	0.0551	0.06206
2022	0.02576	0.0287	0.03717	0.04393	0.0431	0.04981	0.05305	0.06018
2023	0.02489	0.02804	0.03636	0.04303	0.0417	0.04642	0.05244	0.05924
2024	0.02405	0.02741	0.03558	0.04215	0.04041	0.04422	0.05047	0.05875
2025	0.02322	0.0268	0.03482	0.0413	0.03915	0.04327	0.04998	0.05767
2026	0.02243	0.02621	0.03308	0.04047	0.03841	0.04257	0.04793	0.05587
2027	0.02165	0.02564	0.03237	0.03967	0.03747	0.03962	0.04637	0.05432
2028	0.0209	0.02509	0.03168	0.0389	0.03603	0.03876	0.04401	0.05259
2029	0.02017	0.02455	0.03034	0.03815	0.03556	0.03735	0.04328	0.05178
2030	0.01946	0.02404	0.030035	0.03743	0.0348	0.03683	0.04002	0.04939

Fig 3a: Projected IMR Urban (male, female) and Rural (Male, Female) of Madhubani

Fig 3b: Projected U3MR Urban (male, female) and Rural (Male, Female) of Madhubani

Table 5: Estimated/Projected U5MR and LEB for Madhubani, from 2010-30

Year	Projected U5MR Urban		Projected U5MR Rural		Projected LEB Urban		Projected LEB Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.07925	0.0836	0.08925	0.09004	61.17	62.08	59.16	58.11
2011	0.07668	0.07823	0.08668	0.08997	61.56	62.19	59.33	58.46
2012	0.07201	0.07827	0.08452	0.08775	61.95	62.77	59.63	58.8
2013	0.07045	0.07655	0.08258	0.08655	62.33	63.99	59.87	59.15
2014	0.07005	0.07364	0.07905	0.08405	62.7	64.27	60.45	59.49
2015	0.06581	0.07156	0.07858	0.08363	63.07	64.53	61.64	59.84
2016	0.06529	0.07001	0.07788	0.08245	63.43	64.79	62.63	60.2
2017	0.06415	0.06999	0.07553	0.08004	63.79	65.03	62.73	60.55
2018	0.06321	0.06889	0.07259	0.07925	64.14	65.26	62.84	60.91
2019	0.06207	0.06757	0.07078	0.078723	64.49	65.48	63.15	61.26
2020	0.06059	0.06698	0.06957	0.076298	64.83	65.69	63.66	61.62
2021	0.05987	0.06506	0.06754	0.07506	65.16	66.88	63.69	61.99
2022	0.05655	0.06363	0.06577	0.07334	66.32	67.35	64.49	62.07
2023	0.0524	0.06204	0.06477	0.07181	66.95	67.72	64.81	62.24
2024	0.05163	0.06114	0.06329	0.06978	67.59	68.08	65.12	62.4
2025	0.04968	0.06045	0.06204	0.06838	67.64	68.45	65.43	62.54
2026	0.047	0.05848	0.06138	0.06789	67.89	68.83	65.73	63.68
2027	0.04549	0.05678	0.06097	0.06412	68.55	69.62	66.03	64.8
2028	0.04328	0.054636	0.05922	0.06378	69.12	70.58	66.32	64.91
2029	0.04206	0.05336	0.05779	0.06077	69.89	71.95	67.61	65.01
2030	0.04166	0.052451	0.05693	0.05901	70.56	72.33	67.89	65.1

Fig 3c: Projected U5MR Urban (male, female) and Rural (Male, Female) of Madhubani

Fig 3d: Projected LEB Urban (male, female) and Rural (Male, Female) of Madhubani

In Table 4 and 5 the estimated/projected IMR, U3MR, U5MR and LEB are given for the Madhubani district of Bihar from 2010-2030. While inspecting IMR for Madhubani district of Bihar, gender gap is higher in rural areas than the urban. In case of U3MR of Madhubani district female shows higher in U3MR than that of male, the hazards of death of female children population are more than the male children population for this district. While inspecting U5MR, it is found highest for rural female children. On the other hand, LEB for Madhubani district has shown that female life expectancy is better than that of the male.

