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A new area biased two parameter Pranav distribution and its application to medical sciences

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Abstract
The area biased two parameter Pranav distribution is a new variation of the two parameter Pranav distribution that is used in this paper. Its various structural properties such as moments, hazard rate function, survival function, reverse hazard rate function, order statistics, harmonic mean, bonferroni and Lorenz curves have been discussed. The estimation of parameters of new distribution has also been discussed through method of maximum likelihood estimation. Finally, the two real life data sets have been fitted to demonstrate the supremacy of a newly proposed distribution.

Keywords: Two parameter Pranav distribution, weighted distribution, survival analysis, order statistics, moments, maximum likelihood estimation

Introduction

The two parameter Pranav distribution is a newly proposed lifetime model presented by Umeh and Ibenegbu (2019) [19], which is a special case of one parameter Pranav and Ishita distribution. The proposed distribution has two parameters θ and α, where θ is a scale parameter and α is a shape parameter. Its different mathematical and statistical properties including its moments, stochastic ordering, coefficient of variation, skewness, kurtosis, index of dispersion, hazard rate function, mean residual life function, mean deviation, Bonferroni and Lorenz curves have been discussed. Its parameters have been estimated by using the method of moments and maximum likelihood estimation. Shukla (2018) [17] introduced one parameter Pranav distribution and discuss its different structural properties. The two parameter Pranav distribution is a mixture of exponential (θ) and gamma ($4, \theta$). In comparison to the two parameter Akash, Lindley, and one parameter Pranav, Ishita, Akash, Shanker, Sujatha, and exponential distributions, the two parameter Pranav distribution's goodness of fit has been found good.

The two parameter Pranav (TPP) distribution's probability density function is given by

$$f(x;\theta,\alpha) = \frac{\theta^4}{\alpha\theta^4+6}(\alpha\theta + x^3)e^{-\theta x};\ x > 0, \theta > 0, \alpha \geq 0 \tag{1}$$

and the cumulative distribution function of two parameter Pranav distribution is given by

$$F(x;\theta,\alpha) = 1 - \left(1 + \frac{\theta x(\theta^2x^2+3\theta x+6)}{\alpha\theta^4+6}\right)e^{-\theta x}; x > 0, \theta > 0, \alpha \geq 0 \tag{2}$$

Area Biased Two Parameter Pranav (ABTPP) Distribution

The concept of weighted distributions is applied in various fields like biomedicine, ecology and reliability for the development of proper statistical models. When the standard distributions are inappropriate, weighted distributions have set a new benchmark for effective statistical data modelling and prediction. When samples are collected from both the original distribution and the generated distribution, weighted distributions offer a method for fitting the model to the unknown weight function. To represent the ascertainment bias, Fisher (1934) [6] developed the idea of weighted distributions.

When the standard distributions were unable to accurately represent these observations with equal probabilities in the statistical data model, Rao (1965) [14] later developed this idea in a unified manner. The weighted distributions were initially used in relation with sampling wood cells by Warren (1975) [21]. Patil and Rao (1978) [12] introduced the concept of size biased sampling and weighted distributions by identifying some of the situations where the underlying model retain their form. When the weight function only takes the length of the units of interest into account, the weighted distribution becomes a length biased distribution. Cox (1962) [4] first established the idea of a length biased distribution within the framework of the renewal theory. More generally, a distribution is said to be size biased when the sampling process chooses units with a probability proportional to the measure of the unit size. Weighted distributions, a more general variant, are a specific example of size biased distributions. The difficulty of model specification and challenges with data interpretation can be collectively accessed using the weighted distributions.

The numerous weighted probability models have been examined and investigated by various scholars, who have also shown how they can be applied in various disciplines Van Deusen (1986) [20] independently developed the size biased distribution theory when fitting distributions of diameter at breast height (DBH) data arising from horizontal point sampling (HPS) (Grosenbaugh) inventory. Data on HPS diameter increment was then analyzed using weighted distributions by Lappi and Bailey (1987) [9]. Gove (2003) [8] talked about some of the more recent findings about size-biased distributions in parameter estimation for forestry. Chouia *et al.* (2021) [3] presented the size biased Zeghdoudi distribution and discuss its various statistical properties and application. Size-biased sampling and the associated form-invariant weighted distribution were investigated by Patil and Ord in 1975 [12]. Ahmad *et al*. (2016) [2] talked about the statistical characteristics and use of the length biased weighted Lomax distribution. The length and area-biased Maxwell distributions were examined by Sharma *et al*. (2018) [16]. The length biased weighted quasi gamma distribution with attributes and applications was researched by Ganaie and Rajagopalan (2020) [7]. Based on forest inventories, Oluwafemi and Olalekan (2017) [11] provided a length and area biased exponentiated weibull distribution. Perveen (2016) [13] studied the Area biased weighted weibull distribution with applications. Osowole *et al.* (2020) [10] presented Area biased quasi-Transmuted uniform distribution. Rao and Pandey (2020) [15] investigated area-biased Rayleigh distribution parameter estimation. Recently, Ade *et al.* (2021) [1] discussed the characterization and estimation of Area biased quasi-Akash distribution.

