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**S Santhiya Sundararajan**  
M.Sc. Research Scholar,  
Department of Mathematics,  
PRIST Deemed to be University,  
Thanjavur, Tamil Nadu, India

**Dr. S Subramanian**  
Ph.D., Department of  
Mathematics, PRIST Deemed to  
be University, Thanjavur, Tamil  
Nadu, India

**Corresponding Author:**  
**S Santhiya Sundararajan**  
M.Sc. Research Scholar,  
Department of Mathematics,  
PRIST Deemed to be University,  
Thanjavur, Tamil Nadu, India

## Numerical simulation of plasticity identification and discussion of parameters

**S Santhiya Sundararajan and Dr. S Subramanian**

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### Abstract

Numerical studies play a major role in the understanding and prediction of plasticity. The numerical models simplify the reality by considering discrete crack propagations, a relatively high fatigue crack growth, sharp cracks and that propagation occur at a well-defined load. Besides, there are a great number of numerical parameters affecting the accuracy of the predictions. The aim of the present paper is to discuss the numerical simulation of PICC. The numerical parameters, affecting the accuracy of the numerical simulations, and the dependent parameters, adequate to characterise the plastic wake and the closure level, are identified. The influence of the parameters of the numerical algorithm, the radial size of crack front elements and the extent of crack propagation for stabilization is analysed. An intrinsic uncertainty is associated with the number of load cycles between crack increments and the definition of crack closure level. Finally, the model developed is used to study the effect of stress ratio (R) on crack closure level.

**Keywords:** Plasticity induced crack closure, finite element analysis, Middle-crack tension (M(T)), elastic-plastic, numerical parameters

### Introduction

This contact affects the local stress and plastic deformation fields near the crack tip, and therefore, the micro mechanisms responsible for fatigue propagation (cyclic plastic deformation, oxidation, creep, etc.). Therefore, crack closure is an extrinsic mechanism affecting the intrinsic damage mechanisms and the fatigue crack growth rate (FCGR), and must be considered in the design of components. The concept of fracture surface interaction leading to a decrease of stress intensity at the crack tip and to an increase of fatigue life was stated in 1963 discussed the concept in terms of fracture mechanics parameters, promoting a strong research effort into the mechanisms and phenomena associated with fatigue crack closure. According to Elber's understanding of crack closure, as the crack propagates due to cyclic loading, a residual plastic wake is formed. The deformed material acts as a wedge behind the crack tip and the contact of fracture surfaces is forced by the elastically deformed remote material. Under plane stress conditions, i.e., near the surface, this phenomenon happens due to the transportation of material to the interior of the sub-surface region. In this plasticity mechanism, both the current crack tip plastic zone and the residual plastic wake, from previous crack front positions, play a part. Crack closure seems to be able to explain the influence of mean stress in both regimes I and II of crack propagation and the transient crack growth behaviour following overloads among other aspects. identified the main closure mechanisms, which are plasticity induced crack closure oxide-induced crack closure and roughness induced crack closure. Additional mechanisms, such as viscous-fluid induced crack closure transformation-induced crack closure and graphite induced crack closure have been observed to operate in susceptible materials and environments. Anyway, there are researchers arguing that PICC does not exist (particularly for plain strain conditions), even suggesting that the plastic wake is responsible for crack opening and not for crack closure These authors proposed another approach of Keff, called Unified Approach. According to the Unified Approach the crack closure effects are only 20% of that estimated by ASTM E 647.

On the other hand, there is a huge amount of experimental, numerical and analytical work supporting the existence of crack closure and its influence on fatigue crack propagation and various methods have been employed to measure crack opening and crack closure levels experimentally (see Kumar, 1992, for example). Because the experimental techniques only give average values, the numerical approaches become an interesting tool to study crack closure along a 3D crack front. Furthermore, once the numerical procedure is optimized, it is relatively simple to adapt it to new load conditions, materials, crack lengths, etc. Nevertheless, the finite element models must be correctly defined and their limitations understood.

The numerical simulation of PICC is complicated due to the difficulties in modelling the complex material behaviour, the contact between crack faces during crack closure and the crack propagation rate. These complexities, and the large number of numerical and physical independent parameters affecting PICC, justify contradictory literature results. The main objective of present paper is to discuss the numerical study of PICC.

The numerical parameters, affecting the accuracy of the numerical simulations, and the dependent parameters, adequate to characterise the plastic wake and the closure level, are identified. The influence of the parameters of the numerical algorithm, the finite element discretization and the stabilization of the closure level on the quality of the numerical results is detailed discussed. Afterwards, the intrinsic uncertainty of the numerical predictions was studied, by analysing the influence of the closure definition and the number of load cycles between increments, on crack closure measurements. Finally, the influence of stress ratio on crack closure was studied.

