International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(4): 196-203

www.mathsjournal.com
Received: 08-05-2022
Accepted: 12-06-2022

Kurui Godfrey Kipkoech
School of Science and Technology, University of Kabianga, Kericho, Kenya

Corresponding Author:
Kurui Godfrey Kipkoech
School of Science and Technology, University of Kabianga, Kericho, Kenya

An analysis of credit risk measurement and management using a multivariate Markov chain model

Kurui Godfrey Kipkoech

Abstract
A multivariate Markov chain describes the dependency of credit-risk of assets in a portfolio. The application of transition from the credit bureau using transition matrix from consumer performance and experience is combined well in credibility theory. The application of logistic regression has been done on classification and determination of factors which affects the behavioral score of the consumer. One type of logistic regression, the cumulative logistic regression has a latent variable link that links function determining the dynamic of consumers' behavioral score.

Keywords: Logistic regression, asset, portfolio, credibility theory, transition matrix

1. Introduction
Most firms that provide credit has a high a growth of start-ups, they must develop, do tests and utilize models that can assess the risks over a very short time period. One of the main objectives for borrowers is to build their business with not only very high level of automation but also with a greater degree of cautiousness and management of risk. The borrowers are therefore required to build a registered analytics that they can use to apply to their data so that this will enable them maintain a competitive edge in their target markets and compete fairly with already established lenders. Therefore the models being developed must be less costly and should effectively respond to loan applications. All these considerations makes credit-risk modeling have a vital competitive advantage.
These credit-risk models should also be able to perform credit analysis, credit-fraud identification along- side prevention, credit - pricing, collections and asset management. The lenders should therefore analyze and collect their own customer specification of data and use it to build a unique capabilities, efficiencies and competitive gains. This process of gathering and analyzing of data may be a lengthy process, complex and error-prone therefore, there is a need to develop model that facilitate the analysis of commercial data that is timely and efficient in a required manner.

2. Preliminaries
2.1 Transition graph
It is a representation of Markov chain graphically where the states are represented by circles and each arrow represents possible transitions. The transitional probabilities between two states are represented by transition matrix (i, j) with entry of i^{th} row and j^{th} column having probabilities of moving within one step from state i to state j with each row adding up to one at a time.

2.2 Chapman Kolmogorov
It is a formation of equations that can allow calculations of general transitional probabilities in terms of one step probabilities $P_{i,j}^{(n,n+1)}$.
Let $P_{i,j}^{(m,n)}$ be the transitional probability of being in a state j at time n having being been in a state i at time m

$\Pr{}^{X_{m+1=j}}/_{X_{m=i}} = P_{i,j}^{(m,m+1)}$

The transitional probability from a discrete Markov chain do obey Chapman-Kolmogorov equation. With one step transitional probability $P_{i,j}^{(n,n+1)}$ with initial probability distribution given as $q_{k=P(X_0=k)}$ is used in deducing the probability of any path.

2.3 Homogeneous-Time Markov Chain

Homogeneous-time Markov chain is one where one step transitional probabilities are independent of time and is given as

$$P_{i,j}^{j,n+1} = P_{i,j}$$

Therefore the Chapman Kolmogorov equations is given as

$$P_{i,j}^{(n-m)} = \sum_{k \epsilon S} P_{1k}^{(i-m)} P_{kj}^{(n-1)}$$

With normalization condition given as $\sum_{j \epsilon S} P_{ij} = 1$ for all i implying that each row of P must add to one, that is to say

$\sum_{j \epsilon S} P_{ij} = 1$, for all i

2.4 Time non - homogeneous

Absolute values of time determines transitional probabilities but not just time difference hence not only the length of time but also the start process.

2.5 The period

A periodic state i occurs if $d > 1$ and if a return to i is possible if and only if there is a number of steps that is a multiple of d. There exists $\lim_{n \to \infty \forall} P_{ii}^{(n)}$ for which the state is periodic.

3. Main results

3.1 Research Design

We apply the descriptive design that involves obtaining information about the current status of situations in describing "what Exists" in respect to variables used in a situation.