Table 6: Estimated/Projected IMR, U3MR for Muzaffarpur, from 2010-30

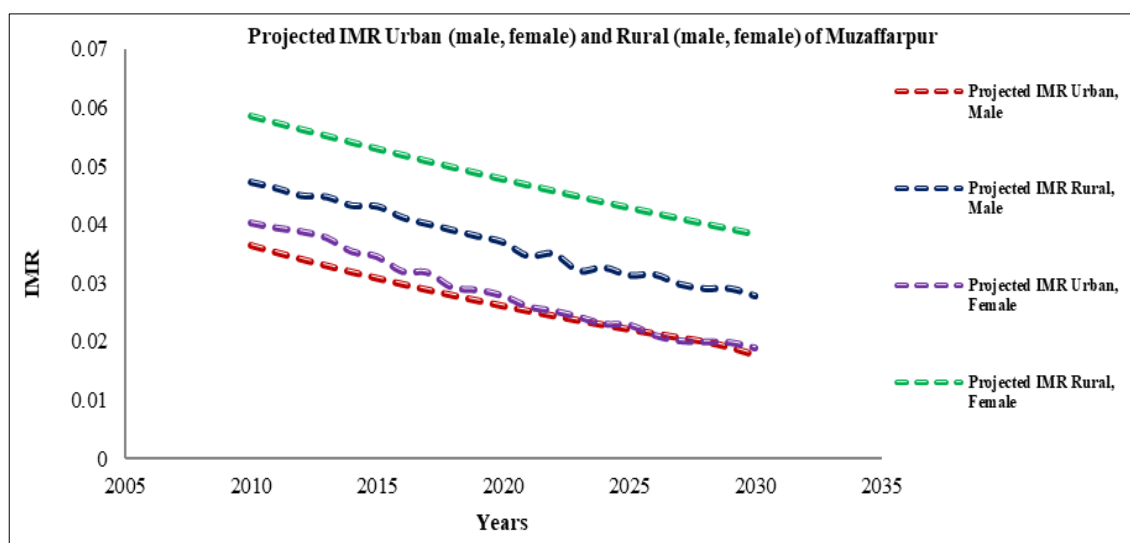| Year	Projected IMR Urban		Projected IMR Rural		Projected U3MR Urban		Projected U3MR Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.03651	0.04035	0.04737	0.05865	0.05998	0.0664	0.06998	0.0764
2011	0.03531	0.0395	0.04634	0.0575	0.05735	0.06566	0.06855	0.07656
2012	0.03415	0.03888	0.04507	0.05636	0.05563	0.06472	0.06729	0.07229
2013	0.03303	0.03782	0.04484	0.05524	0.052093	0.06356	0.06666	0.07167
2014	0.03194	0.03547	0.04336	0.05413	0.049924	0.06234	0.06512	0.07012
2015	0.03089	0.03462	0.04325	0.05304	0.049109	0.06033	0.06467	0.06988
2016	0.02988	0.03209	0.04133	0.05196	0.04836	0.06019	0.06387	0.06887
2017	0.0289	0.03189	0.04022	0.05089	0.04713	0.05946	0.06285	0.06659
2018	0.02795	0.02933	0.03914	0.04984	0.047006	0.05855	0.06154	0.06549
2019	0.02703	0.02889	0.03809	0.0488	0.046979	0.057633	0.06023	0.06423
2020	0.02614	0.027905	0.03707	0.04778	0.04632	0.05611	0.05914	0.06314
2021	0.02528	0.02608	0.03471	0.04677	0.04563	0.05534	0.05826	0.06226
2022	0.02445	0.02535	0.03512	0.04578	0.044981	0.054551	0.05768	0.061668
2023	0.02365	0.02433	0.03218	0.0448	0.04448	0.053855	0.05639	0.06039
2024	0.02288	0.02311	0.03273	0.04383	0.04221	0.052502	0.05508	0.059008
2025	0.02213	0.02296	0.03139	0.04288	0.0402	0.05189	0.054	0.05897
2026	0.0214	0.02119	0.03154	0.04194	0.03976	0.050086	0.053974	0.05774
2027	0.0207	0.02025	0.02987	0.04102	0.03911	0.048276	0.052379	0.056789
2028	0.02002	0.020048	0.02912	0.04011	0.03676	0.04732	0.050664	0.05566
2029	0.019037	0.019958	0.02914	0.03922	0.03617	0.04695	0.04952	0.054552
2030	0.01773	0.019015	0.02785	0.03834	0.03512	0.04514	0.04854	0.05354

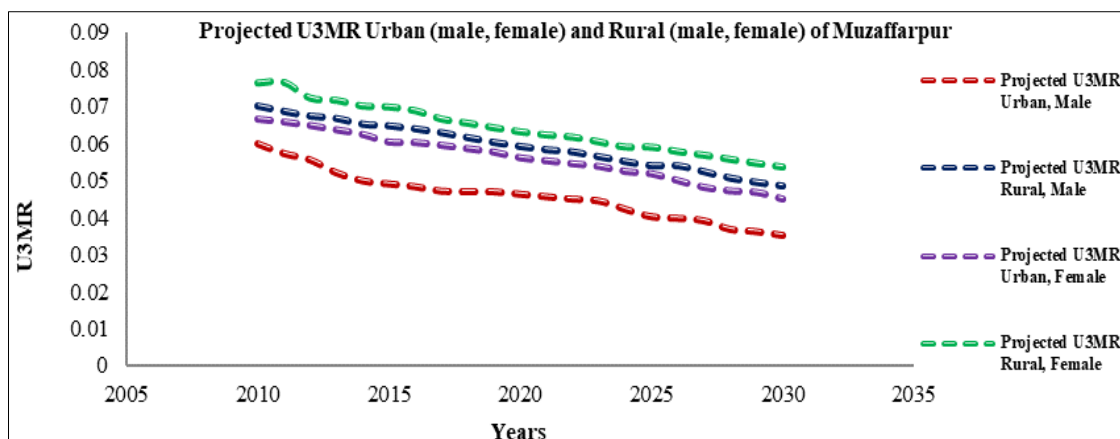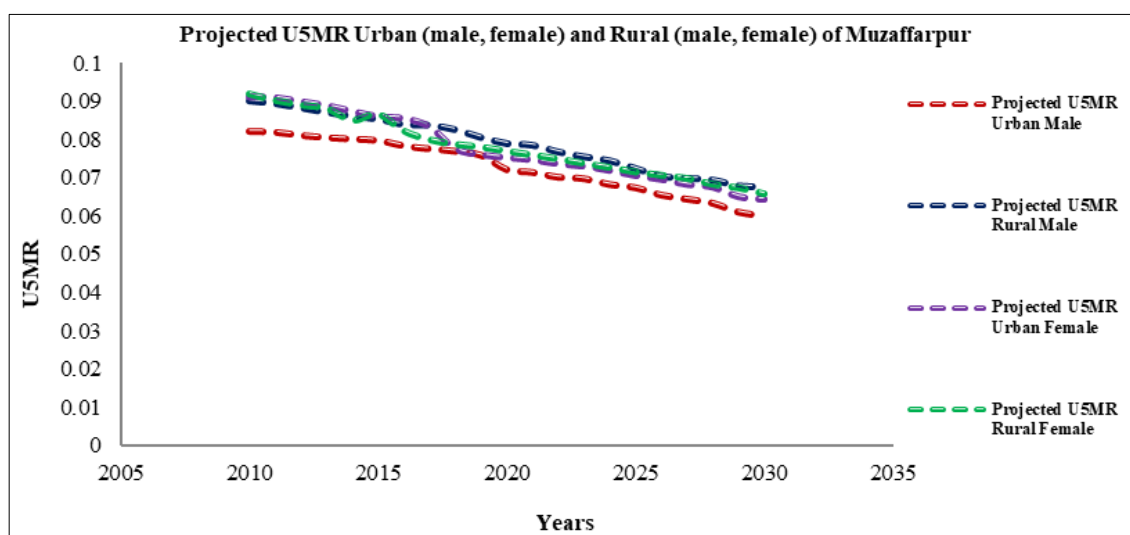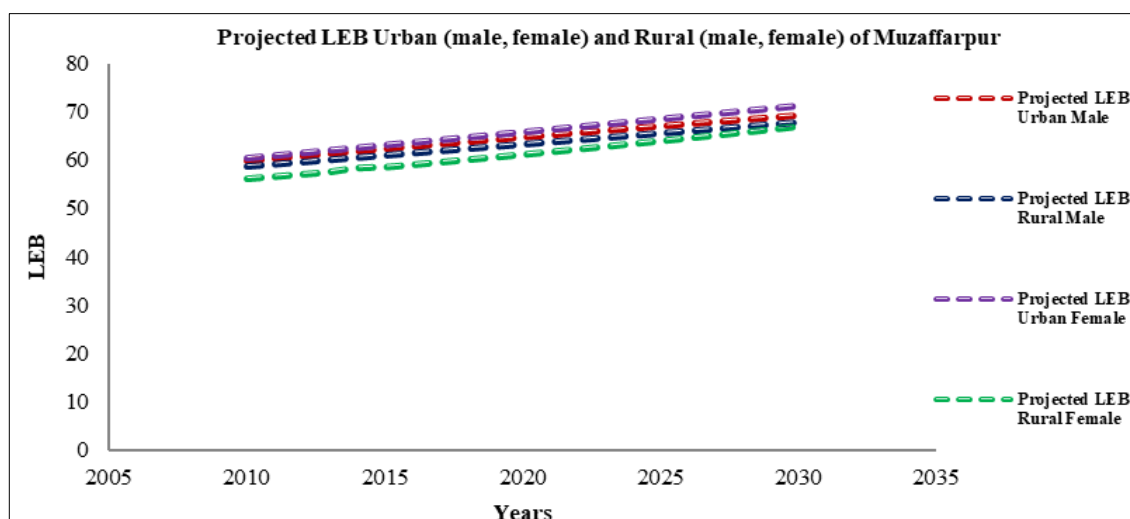**Fig 4a:** Projected IMR Urban (male, female) and Rural (Male, Female) of Muzaffarpur