### Objective

- There are several models that try to relate the occurrence of cyclic plastic deformation at the crack tip, with crack propagation and the formation of striations usually observed on the fracture surface of ductile materials.
- All the models are based on the fact that the fatigue crack propagation process is repetitive, so each of them try to explain the mechanism of crack propagation by explaining the process that happens during a single load cycle.
- The model of striation formation by crack tip plastic blunting of Laird is largely accepted as a general description of the propagation mechanism of fatigue cracks in regime II of. According to this model, plastic deformation at the crack tip is highly concentrated at 45°, producing blunting and creation of new fracture surfaces.
- Crack tip compression stresses reverses slipping, the fracture surfaces approach, but the new surface cannot be removed by re-connection of the atomic bonds, which is in accordance with the entropy law of thermodynamics.

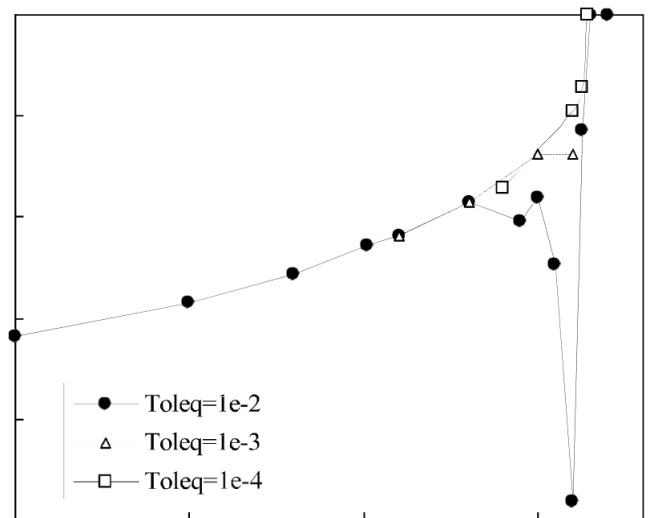
### Material Modelling

The elasto-plastic behaviour of the material was modelled as corresponding to the AA6016-T4 aluminium alloy. Three types of mechanical tests have been performed in order to study the hardening behaviour of this material: Uniaxial tensile tests and monotonic and Bauschinger simple shear tests. From the experimental data and curve fitting results, for different constitutive models, it was determined that the mechanical behaviour of this alloy is better represented using a isotropic hardening model described by a Voce type law combined with a kinematic hardening model described by a

saturation law (Bouvier, 2001). Material parameters were also determined considering pure kinematic and pure isotropic hardening models, for comparison with the mixed hardening model results.

### Framework

The optimum values for most of the numerical parameters of the DD3IMP implicit algorithm, used to implement the PICC model, have been well established in previous works concerning the numerical simulation of sheet metal forming processes (Menezes, 2000; Oliveira *et al.* 2004) and are adequate to be used in the study of the plastic deformation process occurring at crack front. The only numerical parameter optimised in this work was a user defined constant (Toleq) that determines the precision of the numerical results, and consequently, the velocity of the numerical simulation. Figure 6 presents the influence of Toleq on crack closure numerical results, expressed by U. The parameter optimization task was performed by determining the crack closure level (U) for a set of loading conditions characterized by increasing values of R (mean stress) and constant K. Analysing the figure it is possible to conclude that, as could be expected, the increase of R, fixing K, produces an increase of U, i.e., a reduction of closure load. However, the increase of  $K_{max}$ , associated with the increase of U, enhances plastic deformation at the crack tip. Therefore, more precision is required in order to correctly captures the stress- strain gradients at the crack tip, which means that lower values of Toleq as to be used, as figure 6 shows. The results in this figure reinforce the importance of a careful control of the numerical algorithm to ensure the quality of numerical results. Anyway, the results at crack tip elements, and particularly, at the crack tip node, are always affected by errors resulting from the singular character of that point.



Closure level (U) versus stress ratio (R) after 15 propagation cycles ( $a=6.004$  mm;  $K=5.2$  MPa.m<sup>1/2</sup>;  $L_1=16$  m).

### Need

A major aspect in the numerical study of PICC is the parameter used to quantify crack closure. Different parameters can be considered, namely first contact of crack flank. This is the conventional definition proposed by Elber 1970. Within FEM analysis, this corresponds to the contact of first node behind current crack tip. This concept has been widely used (Jiang, 2005, etc.) but results are mesh dependent. In fact, the approximation of the first node to the crack tip increases the crack opening level; first contact of

other nodes behind crack tip. Pommier considered the second node behind crack tip and assumed that the crack was open when being the crack opening. Roychowdhury *et al.* 2003 also considered the second node; occurrence of compressive stresses at the crack tip. Important differences in the crack closure levels were obtained between classical definition and stress inversion. Sehitoglu, Current definition gives higher values of crack closure/opening stresses, because when crack closure is determined according to the conventional definition, a compressive stress state still exists at the crack tip. However, according to stress inversion definition the crack driving force is only related to the portion of the loading cycle during which the crack tip is submitted to tensile stresses. In fact, the contact of the first node behind current crack tip does not indicate that the crack is fully open, i.e., that the crack is open between this node and the current crack tip node. The reduction of  $L_1$  is expected to reduce the difference between both definitions of crack closure. However, the conventional definition correlates better with experimental results based on compliance changes Wei, 2000. On the other hand, the stress inversion is less sensitive to the size of crack tip elements; parameters based on remote measurements of displacements or strains. This definition is similar to most of the experimental procedures. Borrego (2001) experimentally measured the crack closure level in MT specimens with 3 mm thickness of 6082-T6 aluminum alloy, using a pin extensometer placed at mid-section. This remote measurement gives an average closure value. Antunes *et al.* (2004) applied this concept to numerical results. A load-displacement plot was obtained and the crack opening was obtained by maximization of the correlation coefficient (Allison, 1988) [2]. Small differences (<2%) were found relatively to classical definition of closure; load corresponding to a zero stress intensity factor. Shterenlikht *et al.* (2006) applied optical techniques to determine the stress intensity factor and considered that crack closure occurs when  $K=0$ ; the stress intensity factor required to open the crack, computed using the contact stresses along the closed or partially closed crack under minimum load. The contact stress method overcomes the limitation of focusing attention on a single node, considering instead the global behaviour of the entire crack surface.

