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Difference between confidence interval and interval of convergence: Poisson distribution example

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Abstract
The relationship between Mathematics and Statistics is Overlapping but mathematics is the broad domain of study, encompassing primarily all quantitative disciplines whereas Statistics is a scientific discipline, specificaly associated with the Applied Mathematics. Statistics is a strictly bounded discipline limited to physical data and its interpretation, hence it has limited scope. Mathematics, however also deals with the abstract concepts which can be metaphysical in nature. This paper tries to explore the differences between Intervals of confidence and convergence in statistics field, by take Poisson distribution case study to understand the Conceptions.

Keywords: Confidence interval, interval of convergence, mathematics

Introduction
In statistical studies, it is imagined that there may be a certain order and pattern for a series of experiments and events that may seem random and unexpected, and this may give them a pattern and order that arise in different forms and this begins when random events approach to a fixed value and this leads to the emergence of similarities with some mathematical functions [1], there is no doubt that there are formal forms that can be represented by convergence, and this helps to know the difference between the confidence interval and interval of convergence, and the decrease in variance for later results can be described by a random variable by means of convergence, which is known as the distance of the results of a series of calculations indicating the outcomes that approach zero [2].

Importance and Objectives of study
The rationality of this study lies in the linkage which connects the aspects of mathematics and statistics through mathematical statistical concepts, as it centers around the power series distribution by studying one of the distributions and studying it from the side of the confidence interval and interval of convergence [3].

Operational Definations
Confidence Interval
Confidence interval is a kind of estimate computed from the statistics of the observed data, defined by its lower and upper bounds. The confidence interval is expressed as a percentage (the most frequently quoted percentages 90\%, 95\%, 99\%). The percentage indicates the confidence level [4].
The basic statement of confidence interval among the most fundamental theorems in statistics. this have many practical applications in statistics and economics and it can be stated as:
If we assum $X_1, X_2, \ldots, X_n$ is a set of random observations from a population with mean μ and standared deviation σ By the central limit theorem as $n \to \infty$ with $\mu = 0$ and $\sigma = 1$
CI can be writtin as

$$|\mu - \bar{x}| \geq Z\sigma$$

while can be said it came from The Law of Large Numbers (Weak Form)

$$\lim_{x\to\infty} P(|\overline{x} - \mu| \geq \epsilon) = 0$$

The above equation states that with n tending to infinity it becomes more and more observed that the mean of the sample of random variable $\overline{x}$ will become increasingly similar to the identical mean of individual random variables μ [5].

Interval of Convergence

An interval associated with a given power series in such a way that the series converges for all values of the variable inside the interval and diverges for all values outside it.the convergence of the series will now depend upon the values of x that we put into the series. more specific assume R is a number that the power series will converge for, so

$$|X - a| \geq R$$

R This number is called the radius of convergence for the series, also there are many ways to find interval of convergence such as ratio test, etc [6] so,

$$L = \lim_{n\to\infty} \left|\frac{an+1}{an}\right|$$

- The series converges absolutely if L less than 1.
- The series diverges if L greater than 1 or if L is infinite.
- The test is inconclusive if L=1.

Application and conclusion

Confidence interval as One of the parts of the central limit theorem is a popular theory in probability and statistics, which indicates the probability that a parameter lies between a pair of values around the mean and all results can be applied in distributions that were not normally distributed in the original, while the congruence period gives the researcher the period in which the distribution is then congruent with the normal distribution [7].

In practice, we will take Poisson distribution as an example to understand the difference between two concepts (confidence interval and interval of convergence)

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

where

x: number of event,

lambda: mean,

e: Euler constant approxematly equal 2.71828.

Interval of convergence for poisson distribution

$$L = \lim_{n\to\infty} \left|\frac{an+1}{an}\right|$$

$$L = \lim_{x\to\infty} \left|\frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!} \frac{x!}{e^{-\lambda}\lambda^{x}}\right|$$

Then when use ratio test the result will be

$$\lim_{x\to\infty} \left|\frac{\lambda}{x+1}\right|$$

its convergence if

$$L = \lim_{x\to\infty} \left|\frac{\lambda}{x+1}\right| < 1$$

$$-1 < \frac{\lambda}{x+1} < 1$$

$$-\lambda - 1 < x < \lambda - 1$$

While the confidence interval for mean in poisson distribution was

$$\lambda \pm 1.96\sqrt{\frac{\lambda}{n}}$$

The above result clearly indicates the diference between confidence interval and intervel of convergence as so significantly close that it gives us further ideas/indications to do more research about the application to find the significance of the relationship.

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