Fig 4b: Projected U3MR Urban (male, female) and Rural (Male, Female) of Muzaffarpur

Table 7: Estimated/Projected U5MR and LEB for Muzaffarpur, from 2010-30

Year	Projected U5MR Urban		Projected U5MR Rural		Projected LEB Urban		Projected LEB Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.08206	0.09115	0.0903	0.0921	59.86	60.52	58.75	56.07
2011	0.08185	0.09097	0.0896	0.0904	60.36	61.06	59.21	56.54
2012	0.08102	0.08989	0.0885	0.0892	60.86	61.6	59.66	57.02
2013	0.08045	0.08888	0.0873	0.0883	61.35	62.14	60.12	57.51
2014	0.08005	0.08735	0.0863	0.0852	61.84	62.68	60.57	58.34
2015	0.07971	0.086	0.08555	0.0868	62.33	63.22	61.02	58.5
2016	0.07829	0.08548	0.08412	0.0824	62.81	63.75	61.48	59.01
2017	0.07753	0.08333	0.08397	0.0801	63.29	64.29	61.93	59.53
2018	0.07689	0.07755	0.082775	0.0789	63.76	64.83	62.38	60.06
2019	0.07583	0.07601	0.080745	0.0782	64.23	65.37	62.83	60.59
2020	0.07212	0.07525	0.07915	0.0771	64.7	65.91	63.28	61.13
2021	0.07139	0.074666	0.0787	0.0761	65.16	66.45	63.73	61.68
2022	0.07012	0.073609	0.07694	0.07504	65.61	66.99	64.18	62.24
2023	0.06975	0.072979	0.075678	0.0739	66.07	67.53	64.63	62.81
2024	0.06823	0.071872	0.074652	0.07282	66.52	68.07	65.07	63.38
2025	0.06739	0.070533	0.072644	0.07164	66.96	68.61	65.52	63.96
2026	0.06548	0.06955	0.070624	0.0708	67.41	69.15	65.97	64.55
2027	0.06442	0.068229	0.070061	0.06994	67.84	69.69	66.41	65.15
2028	0.06336	0.06752	0.069602	0.0684	68.28	70.23	66.86	65.76
2029	0.06105	0.065008	0.0681	0.0675	68.71	70.77	67.3	66.37
2030	0.06002	0.064283	0.0678	0.06589	69.13	71.31	67.75	66.99

Fig 4c: Projected U5MR Urban (male, female) and Rural (Male, Female) of Muzaffarpur

Fig 4d: Projected LEB Urban (male, female) and Rural (Male, Female) of Muzaffarpur

In Table 6 and 7 the estimated/projected IMR, U3MR, U5MR and LEB are given for the Muzaffarpur district of Bihar from 2010-2030. While inspecting IMR and U3MR for Muzaffarpur district of Bihar, gender gap is higher in rural areas than the urban. In case of U3MR of Muzaffarpur district female population is shown higher in U3MR than that of male, the hazards of death of female population are more than the male population for this district. While inspecting U5MR, rural female population is highest. For LEB of Muzaffarpur district, it is found that female life expectancy in urban area is found better than that of all other categories.