## Scope

### The numerical program

The numerical simulations were performed with the Three-Dimensional Elasto-plastic Finite Element program (DD3IMP) that follows a fully implicit time integration scheme. The use of an implicit algorithm guarantees the accuracy of the state variables during the numerical simulation making the numerical code very robust. The mechanical model and the numerical methods used in the finite element code DD3IMP, specially developed for the numerical simulation of metal forming processes, takes into account the large elasto-plastic strains and rotations that are associated with large deformation processes. To simulate the contact with friction, the program makes use of an augmented Lagrangian method, leading to a mixed formulation where the final unknowns are displacements and contact forces. The obtained non-linear system is solved by a Newton-Raphson method that can exhibit quadratic convergence in the vicinity of the solution. The global convergence of the Newton-Raphson method, when applied to simulate large deformation processes, depends mainly on parameters like: numerical variables, the mechanical behaviour laws, the friction

coefficient, the discretization of the deformable body, the loading conditions and the initial solution for the equilibrium iterative loop. The results of numerical tests undertaken showed that it is always possible to overcome the problems of convergence, acting on specific numerical parameters of the algorithm. It was also proved that changing these numerical parameters during the numerical simulation has no influence on the quality of the solution, making the implicit program highly sound and reliable. Since in large deformation processes the elastic part of the transformation leads to small strains as compared with unity, the hypothesis of small elastic strains is assumed in DD3IMP mechanical model, regardless of the high elastic rotations that can occur. The accuracy of the solutions, provided by this numerical program, strongly rely on the models used to describe the strong non-linear material behaviour and the contact with friction. In the last years, an enormous effort has been concentrated on sheet metals behaviour, concerning the improvement of both the behaviour laws and the initial yield locus description. The DD3IMP finite element code follows the same tendency and, nowadays, presents several isotropic and anisotropic constitutive models (seven isotropic/kinematic hardening laws and eight yield criteria). The Coulomb law and the Signorini conditions are used to describe the contact with friction between the deformable body and the rigid surfaces. The evolution of the contact conditions is controlled by a mixed formulation, using an augmented Lagrangian approach [Oliveira *et al.*, 2004].

## Conclusions

Present paper is a numerical study on the main parameters affecting plasticity induced crack closure. The main aspects and conclusions of this study are a numerical model of PICC was developed and implemented in DD3IMP home code. The model maintains main limitations of literature models, namely, discrete crack propagations, a relatively high fatigue crack growth, sharp cracks and consider that propagation occurs at a well-defined load. However, contrary to most literature studies a mixed hardening model was assumed, consisting of Voce type law combined with kinematic hardening described by a saturation law the physical parameters and the numerical parameters were systematically identified. The dependent parameters used in literature to quantify crack closure level and crack tip phenomena were also discussed the numerical parameters were divided into two main groups: those affecting the accuracy of predictions (parameters of numerical algorithm, radial size of crack tip elements and minimum crack propagation for stabilization) and those responsible for the intrinsic uncertainty of the numerical modelling of PICC; a study was developed to optimize the accuracy of numerical predictions. The parameters of the numerical algorithm were identified and optimized. The minimum radial size of crack tip elements,  $L_1$ , necessary to model adequately reversed plasticity was studied. The analysis of the stress-strain curve for a Gauss point close to the crack tip was found adequate to ensure that reversed plastic deformation is being adequately modelled and to define an upper bound for  $L_1$ . The convergence of closure values with crack propagation was studied, and extrapolation models compared. The study of these 3 parameters is fundamental to ensure feasible results, and most be done for each set of physical parameters; although the emphasis on quality, the closure predictions will always be affected by an intrinsic uncertainty. Main parameters affecting this intrinsic uncertainty are the number of load cycles applied to near

crack tip points (NLC) and the definition of closure. The NLC depends on L1 and on the number of load cycles between crack propagations. A convergence of the opening values was found with the increase of NLC, explained by the stabilization of cyclic curves. The definition of closure is probably the main source of uncertainty; the numerical models are ideal to study the influence of physical parameters. For that the parameters affecting the accuracy must be optimized, and the parameters of uncertainty fixed. The effect of stress ratio was studied. A reasonable agreement was found with empirical models found in literature.

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