3.2 Data Description

We consider data from a loan issuing firm with successful application of 30,000 customers alongside demographic features with behavioral characteristics as per terms of repayment status, limit of loan and the payment amount in three months. We compute a logistic regression from a sample of 220 using the R program.
The resulting logistic regression is summarized in the table below;

Coefficients

Coefficients	Estimate	p-value
Intercept	-0.9489	0.3427
Sex	-0.5439	0.5866
Education	0.5859	0.5575
Marriage	-0.389	0.6968
Payment status	-4.5901	0.4439
Outstanding Balance	-1.1716	0.2409
Bill	1.1146	0.2648
Age	0.8386	0.4008

3.3 Data Analysis

Using Wald test on significance we compare the p value and conclude that any p value less than 0.5 is significant while the one greater than 0.5 is ignored hence we again carry out the test. The gender value, status of education and status of marriage as predictors are all eliminated.
The second regression contain only factors that are significant and shows how each relate to response variable at hand.

3.4 Illustration.

The cumulative logistic regression of marginal probability is therefore given by

$$p(y_i = j) = \begin{cases} F(\delta_1)\, if\ j = 1 \\ F(\delta_1) - F(\delta_1)\, if\ j = 1\, nad\ j - 1 \\ F(\delta_1) \leq J \end{cases}$$

Assuming that given the output for a fitted cumulative logit model for different categories.
J = 1, 2, 3, 4 at the time T= t+1

Category	**J = 1**		**J = 2**		**J = 3**		**J = 4**	
	Y/X=1		Y/X=2		Y/X=3		Y/X=4	
	Est	S.E	Est	S.E	Est	S.E	Est	S.E
α_1	-0.089	0.522	0.909	0.490	-2.235	0.436	-0.561	0.385
α_2	1.478	-0.621	2.377	0.623	-0.083	0.330	-0.926	0.288
α_3	3.0007	1.058	3.397	0.833	2.592	0.610	0.317	0.272
$\boldsymbol{\beta}$	-0.51	0.765	0.792	0.651	-1.705	0.477	-0.161	0.381

At Y and where J = 1,

$$\ln\left[\frac{\Pr(Y_i \le b)}{\Pr(Y_i > b)}\right] = \alpha_b - \sum_{k=1}^{K} \beta_k X_{ik} \quad b = 1$$

$$\ln\left[\frac{\Pr(Y_i \le 1)}{\Pr(Y_i > 1)}\right] = \alpha_1 - \sum_{k=1}^{K} \beta_k X_{ik} \quad b = 1$$

β = -0.507 fixed for J = 1

$$\pi = \frac{e^{\alpha-\beta X}}{1+e^{\alpha-\beta X}} =$$

$$\pi = \frac{e^{-0.09\mp 0.51}}{1+e^{-0.09\mp 0.51}} = \pi = \frac{e^{0.42}}{1+e^{0.42}} = 0.60$$

$$\pi = \frac{e^{-0..48\mp 0.51}}{1+e^{-0.48\mp 0.51}} = \pi = \frac{e^{1.99}}{1+e^{1.99}} = 0.88$$

$$\pi = \frac{e^{-3.0\mp 0.51}}{1+e^{-3.0\mp 0.51}} = \pi = \frac{e^{3.51}}{1+e^{3.51}} = 0.97$$

J = 4= 1- $\pi = 1 - 0.97$ =.03

The marginal probabilities are below
j = 1, 0.60, j = 2, 0.88 – 0.60 =0.28, j = 3, 0.97 -0.88 =0.09, j = 4, 1-0.97 =0.03

β = -0.792 fixed for J = 2,

j = 1, 0.53, j = 2, 0.83– 0.53 =0.3, j = 3, 0.93 -0.83 =0.1, j = 4, 1-0.93 =0.07

β = -1.71 fixed for J = 3

j = 1, 0.371, j = 2, 0.836– 0.371 = 0.465, j = 3, 0.986 -0.836 =0.1, j = 4, 1 - 0.836 =0.07

β = -0.171 fixed for J = 4

j = 1, 0.278, j = 2, 0. 316 – 0.618 = 0.038, j = 3, 0.618 -0.316 =0.302, j = 4, 1 - 0.618 =0.302

The behavioral transition matrix estimates for this consumers is given as

$$N = \begin{bmatrix} 0.60 & 0.28 & 0.09 & 0.03 \\ 0.07 & 0.10 & 0.30 & 0.53 \\ 0.371 & 0.465 & 0.015 & 0.014 \\ 0.278 & 0.038 & 0.302 & 0.382 \end{bmatrix},$$

