International Journal of Statistics and Applied Mathematics 2022; 7(4): 105-109

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(4): 105-109

www.mathsjournal.com
Received: 07-04-2022
Accepted: 11-05-2022

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Nonsplit domination number in vertex semi-middle graph

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DOI: https://doi.org/10.22271/maths.2022.v7.i4b.850

Abstract
Let $G(p,q)$ be a connected graph and $M_v(G)$ be its corresponding vertex semi middle graph. A dominating set $D \subseteq V[M_v(G)]$ is Nonsplit dominating set $\langle V[M_v(G)] - D\rangle$ is connected. The minimum size of D is called the Nonsplit domination number of $M_v(G)$ and is denoted by $\gamma_{ns}[M_v(G)]$. In this paper we obtain several results on Nonsplit domination number.

Keywords: Domination number, Independent domination number, Nonsplit domination number, Vertex semi-middle graph

1. Introduction

Domination is an area in graph theory with an extensive research activity. We consider simple, finite, undirected, non-trivial and connected graphs for our study. In literature, the concept of graph theory terminology not presented here can be found in Harary [1]. In a graph G, a set $D \subseteq V$ is dominating set of G if every vertex in $V - D$ is adjacent to some vertex in D. The domination number of a graph G is the minimum size of D. Some studies on domination and other graph valued functions in graph theory were studied in [2-14, 16-24, 27-29]. The vertex semi middle graph $M_v(G)$ of a graph G was studied in [25] and is defined as follows. The vertex semi-middle graph of a graph G, denoted by $M_v(G)$ is a planar graph whose vertex set is $\mathrm{V(G)} \cup \mathrm{E(G)} \cup \mathrm{R(G)}$ and two vertices of $M_v(G)$ are adjacent if and only if they corresponds to two adjacent edges of G or one corresponds to a vertex and other to an edge incident with it or one corresponds to a vertex other to a region in which vertex lies on the region. Let $\mathrm{R}' = \{\mathrm{r}'_1, \mathrm{r}'_2, \dots \mathrm{r}'_m\} \subseteq \mathrm{V}[M_v(G)]$ for the region set $\{\mathrm{r}_1, \mathrm{r}_2, \dots \mathrm{r}_m\}$ of G. Let $\mathrm{V}' = \{\mathrm{v}'_1, \mathrm{v}'_2, \dots \mathrm{v}'_p\} \subseteq \mathrm{V}[M_v(G)]$ for the vertex set $\{\mathrm{v}_1, \mathrm{v}_2, \dots \mathrm{v}_p\}$ of G. Let $\mathrm{E}' = \{\mathrm{e}'_1, \mathrm{e}'_2, \dots \mathrm{e}'_q\} \subseteq \mathrm{V}[M_v(G)]$ for the edge set $\{\mathrm{e}_1, \mathrm{e}_2, \dots \mathrm{e}_q\}$ of G such that $\mathrm{V}[M_v(G)] = \mathrm{V}' \cup \mathrm{E}' \cup \mathrm{R}'$. The study of domination number of jump graph [15] motivated us to introduce nonsplit domination number in vertex semi middle graph.

2. Preliminaries

Theorem 2.1: [26] For the path $P_n, \gamma[M_v(P_n)] = \gamma[L(P_n)] + 1$.

Theorem 2.2: [26] For the cycle $C_n, n \geq 4$,

$$\gamma[M_v(C_n)] = \begin{cases} \frac{n}{3} + 2 \; if \; n = 3k, k \geq 2. \\ \left\lceil \frac{n}{3} + 1 \right\rceil \; if \; n = 3k+1 \; or \; n = 3k+2, k \geq 1. \end{cases}$$

Theorem 2.3: [26] For any graph G, $\gamma[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil$.

3. Nonsplit domination number in vertex semi-middle graph

A dominating set D of $M_v(G)$ is a nonsplit dominating set if $\langle V[M_v(G)] - D \rangle$ is connected. The minimum cardinality of D is called nonsplit domination number of $M_v(G)$ and is denoted by $\gamma_{ns}[M_v(G)]$. A minimum nonsplit dominating set is denoted by $\gamma_{ns} - set$.