Table 8: Estimated/Projected IMR, U3MRfor Patna, from 2010-30

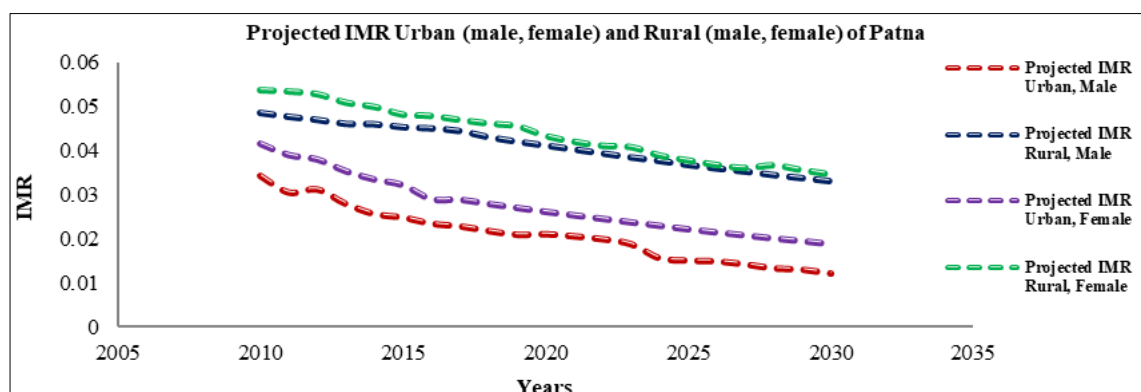| Year	Projected IMR Urban		Projected IMR Rural		Projected U3MR Urban		Projected U3MR Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.0344	0.0417	0.0485	0.0537	0.05622	0.05959	0.06005	0.06496
2011	0.0306	0.0391	0.04763	0.0534	0.05533	0.05863	0.05907	0.06275
2012	0.0313	0.0381	0.04694	0.0528	0.05425	0.05789	0.05887	0.06215
2013	0.028	0.0354	0.04603	0.0509	0.05021	0.05649	0.057936	0.06109
2014	0.0257	0.0335	0.045932	0.04994	0.04928	0.05602	0.05495	0.06015
2015	0.0249	0.03222	0.045261	0.0482	0.04976	0.05567	0.05389	0.05977
2016	0.0235	0.029	0.045	0.0479	0.04867	0.05479	0.05222	0.05808
2017	0.02289	0.0289	0.04439	0.04699	0.04678	0.05357	0.05005	0.05739
2018	0.02183	0.0279	0.043	0.04618	0.04582	0.05243	0.04924	0.05637
2019	0.020977	0.027	0.042	0.0456	0.04436	0.05048	0.04725	0.05418
2020	0.0211	0.0261	0.0411	0.0434	0.04325	0.04913	0.04424	0.0522
2021	0.0206	0.0253	0.0402	0.0421	0.04155	0.0487	0.043	0.05099
2022	0.01996	0.0245	0.0393	0.0412	0.04018	0.04646	0.04266	0.04913
2023	0.0188	0.0237	0.0384	0.0409	0.03891	0.04596	0.0408	0.04808
2024	0.0156	0.0229	0.0376	0.039	0.03702	0.0439	0.03902	0.04712
2025	0.01506	0.0221	0.0367	0.0379	0.03575	0.04089	0.03757	0.04525
2026	0.0149	0.0214	0.0359	0.03689	0.03414	0.03963	0.03523	0.0442
2027	0.0142	0.0207	0.0352	0.0362	0.03387	0.03757	0.03407	0.04329
2028	0.0133	0.02	0.0344	0.0368	0.03242	0.0363	0.03312	0.03929
2029	0.013	0.0194	0.0337	0.0357	0.03014	0.03557	0.03224	0.03798
2030	0.0121	0.0187	0.033	0.03473	0.02968	0.03456	0.03252	0.03654