And the empirical transition estimates for the matrices as,

$$\dot{M}^{(11)} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{7}{9} & \frac{1}{8} & 0 \\ 0 & \frac{2}{9} & \frac{7}{8} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}, \quad \dot{M}^{(12)} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{6}{11} & \frac{1}{6} & 0 \\ 0 & \frac{5}{11} & \frac{5}{6} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}, \ldots\ldots, \quad \dot{M}^{(22)} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{2}{11} & \frac{1}{6} & 0 \\ 0 & \frac{9}{11} & \frac{5}{6} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix},$$

The probability state for the following states in the long run is given as;

$$X^{(1)} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \\ 0 \end{bmatrix}, \quad X^{(2)} = \begin{bmatrix} 0 \\ 0.61 \\ 0.39 \\ 0 \end{bmatrix}$$

$$X_{t+1}^{(j)} = \sum_{j=1}^{2} \left(\tilde{\lambda}_{jk}^{1} \widehat{N}^{(jk)} + \tilde{\lambda}_{jk}^{2} M^{(jk)}\right) X^{(k)}$$

$$X_{t+1}^{(1)} = \sum_{j=1}^{2} \left(\tilde{\lambda}_{1k}^{1} \widehat{N}^{(1k)} + \tilde{\lambda}_{1k}^{2} M^{(1k)}\right) X^{(k)}$$

$$= \left(\tilde{\lambda}_{11}^{1} \widehat{N}^{(11)} + \tilde{\lambda}_{11}^{2} M^{(11)}\right) X^{(1)} + \left(\tilde{\lambda}_{12}^{1} \widehat{N}^{(12)} + \tilde{\lambda}_{12}^{2} M^{(12)}\right) X^{(2)}$$

$$= X_{t+1}^{(1)} = \tilde{\lambda}_{11}^{1} \widehat{N}^{(11)} X^{(1)} + \tilde{\lambda}_{11}^{2} M^{(11)} X^{(1)} + \tilde{\lambda}_{12}^{1} \widehat{N}^{(12)} X^{(2)} + \tilde{\lambda}_{12}^{2} M^{(12)} X^{(2)}$$

We therefore formulate our estimation problem as follows

$$\min_{\tilde{\lambda}_{jk}^{1} \tilde{\lambda}_{jk}^{2}} \max_{i} \left[\tilde{\lambda}_{11}^{1} \widehat{N}^{(11)} X^{(1)} + \tilde{\lambda}_{11}^{2} M^{(11)} X^{(1)} + \tilde{\lambda}_{12}^{1} \widehat{N}^{(12)} X^{(2)} + \tilde{\lambda}_{12}^{2} M^{(12)} X^{(2)} - X^{(1)}\right]^{i}$$

Subject to: $\tilde{\lambda}_{11}^{1} + \tilde{\lambda}_{11}^{2} + \tilde{\lambda}_{12}^{1} + \tilde{\lambda}_{12}^{2} = 1$

$\tilde{\lambda}_{11}^{1}, \tilde{\lambda}_{11}^{2}, \tilde{\lambda}_{12}^{1}, \tilde{\lambda}_{12}^{2} \geq 0$

The above equation can be re - written as;

$$\text{Let } O_j = \max_i [\tilde{\lambda}_{11}^{1} \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0.78 & 0.125 & 0 \\ 0 & 0.22 & 0.875 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0. \\ 0.5 \\ 0.5 \\ 0 \end{bmatrix} + \tilde{\lambda}_{11}^{2} \begin{bmatrix} 0.60 & 0.28 & 0.09 & 0.03 \\ 0.07 & 0.10 & 0.30 & 0.53 \\ 0.371 & 0.465 & 0.015 & 0.014 \\ 0.278 & 0.038 & 0.302 & 0.382 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5 \\ o.5 \\ 0 \end{bmatrix} +$$