Let the random variable X follows non-negative condition has probability density function $f(x)$. Let its non-negative weight function be *w(x)*, then the probability density function of weighted random variable X_w is given by

$$f_w(x) = \frac{w(x)f(x)}{E(w(x))}, x > 0.$$

Where its non-negative weight function be$w(x)$and $E(w(x)) = \int w(x)f(x)dx < \infty$.

Depending upon the various choices of weight function especially when $w(x) = x^c$, result is called weighted distribution. In this paper, we have to obtain the Area biased version of two parameter Pranav distribution, so we will consider as $w(x) = x^2$ to obtain the Area biased two parameter Pranav distribution. Then, the probability density function of Area biased distribution is given by

$$f_a(x) = \frac{x^2 f(x)}{E(x^2)} \tag{3}$$

Where

$$E(x^2) = \int_0^\infty x^2 f(x)dx$$

$$E(x^2) = \frac{2\alpha\theta^4+120}{\theta^2(\alpha\theta^4+6)} \tag{4}$$

By substituting equations (1) and (4) in equation (3), we will get the probability density function of Area biased two parameter Pranav distribution

$$f_a(x) = \frac{\theta^6}{2\alpha\theta^4+120} x^2(\alpha\theta + x^3)e^{-\theta x} \tag{5}$$

and the cumulative distribution function of Area biased two parameter Pranav distribution can be obtained as

$$F_a(x) = \int_0^x f_a(x)dx$$

$$F_a(x) = \int_0^x \frac{\theta^6}{2\alpha\theta^4 + 120} x^2(\alpha\theta + x^3)e^{-\theta x}\, dx$$

$$F_a(x) = \frac{1}{2\alpha\theta^4+120} \int_0^x x^2\theta^6(\alpha\theta + x^3)e^{-\theta x}dx$$

$$F_a(x) = \frac{1}{2\alpha\theta^4+120} \left(\alpha\theta^7 \int_0^x x^2 e^{-\theta x}dx + \theta^6 \int_0^x x^5 e^{-\theta x}dx\right) \tag{6}$$

Put $\theta x = t \Rightarrow \theta dx = dt \Rightarrow dx = \frac{dt}{\theta}$, when$x \to x, t \to \theta x, x \to 0, t \to 0$

After simplification of equation (6), we will obtain the cumulative distribution function of Area biased two parameter Pranav distribution as

$$F_a(x) = \frac{1}{2\alpha\theta^4+120}(\alpha\theta^4\gamma(3,\theta x) + \gamma(6,\theta x)) \tag{7}$$