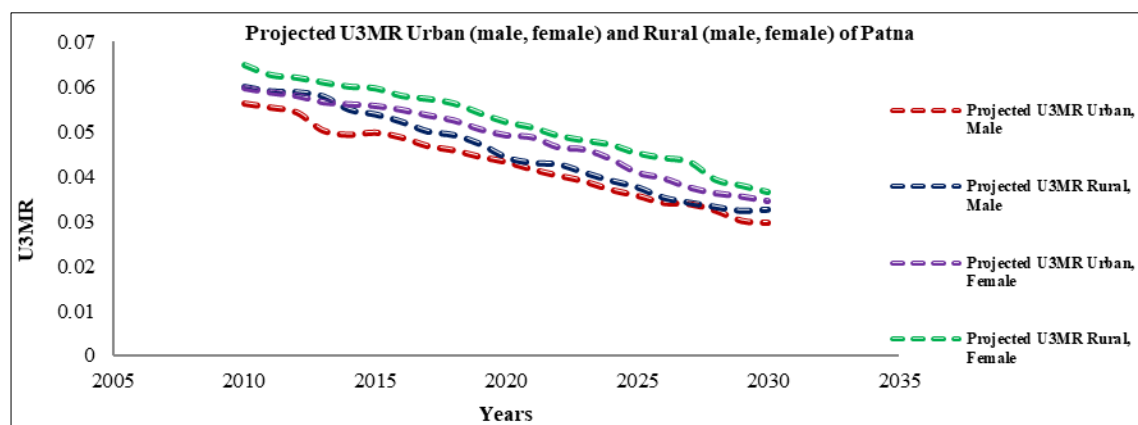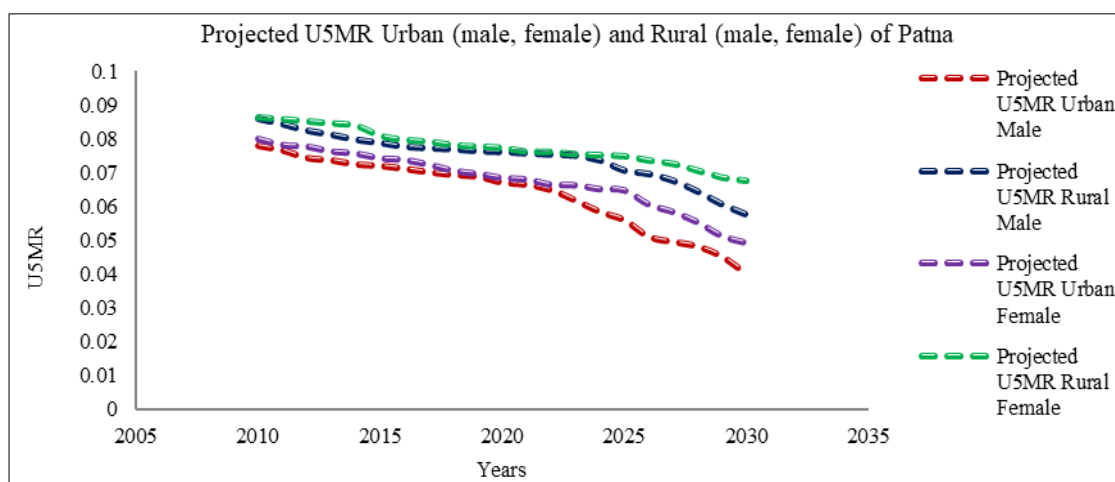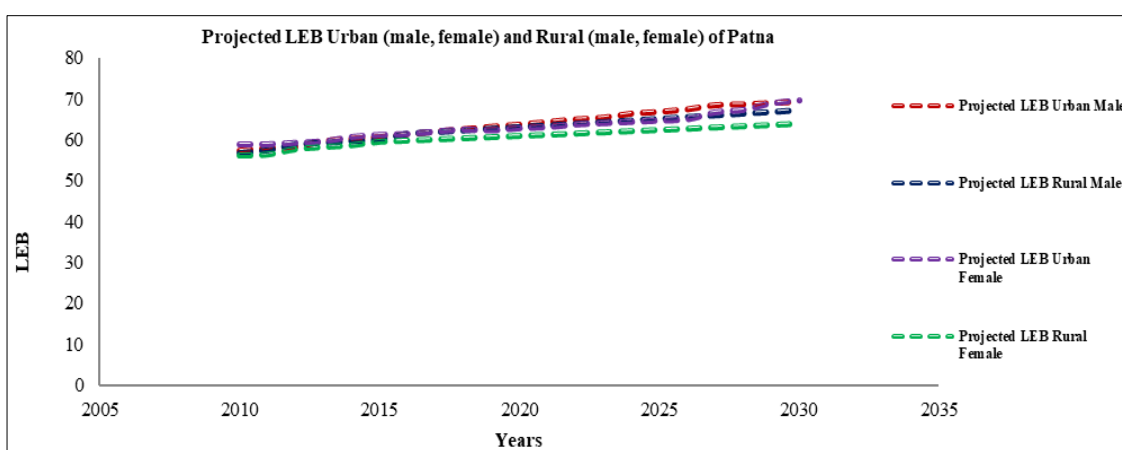**Fig 5a:** Projected IMR Urban (male, female) and Rural (Male, Female) of Patna

Fig 5b: Projected U3MR Urban (male, female) and Rural (Male, Female) of Patna

Table 9: Estimated/Projected U5MR and LEB for Patna, from 2010-30

Year	Projected U5MR Urban		Projected U5MR Rural		Projected LEB Urban		Projected LEB Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.07786	0.07985	0.0858	0.08623	57.3	58.91	56.86	56.11
2011	0.07647	0.07807	0.0842	0.0857	57.88	58.94	57.56	56.38
2012	0.07426	0.07758	0.0823	0.085078	58.22	59.31	58.96	57.65
2013	0.07356	0.07616	0.08107	0.084479	59.61	59.67	59.37	58.19
2014	0.07235	0.07556	0.0796	0.083777	60.17	60.73	59.77	58.62
2015	0.07194	0.07415	0.07863	0.080779	60.58	61.24	60.17	59.47
2016	0.07105	0.07366	0.07752	0.07955	61.28	61.47	61.58	59.75
2017	0.07002	0.07225	0.0771	0.079033	61.93	62.08	61.99	60.03
2018	0.06926	0.07046	0.07678	0.07801	62.51	62.56	62.39	60.31
2019	0.06874	0.06955	0.07623	0.077626	63.09	62.34	62.8	60.6
2020	0.067023	0.06831	0.07596	0.077163	63.62	62.86	63.21	60.88
2021	0.06636	0.067856	0.07558	0.07609	64.24	63.11	63.62	61.17
2022	0.06479	0.06633	0.07522	0.076009	64.87	63.77	64.03	61.46
2023	0.061679	0.06615	0.07489	0.07534	65.31	64.09	64.44	61.75
2024	0.05834	0.065008	0.073483	0.075048	66.27	64.41	64.85	62.05
2025	0.05588	0.064654	0.070478	0.07473	66.69	64.73	65.26	62.34
2026	0.05101	0.060444	0.069455	0.073467	67.31	65.04	65.68	62.64
2027	0.04946	0.05824	0.067448	0.072456	68.33	66.77	66.09	62.94
2028	0.04814	0.055201	0.06426	0.070453	68.49	67.34	66.51	63.24
2029	0.04512	0.05113	0.060421	0.068425	68.82	68.95	66.92	63.55
2030	0.04002	0.049105	0.057422	0.067427	69.17	69.66	67.34	63.85

Fig 5c: Projected U3MR Urban (male, female) and Rural (Male, Female) of Patna

Fig 5d: Projected LEB Urban (male, female) and Rural (Male, Female) of Patna

In Table 8 and 9, estimated/projected IMR, U3MR and U5MR for Patna district have shown a decreasing trend for future in the three cases. While inspecting the future trend of IMR and U3MR in between rural and urban it has been indicated that the gender gap is higher in 2018 for IMR in urban areas and it has shown highest gap in gender in rural areas in 2015. While comparing with the projected years of Patna district, in case of U3MR, highest gap between genders has been shown in urban areas in 2016, whereas in 2013 highest gap in gender is shown rural areas. While inspecting life expectancy at birth (LEB), male LEB of Patna district is higher than the female for both rural and urban.