$$\tilde{\lambda}_{12}^{1} \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0.55 & 0..33 & 0 \\ 0 & 0.45 & 0.67 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0.61 \\ 0.39 \\ 0 \end{bmatrix} + \tilde{\lambda}_{12}^{2} \begin{bmatrix} 0.60 & 0.28 & 0.09 & 0.03 \\ 0.07 & 0.10 & 0.30 & 0.53 \\ 0.371 & 0.465 & 0.015 & 0.014 \\ 0.278 & 0.038 & 0.302 & 0.382 \end{bmatrix} \begin{bmatrix} 0 \\ 0.61 \\ .0.39 \\ 0 \end{bmatrix}]$$

$$\min_{\tilde{\lambda}_{jk}^{1} \tilde{\lambda}_{jk}^{2}} O_j$$

Subject to:

$$O_j \geq (X^{(j)}) - B_j \begin{pmatrix} \tilde{\lambda}_{11}^1 \\ \tilde{\lambda}_{11}^2 \\ \tilde{\lambda}_{12}^1 \\ \tilde{\lambda}_{12}^2 \end{pmatrix}, O_j \geq (-X^{(j)}) + B_j \begin{pmatrix} \tilde{\lambda}_{11}^1 \\ \tilde{\lambda}_{11}^2 \\ \tilde{\lambda}_{12}^1 \\ \tilde{\lambda}_{12}^2 \end{pmatrix}$$

$$O_j \geq 0$$

$$\sum_{k=1}^{n} (\tilde{\lambda}_{jk}^1 + \tilde{\lambda}_{jk}^2) = 1 \; \tilde{\lambda}_{jk}^1, \tilde{\lambda}_{jk}^2 \geq 0 \; \forall \, j, k$$

B_j *is given by* $Q^{(j1)}\hat{X}^{(1)}P^{(j1)}/Q^{(j2)}\hat{X}^{(2)}/P^{(j2)}\hat{X}^{(2)}/ \ldots /Q^{(jn)}\hat{X}^{(n)}/P^{(jn)}\hat{X}^{(n)}$

For the example below we have $B_j = Q^{(11)}\hat{X}^{(1)}P^{(11)}\hat{X}^{(1)}/Q^{(12)}\hat{X}^{(2)}/P^{(12)}\hat{X}^{(2)}/\ Q^{(12)}\hat{X}^{(2)}/P^{(12)}\hat{X}^{(2)}$
Given below

$$M^{(11)}X^{(1)} = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0.78 & 0.125 & 0 \\ 0 & 0.22 & 0.875 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.451389 \\ 0.548611 \\ 0 \end{bmatrix}$$

$$N^{(11)}X^{(1)} = \begin{bmatrix} 0.60 & 0.07 & 0.371 & 0.278 \\ 0.28 & 0.10 & 0.465 & 0.038 \\ 0.09 & 0.30 & 0.15 & 0.302 \\ .03 & 0.53 & 0.014 & 0.382 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.4505 \\ 0.3825 \\ 0.125 \\ 0.042 \end{bmatrix}$$

$$M^{(12)}X^{(2)} = \begin{bmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 0.55 & 0.33 & 0 \\ 0 & 0.45 & 0.67 & 0 \\ 0.5 & 0 & 0 & 0.5 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.61 \\ 0.39 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.564815 \\ 0.435185 \\ 0 \end{bmatrix}$$

$$N^{(12)}X^{(2)} = \begin{bmatrix} 0.60 & 0.07 & 0.371 & 0.278 \\ 0.28 & 0.10 & 0.465 & 0.038 \\ 0.09 & 0.30 & 0.15 & 0.302 \\ 0.03 & 0.53 & 0.014 & 0.382 \end{bmatrix} * \begin{bmatrix} 0 \\ 0.61 \\ 0.39 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.468167 \\ 0.364167 \\ 0.119444 \\ 0.048222 \end{bmatrix}$$

This formulates the problem as shown below,

$$\min_{\tilde{\lambda}_{jk}^1 \tilde{\lambda}_{jk}^2} O_j$$

Subject to

$$(O_1) \geq 0 - 0.\tilde{\lambda}_{11}^1 - 0.4505\tilde{\lambda}_{11}^2 - 0.\tilde{\lambda}_{12}^1 - 0.468167\tilde{\lambda}_{12}^2$$

$$(O_1) \geq \frac{1}{2} - 0.451389\tilde{\lambda}_{11}^1 - 0.3825\tilde{\lambda}_{11}^2 - 0.564815\tilde{\lambda}_{12}^1 - 0.364167\tilde{\lambda}_{12}^2$$

$$(O_1) \geq \frac{1}{2} - 0.548611\tilde{\lambda}_{11}^1 - 0.125\tilde{\lambda}_{11}^2 - 0.435185\tilde{\lambda}_{12}^1 - 0.119444\tilde{\lambda}_{12}^2$$

$$(O_1) \geq 0 - 0.0\tilde{\lambda}_{11}^1 - 0.042\tilde{\lambda}_{11}^2 - 0.0\tilde{\lambda}_{12}^1 - 0.042822\tilde{\lambda}_{12}^2$$