In the Fig 1, the nosplit dominating set of $M_v(G)$ is D = {3, r_1', r_2'}, $\gamma_{ns}[M_v(G)] = 3$.

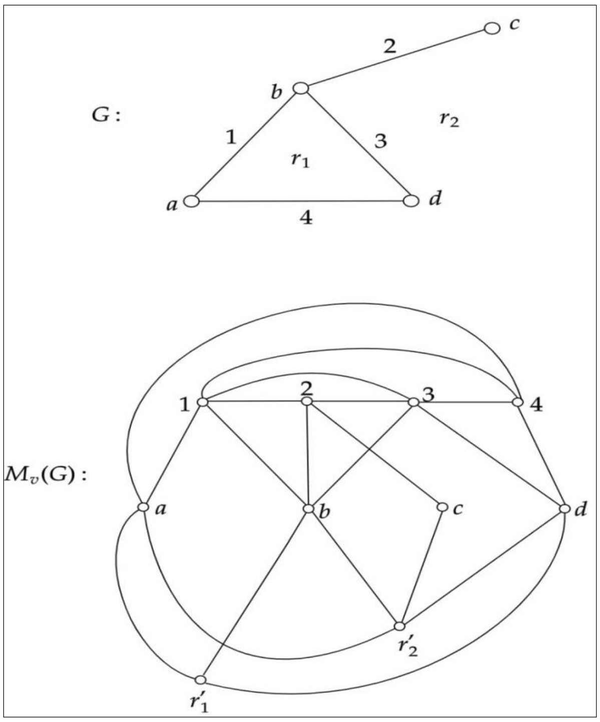

Fig 1: The Graph G and its $M_v(G)$

We begin with some observations.

Observation 3.1

For the path $P_2, \gamma_{ns}[M_v(P_2)] = 2$.

Observation 3.2

For any cycle $C_n, \gamma_{ns}[M_v(C_n)] = \gamma[M_v(C_n)]$.

Observation 3.3

For the path $P_n, i[M_v(P_n)] = \gamma[M_v(P_n)]$.

4. Main Results

Theorem 4.1: For the path $P_n, n \geq 3, \gamma_{ns}[M_v(P_n)] = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Proof: Consider $G = P_n$ and $V(P_n) = \{v_i, 1 \leq i \leq n\}$ for $n \geq 3$. Let D be the dominating set of $M_v(P_n)$. For P_3, P_5, D itself is a $\gamma_{ns} - set$. Let D′ be the dominating set for $P_n, n \neq 3{,}5$ and define it as follows.

$$D' = \begin{cases} r_1', v_2', v_4' \ldots \ldots \ldots v_n' \; if \; n = 2l, l \geq 2. \\ r_1', v_2', v_4', \ldots \ldots \ldots v_{n-1}' \; if \; n = 2l+1, l \geq 1. \end{cases}$$

Clearly, $\langle V[M_v(P_n) - D'] \rangle$is connected. Therefore $|D'| = \left\lfloor \frac{n}{2} \right\rfloor + 1$. It follows that $\gamma_{ns}[M_v(P_n)] = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Theorem 4.2: For every graph $G(p,q)$, $\gamma_{ns}[M_v(G)] \geq \gamma[M_v(G)]$.

Proof: Consider $V[M_v(G)] = V' \cup E' \cup R'$ and $D = \{u_i' / u_i' \in V[M_v(G)]\}$ be the dominating set of $M_v(G)$.

Here we have to consider four cases.

Case 1: Let $v_i$be the non pendant vertex in P_n then the corresponding vertex in $M_v(G)$ is v_i'. Consider $G = P_n$. By the Theorem 4.1, D is not non-split dominating set. Further, Consider D′ = D ∪ $\{v_i'\}$ such that $V[M_v(P_n)] - D'$ is connected. It follows that |D′| ≥ |D|. Hence $\gamma_{ns}[M_v(P_n)] \geq \gamma[M_v(P_n)]$.

Case 2: Let G be a tree. By Case 1, it is obvious that,

$\gamma_{ns}[M_v(T)] \geq \gamma[M_v(T)]$.