Table 10: Estimated/Projected IMR, U3MR for Purba Champaran, from 2010-30

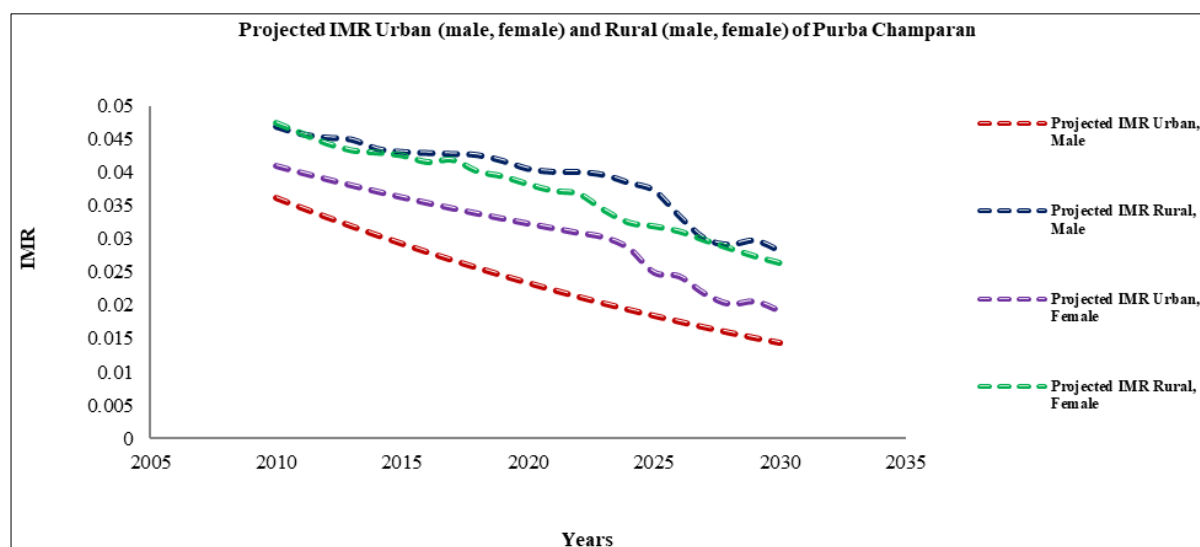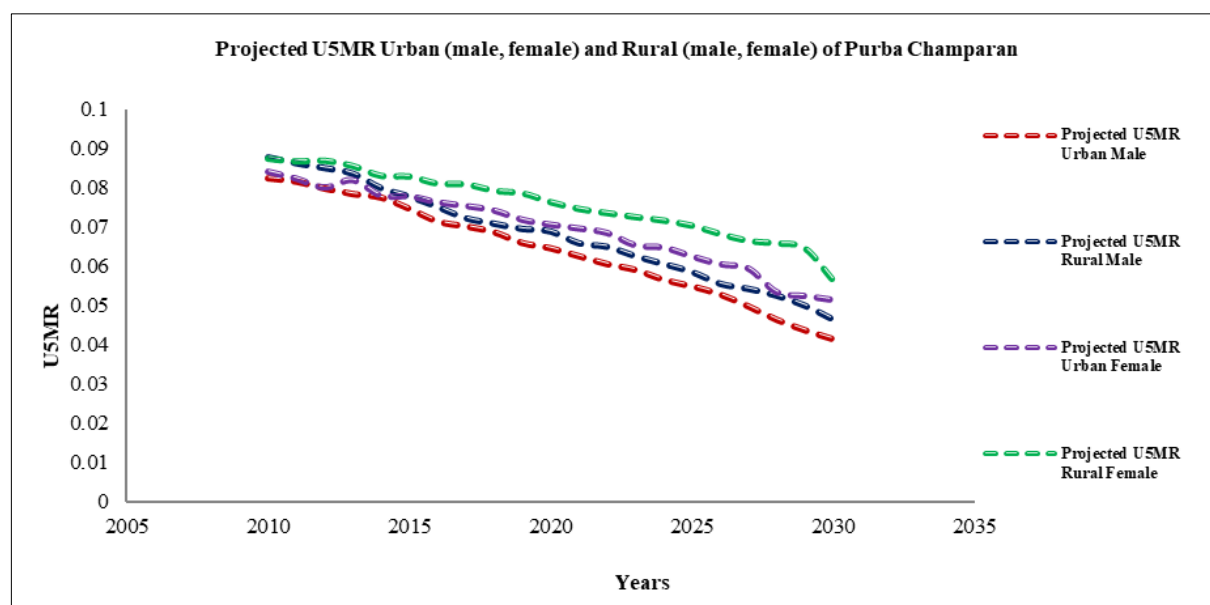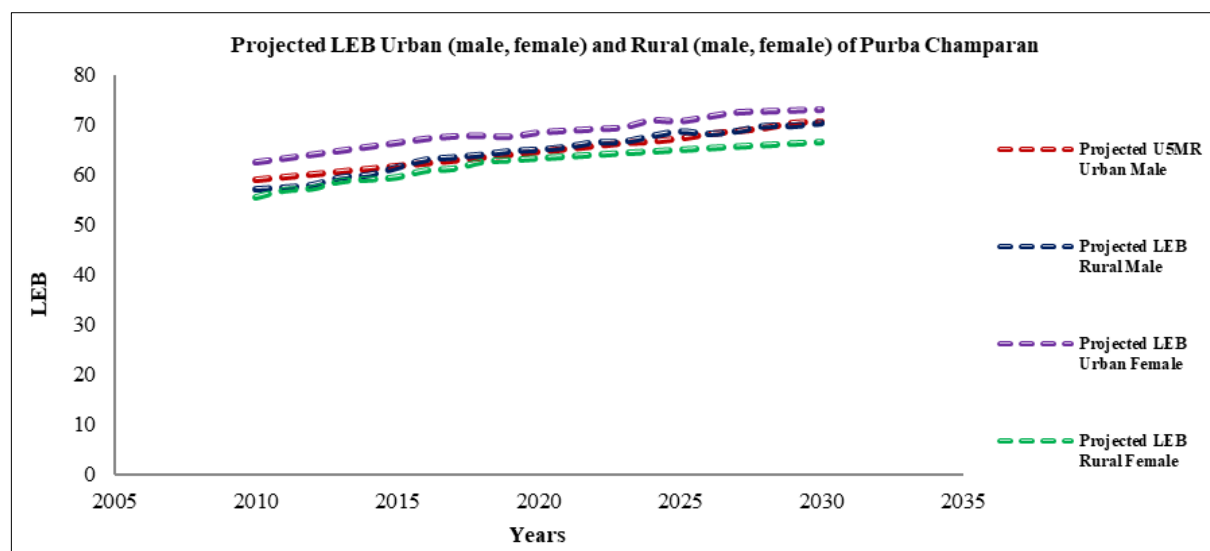| Year	Projected IMR Urban		Projected IMR Rural		Projected U3MR Urban		Projected U3MR Rural	
	Male	Female	Male	Female	Male	Female	Male	Female
2010	0.03621	0.0409	0.0469	0.04749	0.06522	0.06996	0.0659	0.06954
2011	0.03474	0.0399	0.04579	0.04594	0.06245	0.06657	0.06348	0.06826
2012	0.03332	0.03893	0.04526	0.04442	0.06026	0.06249	0.06258	0.06749
2013	0.03194	0.03799	0.04496	0.04334	0.05875	0.05965	0.06088	0.06634
2014	0.0306	0.03709	0.04358	0.0430092	0.05582	0.05586	0.05956	0.065667
2015	0.0293	0.03622	0.04311	0.04259	0.05225	0.05561	0.05823	0.0646832
2016	0.02804	0.03538	0.04295	0.041569	0.05116	0.05358	0.05586	0.0635456
2017	0.02682	0.03457	0.0428	0.0419031	0.050129	0.05305	0.05145	0.0625339
2018	0.02564	0.03379	0.04264	0.04016	0.04388	0.04896	0.04888	0.061239
2019	0.0245	0.03303	0.04176	0.0394147	0.04256	0.04689	0.04493	0.0615167
2020	0.0234	0.0323	0.04057	0.038234	0.04215	0.04651	0.04422	0.060112
2021	0.02233	0.0316	0.040122	0.03722	0.03847	0.04637	0.04316	0.05996
2022	0.0213	0.03092	0.04009	0.036781	0.03786	0.04565	0.0426	0.0579
2023	0.02031	0.03026	0.03967	0.034353	0.03666	0.04321	0.04081	0.05695
2024	0.01936	0.02863	0.03848	0.0324287	0.03512	0.04237	0.03932	0.0567
2025	0.01844	0.02502	0.037333	0.03187	0.03427	0.03964	0.03776	0.05558
2026	0.01756	0.02443	0.0335243	0.0310965	0.034154	0.03869	0.03637	0.054059
2027	0.01671	0.02186	0.030152	0.0297786	0.03337	0.03779	0.03671	0.053389
2028	0.01589	0.02031	0.02921	0.028545	0.03238	0.03658	0.03438	0.0483834
2029	0.0151	0.02068	0.029833	0.0273415	0.03089	0.03545	0.03209	0.0452338
2030	0.01435	0.01927	0.0283012	0.0263125	0.03136	0.03144	0.03236	0.0445335