$1 = \tilde{\lambda}_{11}^{1} + \tilde{\lambda}_{11}^{2} + \tilde{\lambda}_{12}^{1} + \tilde{\lambda}_{12}^{2}$

$\tilde{\lambda}_{11}^{1}, \tilde{\lambda}_{11}^{2}, \tilde{\lambda}_{12}^{1}, \widetilde{\lambda}_{12}^{2} \geq 0$

The solution that optimizes this problem can be the obtained using the excel solver as follows

$X_{t+1}^{(1)} = 0.09985M^{(11)}X^{(1)} + 0.90015N^{(12)}X^{(2)}$

$X_{t+1}^{(2)} = N^{(12)}X^{(2)}$

3.5 Computation of Credit Value at Risk and Estimated shortfall

We generate the predictive probability distributions using the above model in evaluating credit risk measures.

We let $L_{t+1}\left(Y_{t+1}^{(1)}, Y_{t+1}^{(2)}\right) = L_{t+1}\left(Y_{t+1}^{(1)}\right) + L_{t+1}\left(Y_{t+1}^{(2)}\right)$

Such that each of $j = 1,2$ above has a rating class $j^{(th)}$ and $Y_{t+1}^{(j)}$ at time $t + 1$ we take values in set of unit basis vector$(e_1 e_2 \dots\dots e_4) \in R^4$.

When we consider a unit vector [0,1] and its uniform partition $U_{i=1}^{8} p_i$ where $p_i = \left(\frac{i-1}{4} \frac{i}{4}\right)$,

We assume that for each $j = 1,2$ and $i = 1,2 \dots .4$ contain a loss from the j^{th} asset $Y_{t+1}^{(j)}\left(Y_{t+1}^{(i)}\right)$ given that $Y_{t+1}^{(j)} = e_i$ thus takes the interval p_i for each $i = 1,2 \dots 4$

Implying that $Y_{t+1}^{(j)} = e_i$ contain the loss from the $j^{(th)}$ asset $Y_{t+1}^{(j)}\left(Y_{t+1}^{(i)}\right)$ at time $t + 1$ that can take values in $p_1 = \left[0, \frac{1}{4}\right]$. The simulated results is given below

1	0.1235	0.2672	0.3949	0.3221
2	0.0289	0.1857	0.3453	0.3773

The aggregate losses ordered from the credit portfolio at time $t + 1$ based on the the simulated values below are given above;

0.7529, 0.8509, 0.9419, 1.0028, 1.0427, 1.4346, 1.2357, 1.3466, 1.5478, 1.6796, 1.7767, 1.8027, 1.900, 1.998, 2.038, 2.591, 2.65.

The probability predicted from the equation derived above is as below

.0001, 0.0053, 0.0032, 0.300, 0.035, 0, 0, 0, 0.046, 0.0014, 0.091, 0.48, 0.021, 0.003, 0.002, 0.012

In evaluating the credit value at risk we choose a value K^* which satisfies these two equations

$$(\mathcal{L}_{t+1} \geq / \mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t) = \sum_{k=K^\star}^{M} \rho(\mathcal{L}_{t+1} = \mathcal{L}_{t+1}(\widetilde{k}) \,/\, \mathcal{F}_t) \leq \alpha$$

$$\rho(\mathcal{L}_{t+1} \geq / \mathcal{L}_{t+1}(K^\star) + 1/\mathcal{F}_t) = \sum_{k=K^\star+1}^{M} \rho(\mathcal{L}_{t+1} = \mathcal{L}_{t+1}(\widetilde{k}) \,/\, \mathcal{F}_t) > \alpha$$