Case 3: Let $G = C_n$. By Observation 3.2, $\gamma_{ns}[M_v(C_n)] = \gamma[M_v(C_n)]$. Thus, $V[M_v(C_n) - D]$ is connected by Theorem 2.2 and it follows.

Case 4: Let G be any graph. By the Theorem 4.1, Observation 3.2 and Theorem 2.2, as a result, $\gamma_{ns}[M_v(G)] \geq \gamma[M_v(G)]$.

From the above cases, we can say that $\gamma_{ns}[M_v(G)] \geq \gamma[M_v(G)]$.

Theorem 4.3: For every graph $G(p,q)$, $\gamma_{ns}[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil$.

Proof:
From Theorem 2.3,

$$\gamma[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil \ldots\ldots\ldots\ldots (1)$$

By Theorem 4.2,

$$\gamma_{ns}[M_v(G)] \geq \gamma[M_v(G)] \ldots\ldots\ldots\ldots (2)$$

We have, From equation (1) and equation (2),

$$\gamma_{ns}[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil \ldots\ldots\ldots\ldots (3)$$

Theorem 4.4: Let $G(p,q)$ be a graph, $\gamma_{ns}[M_v(G)] \geq \left\lceil \frac{diam(G)+1}{2} \right\rceil$.

Proof: Let $S = \{e_k, 1 \leq k \leq q\} \subseteq V(G)$ be the diametral path in G. Clearly, |S| = diam(G). Consider D be the dominating set of $M_v(G)$. If $\langle V[M_v(G)] - D \rangle$ is connected, then D itself is a $\gamma_{ns} - set$ of $M_v(G)$. Otherwise, at least one vertex exists with a maximum degree $\{v_i'\} \in V[M_v(G)] - D$ such that $\langle V[M_v(G)] - (D \cup \{v_i'\}\rangle$ has only one component. It explicitly forms a γ – set of $M_v(G)$ that is non-split. Further, since S′⊆ V$[M_v(G)]$ and D∪$\{v_i'\}$ is a $\gamma_{ns} - set$ in $M_v(G)$. The diametral path includes at most $\gamma_{ns}[M_v(G)] - 1$ edges connecting the neighbourhoods of the vertices of $V[M_v(G)] - (D \cup \{v_i'\})$. Hence $\gamma_{ns}[M_v(G)] + \gamma_{ns}[M_v(G)] - 1 \geq diam(G)$. Which gives $\gamma_{ns}[M_v(G)] \geq \left\lceil \frac{diam(G)+1}{2} \right\rceil$.

Theorem 4.5: For every graph $G(p,q), \gamma_{ns}[M_v(G)] \leq diam(G) + \alpha_0(G)$.

Proof: Let $\alpha_0(G)$ be the vertex covering number of G. Let $V(G) = \{v_1, v_2, \ldots\ldots v_n\} \exists\ v_i, v_j \in V(G)$ such that $K = d(v_i, v_j) = \{v_1, v_2, \ldots\ldots v_k\}$ forms a diametral path in G. Let D be the $\gamma - set$ of $M_v(G)$. If $\langle V[M_v(G)] - D \rangle$ is connected then $\langle D \rangle$ itself forms the $\gamma_{ns} - set$ of $M_v(G)$. Clearly, |D| ≤ |K ∪ A|. It follows that $\gamma_{ns}[M_v(G)] \leq diam(G) + \alpha_0(G)$. Otherwise, there exists at least one vertex having maximum degree $\{v_i'\} \in V[M_v(G)] - D$ such that $\langle V[M_v(G)] - (D \cup \{v_i'\}\rangle$ has only one component. Clearly, it forms a $\gamma_{ns} - set$ of $M_v(G)$. Since $V[M_v(G)] - (D \cup \{v_i'\})$ includes diametral path. We have $|V[M_v(G)] - (D \cup \{v_i'\})| \leq |K \cup A|$. Hence $\gamma_{ns}[M_v(G)] \leq diam(G) + \alpha_0(G)$.

Theorem 4.6: For every tree $T, \gamma[M_v(T)] = i[M_v(T)]$.