Fig 6a: Projected IMR Urban (male, female) and Rural (Male, Female) of Purba Champaran
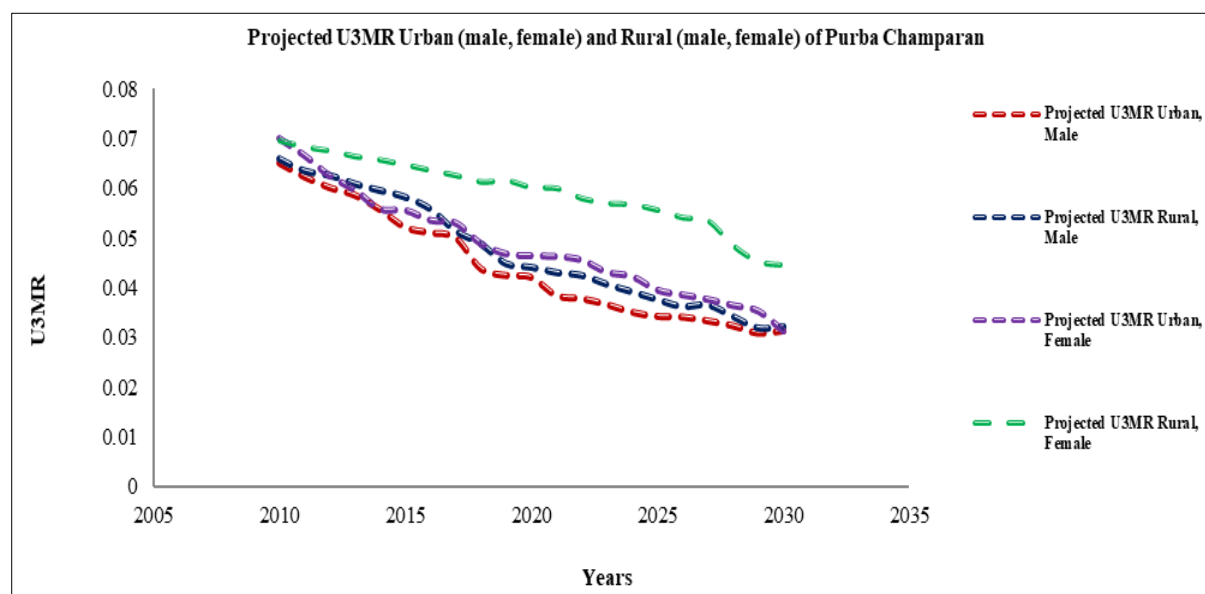
Fig 6b: Projected U3MR Urban (male, female) and Rural (Male, Female) of Purba Champaran