The above illustrations of the two equations are satisfied at the point $K^* = 12$ and thus the

$VaR_{\alpha,}\rho(\mathcal{L}_{t+1}/\mathcal{F}_t) = \mathcal{L}_{t+1}(K^\star)$= 1.9

We can therefore estimate the expected shortfall by;

$ES_\alpha(\mathcal{L}_{t+1}/\mathcal{F}_t) = \frac{1}{\alpha} E_p(\mathcal{L}_{t+1}\boldsymbol{I}_{\{\mathcal{L}_{t+1} \geq / \mathcal{L}_{t+1}(K^\star)\}} + A(\alpha),$

Where adjustment term $A(\alpha)$ is given by

$$A(\alpha) := \mathcal{L}_{t+1}(K^\star)\left[1 - \frac{\rho(\mathcal{L}_{t+1} \geq / \mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t)}{\alpha}\right]$$

$$ES_\alpha(\mathcal{L}_{t+1}/\mathcal{F}_t) = \frac{1}{\alpha}\left[\sum_{k=K^\star}^{M} \mathcal{L}_{t+1}(\widetilde{k})\rho\left(\mathcal{L}_{t+1} = \mathcal{L}_{t+1}(\widetilde{k})/\mathcal{F}_t\right) - \mathcal{L}_{t+1}(K^\star) * \rho(\mathcal{L}_{t+1} \geq / \mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t) - \alpha\right]$$

$$= \frac{1}{\alpha}\left\{\sum_{k=K^\star}^{M} \mathcal{L}_{t+1}(k)\left(\sum_{(i_1 i_2,\dots,i_n)\epsilon I_{t+1,\check{k}}} \prod_{j=1}^{n} P_{t+1/t}^{j i_j}\right) - \mathcal{L}_{t+1}(K^\star) * \left[\sum_{k=K^\star}^{M}\left(\sum_{(i_1 i_2,\dots,i_n)\epsilon I_{t+1,\check{k}}} \prod_{j=1}^{n} P_{t+1/t}^{j i_j}\right) - \alpha\right]\right\}$$

$$\approx \frac{1}{\alpha}\left\{\sum_{k=K^\star}^{M} \mathcal{L}_{t+1}(k)\left\{\sum_{(i_1 i_2,\dots,i_n)\epsilon I_{t+1,\check{k}}}\left\{\left[\sum_{k=1}^{n}\left(\lambda_{jk}^{\sim 1} Q^{(jk)} + \lambda_{jk}^{\sim 2}\hat{P}^{(jk)}\right)X_t^{(k)} \mathrm{I}_{X_t=(e_{i_1},e_{i_2},\dots,e_{i_n})}\right]^{i_j}\right\}\right\} - \mathcal{L}_{t+1}(K^\star)\left\{\sum_{k=K^\star}^{M}\left\{\sum_{(i_1 i_2,\dots,i_n)\epsilon I_{t+1,\check{k}}}\left\{\left[\sum_{k=1}^{n}\left(\lambda_{jk}^{\sim 1} Q^{(jk)} + \lambda_{jk}^{\sim 2}\hat{P}^{(jk)}\right)X_t^{(k)} \mathrm{I}_{X_t=(e_{i_1},e_{i_2},\dots,e_{i_n})}\right]^{i_j}\right\}\right\} - \alpha\right\}\right\}$$

Using $\alpha = 0.05$, and value$K^* = 12$, then

$$= \frac{1}{0.05}\left[\sum_{k=12}^{16} \mathcal{L}_{t+1}(K^\star)\{p(\mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t)\}\right] =$$

$$\sum_{k=12}^{16} \mathcal{L}_{t+1}(K^\star)\{p(\mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t)\} = 0.961024$$

$$=\frac{1}{.05} * 0.961024 = 19.2205$$

$$\frac{1}{0.05}\left[\mathcal{L}_{t+1}(K^\star)\left\{\sum_{k=12}^{16}\{p(\mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t)\} - \alpha\right\}\right]$$

$$\mathcal{L}_{t+1}(K^\star) = 1.900$$

$$\frac{1}{0.05}\left[\mathcal{L}_{t+1}(K^\star)\left\{\sum_{k=12}^{16}\{p(\mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t)\} - \alpha\right\}\right]$$