Proof: Suppose that the minimum vertex set is $D \subseteq E'(T) \cup r_1'$ that is $N(D) = V[M_v(T)]$. Hence D forms the γ - set in $M_v(T)$. Clearly $|D| = \gamma[M_v(T)]$ and also $\langle D \rangle$is totally disconnected. Thus, $\gamma[M_v(T)] = i[M_v(T)]$.

Theorem 4.7: For the cycle C_n,

i) $i[M_v(C_n)] = \left\lfloor \frac{n}{2} \right\rfloor + 1\ if\ 4 \leq n \leq 11$.

ii) $i[M_v(C_n)] = \left\lfloor \frac{n}{3} \right\rfloor + 2\ if\ n \geq 12$.

Proof: Consider $G = C_n$ and $V(C_n) = \{v_i, 1 \leq i \leq n\}\ for\ n \geq 3$. Let $e_i', v_i' \in V[M_v(C_n)]$. Let the dominating set of $M_v(C_n)$ be D.

Case 1: For $n = 4,5,6,7,9$ D itself is a $i - set$. Otherwise, D is not a independent dominating set for $n = 8,10,11$. Let D' be the set and define it as follows.

$$D' = \begin{cases} v_1', e_2', e_4', \dots\dots\ e_{n-2}' v_n' \text{ if } n = 8, 10. \\ r_1', r_2', e_2', e_5' \dots\dots e_n' \text{ if } n = 11. \end{cases}$$

Clearly, $\langle D' \rangle$ is totally disconnected and is $i - set$ of $M_v(C_n)$. Hence $|D'| = \left\lceil \frac{n}{2} \right\rceil + 1$. It follows that $i[M_v(C_n)] = \left\lceil \frac{n}{2} \right\rceil + 1$.

Case 2: For $n \geq 12$, if $\langle D \rangle$ is totally disconnected then D itself is a $i - set$ of $M_v(C_n)$. Otherwise, we consider $D' = D \cup r_2'$ such that D' is totally disconnected. It follows that $|D'| = \left\lceil \frac{n}{3} \right\rceil + 2$. Hence $i[M_v(C_n)] = \left\lceil \frac{n}{3} \right\rceil + 2$.

Theorem 4.8: For every graph $G(p,q), i[M_v(G)] \geq \gamma[M_v(G)]$.

Proof: By the definition $V[M_v(G)] = V' \cup E' \cup R'$. Let the dominating set of $M_v(G)$ be $D = \{u_i' / u_i' \in V[M_v(G)]\}$. In subsequent cases, we shall prove that.

Case 1: Let G be a path. By the Observation 3.3, $\langle D \rangle$ is totally disconnected. Hence $i[M_v(P_n)] = \gamma[M_v(P_n)]$. It follows.

Case 2: Let G be a tree. By the Theorem 4.6, $\langle D \rangle$ is totally disconnected. Hence $i[M_v(T)] = \gamma[M_v(T)]$. It follows.

Case 3: Now we consider the cycle C_n. By the Theorem 2.2 and Theorem 4.7, it follows that $i[M_v(C_n)] \geq \gamma[M_v(C_n)]$.

Case 4: Consider G be any graph. By Observation 3.3, Theorem 4.6, Theorem 2.2 and Theorem 4.7, it follows that $i[M_v(G)] \geq \gamma[M_v(G)]$.

From the above cases, we can say that $i[M_v(G)] \geq \gamma[M_v(G)]$.

Theorem 4.9: For every graph $G(p,q)$, $i[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil$.

Proof

From Theorem 2.3,

$$\gamma[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil \dots\dots\dots\dots (1)$$

By Theorem 4.8,

$$i[M_v(G)] \geq \gamma[M_v(G)] \dots\dots\dots\dots (2)$$

We have, From equation (1) and equation (2),

$$i[M_v(G)] \geq \left\lceil \frac{P}{1+\Delta(G)} \right\rceil \dots\dots\dots\dots (3)$$

5. Conclusions

In this paper we established nonsplit domination results on vertex semi-middle graph. Many bounds on domination number of vertex semi-middle graph are obtained.

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