Table 11: Estimated/Projected U5MR, LEB for Purba Champaran, from 2010-30

Year	U5MR Urban		Rural		Urban		Rural	
	Male	**Female**	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**
2010	0.08248	0.08412	0.08775	0.08741	59.1	62.66	57.15	55.5
2011	0.08164	0.08237	0.08625	0.08669	59.64	63.41	57.5	56.91
2012	0.07986	0.08013	0.085014	0.08689	60.18	64.18	58.01	57.32
2013	0.07845	0.082	0.08358	0.08555	60.73	64.95	59.52	58.72
2014	0.07755	0.07802	0.07988	0.083023	61.28	65.73	60.03	59.11
2015	0.07467	0.07785	0.07774	0.082855	61.84	66.51	61.54	59.55
2016	0.07143	0.07635	0.07508	0.081052	62.4	67.31	63.06	60.88
2017	0.07016	0.07539	0.0722	0.08089	62.96	67.81	63.58	61.26
2018	0.06867	0.07425	0.07079	0.079246	63.53	67.92	64.1	62.63
2019	0.065896	0.07187	0.069448	0.07859	64.1	67.74	64.82	62.99
2020	0.0645575	0.0706489	0.06882	0.076336	64.68	68.57	65.14	63.35
2021	0.0625246	0.06967	0.065778	0.0746018	65.26	68.94	65.66	63.7
2022	0.0605006	0.0685216	0.06488	0.0735498	65.84	69.25	66.59	64.05
2023	0.059025	0.065363	0.0625009	0.0725305	66.43	69.61	66.72	64.39
2024	0.0564829	0.064789	0.0604829	0.0716481	66.72	70.96	67.85	64.72
2025	0.0548	0.0625465	0.0584446	0.0703473	67.32	70.82	68.78	65.05
2026	0.052722	0.0605467	0.0554101	0.0682467	68.22	71.7	68.31	65.37
2027	0.049661	0.0594557	0.05412	0.0664557	68.82	72.58	68.85	65.69
2028	0.04625	0.0534528	0.0524006	0.0658453	69.43	72.77	69.88	66
2029	0.04356	0.0524432	0.04982	0.0645742	70.44	72.97	69.92	66.3
2030	0.04126	0.0514359	0.04626	0.0564189	70.66	73.12	70.46	66.6

Fig 6c: Projected U5MR Urban (male, female) and Rural (Male, Female) of Purba Champaran

Fig 6d: Projected LEB Urban (male, female) and Rural (Male, Female) of Purba Champaran

In Table 10 and 11, estimated/projected IMR, U3MR and U5MR for Purba Champaran district of Bihar, have shown a decreasing trend for future in the three cases. While inspecting the future trend of IMR and U3MR in between rural and urban it has been indicated that the gender gap is higher in 2023 for IMR in urban areas. Also, while inspecting U5MR the gender gap is shown highest in 2019 for urban and in 2012 for rural areas of Patna district. While inspecting life expectancy at birth (LEB), female LEB of Purba Champaran district is higher than the male for both rural and urban.

4. Conclusion

The estimation and projection of a few demographic indicators done in this paper is based on procedural development in which the components of the procedure are highly efficient and most widely used techniques and methods. The data for IMR, U3MR, U5MR and LEB are not available for the said five districts of Bihar except for census data for years 1991, 2001, and 2011. However, we have extracted data for several consecutive years by using Hill's (2013) method which have facilitated the projection. Moreover, in selecting suitable Standard MLT, a procedure is made comprising two effective methods which would enhance precision. Trussell's variant of Brass method along with logit smoothing and use of Weibull survival function for indirect method estimation for LEB based CEB and CS data is also a prominent one. Therefore, the estimates/projections done for a few demographic indicators in the paper are justifiably credible and can be used for planning and implementing government policies, budget and resource allocation, decision-making processes for the said five backward districts of Bihar.

5. References

1. Phukon M, Ahamed IM. A few methods for estimating life expectancy. Journal of Scientific Research. 2019;11(3):321-338.
2. Hill K. Indirect estimation of child mortality, in Moultrie, T, R Dorrington, A. Hill, K, Hill, I. Timaeus and B. Zaba. Tools for Demographic Estimation, International Union for the Scientific Study of Population (IUSSP), France, 2013.
3. Samir KC, Wurzer M, Speringer M, Lutz W. Future population and human capital in heterogeneous India, PNAS. 2018;15(33):8328-8333.
4. Najafi F, Saeedi F, Bashiri Y, Tatari M. Estimating life expectancy at birth using direct method in Torbat Heydariyeh country in Iran during 2015-2016. Journal of Research in Medical and Dental Science. 2018;6(3):382-386.
5. Motlagh ME, Safari R, Karami M, Khoseavi A. Life expectancy at birth in rural areas based on corrected data of Iranian vital horoscope. Iranian J. Publ Health. 2012;41(9):18-24.
6. United Nations. Demographic models. Manual X: Indirect techniques for demographic estimation. Department of International Economic and Social Affairs, Population Studies. ST/ESA/SER.A/81. United Nations Publications, 1983.
7. Coale AJ, Demeny P. Regional model life tables and stable populations. Princeton University Press, Princeton, N J, 1966.