$$= \frac{1}{0.05}\{1.900(0.518 - 0.05)\}$$

$$== 17.784$$

$$ES_\alpha(\mathcal{L}_{t+1}/\mathcal{F}_t) = 19.2205 - 17.784$$

=1.4365

When $\alpha = 0.01$ then

$K^* = 14$

$VaR_{\alpha,}\rho(\mathcal{L}_{t+1}/\mathcal{F}_t) = \mathcal{L}_{t+1}(K^\star)$= 2.048

$$\frac{1}{0.01}\left[\sum_{k=14}^{16} \mathcal{L}_{t+1}(K^\star)\{p(\mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t)\}\right] = 3.6992$$

$$\frac{1}{0.01}\left[\mathcal{L}_{t+1}(K^\star)\left\{\sum_{k=14}^{16}\{p(\mathcal{L}_{t+1}(K^\star)/\mathcal{F}_t)\} - \alpha\right\}\right] = 0.7000$$

$$ES_\alpha(\mathcal{L}_{t+1}/\mathcal{F}_t) = 3.6992 - 0.700$$

=2.9992

4. Conclusion

This model is a combination of transitions based on the behavioral characteristics of other existing consumers and empirical transitions of the new consumers. We have assumed a first order homogeneous Markov chain with a stationary distribution as it is essential feature in estimation unknown parameters. This model has been applied to measuring of credit risks by evaluation of estimated shortfall and credit value at risk and it has been found that the model is consistent with the understanding of credit measures as the both increases as the probability level. Other areas for further research is application of risk - neutral technique from specification of the transition between states of the existing consumers and the impact of extension to higher order Markov chains.

5. References

1. Altman E. Financial ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. Journal of Finance. 1968 Sep 1;23(4):589-609.
2. Andanje Mulambula, Oduor DB, Kwach B. Derivation of Black – Scholes – Merton Logistic Brownian motion differential equation with Jump diffusion process. Int. J. Math. And Appl. 2019;7(3):85-93. ISSN 2347-1557.
3. Andanje Mulambula, Oduor DB, Kwach B. Volatility estimation using European Logistic Brownian motion with Jump diffusion process. Int. J Math. And Appl. 2020;8(2):155-163. ISSN 2347-1557
4. Crouhy MG. A comparative Analysis of Current Credit Risk Models. Journal of Banking and Finance. 2000 Jan 1;24(1-2):59-117.
5. Oduor DB, Omolo Ongati N, Okelo NB, Silas Onyango N. Estimation of market volatility: A case of Logistic Brownian Motion. International Journal of Marketing and Technology 2012 Jan 1;2(1):37. ISSN: 2249-1058.
6. Das S. Correlated Default Risk, Working Paper; c2004.
7. Leland HT. Optimal Capital Structure, Endogeneous Bankruptcy, and Term Structure of Credit Risk of Credit Spreads. Journal of Finance; c1996, p987-819.
8. Opondo MO, Oduor DB, Odundo F. Jump diffusion Logistic Brownian Motion with dividend yielding asset. Int. J. Math. And Appl. 2021;9(4):25-34. ISSN 2347-1557
9. Merton R. On the Pricing of the Corporate Debt: The Risk Structure of Interest Rates. Journal of Finance. 1974, 449-470.
10. Oduor Brian D. Derivation of black Scholes equation using Heston's model with dividend yielding asset. International Journal of Statistics and Applied Mathematics. 2022;7(1):8-12. ISSN: 2456-1452.
11. Oduor Brian D. Formulating Black Scholes equation using a Jump diffusion Heston's model. International Journal of Statistics and Applied Mathematics. 2022;7(1):13-18. ISSN: 2456-1452.
12. Oduor Brian D. A combination of dividend and jump diffusion process on Heston model in deriving Black Scholes equation. International Journal of Statistics and Applied Mathematics. 2022;7(1):8-12. ISSN: 2456-1452.
13. Oduor DB, Onyango SN. A logistic Brownian motion with price of dividend yielding asset, International Journal of Research and Social Sciences. 2012;2(2).
14. Nickell PS. Stability of rating transitions. Journal of Banking and Finance, 2000 Jan 1;24(1-2):203-27.
15. Umberto Cheruibini EL. Copulas Methods in Finance. New York: John Wiley & Sons; c2004.