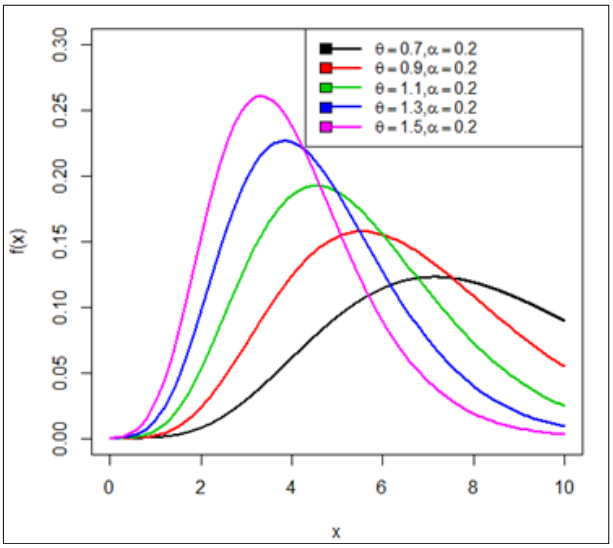

Fig 1: Pdf Plot of area biased two prarameter pranac distribution

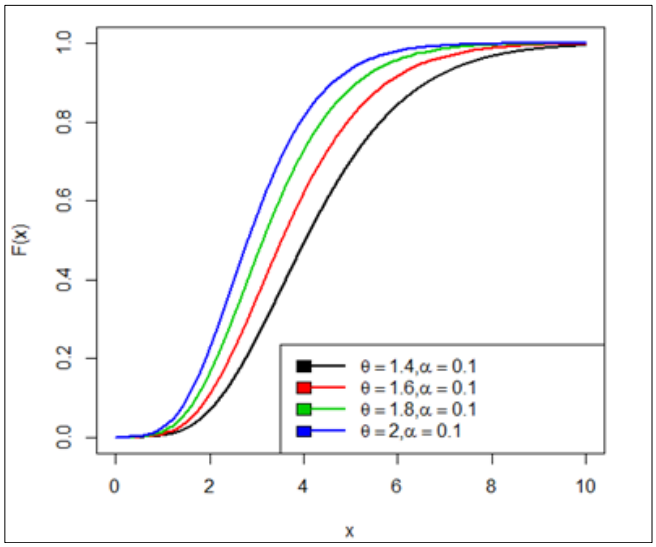

Fig 2: cdf Plot of area biased two prarameter pranac distribution

3. Survival Analysis

In this section, we will discuss the survival function, hazard rate and reverse hazard rate functions of the Area biased two parameter Pranav distribution.

a) Survival function

The survival function is also known as reliability function or compliment of the cumulative distribution function and the survival function of Area biased two parameter Pranav distribution can be obtained as.

$$S(x) = 1 - F_a(x)$$

$$S(x) = 1 - \frac{1}{2\alpha\theta^4+120}(\alpha\theta^4\gamma(3,\theta x) + \gamma(6,\theta x))$$

b) Hazard function

The hazard function is also known as instantaneous failure rate or force of mortality and is given by

$$h(x) = \frac{f_a(x)}{S(x)}$$

$$h(x) = \frac{x^2\theta^6(\alpha\theta+x^3)e^{-\theta x}}{(2\alpha\theta^4+120)-(\alpha\theta^4\gamma(3,\theta x)+\gamma(6,\theta x))}$$

c) Reverse hazard function

The reverse hazard function is given by

$$h_r(x) = \frac{f_a(x)}{F_a(x)}$$

$$h_r(x) = \frac{x^2\theta^6(\alpha\theta+x^3)e^{-\theta x}}{(\alpha\theta^4\gamma(3,\theta x)+\gamma(6,\theta x))}$$

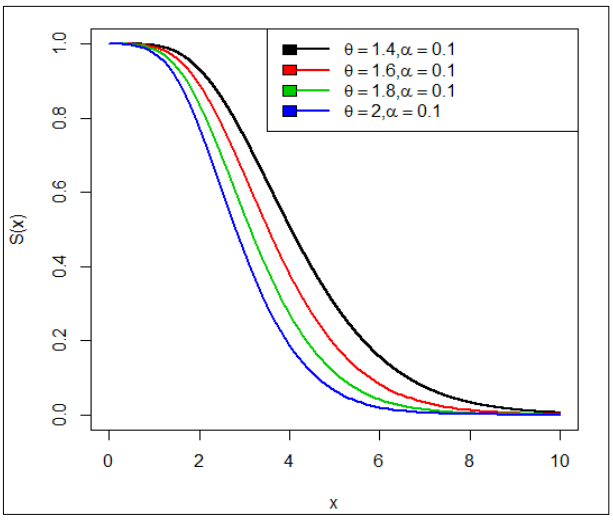

Fig 1: Survival Plot of area biased two prarameter pranac distribution

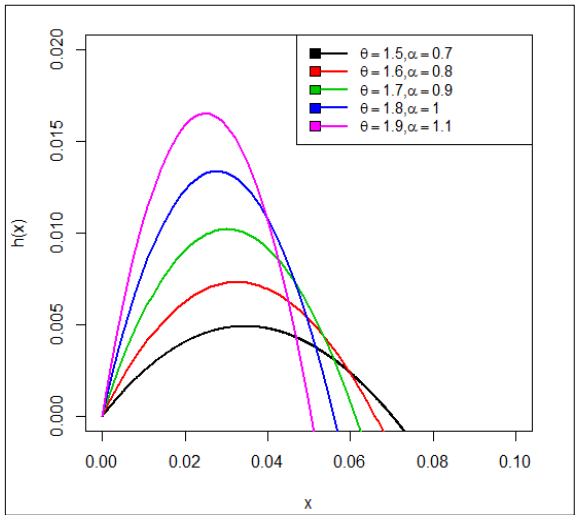

Fig 1: Hazard Plot of area biased two prarameter pranac distribution

4. Statistical Structures

In this section, we will discuss different statistical properties of Area biased two parameter Pranav distribution especially its moments, harmonic mean, MGF and characteristic function.

4.1 Moments

Let the random variable X represents Area biased two parameter Pranav distribution with parameters θ and α, then the rth order moment $E(X^r)$ of X about origin can be obtained as

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_a(x)dx$$

$$E(X^r) = \int_0^\infty x^r \frac{\theta^6}{2\alpha\theta^4+120} x^2(\alpha\theta + x^3)e^{-\theta x}\, dx$$

$$E(X^r) = \frac{\theta^6}{2\alpha\theta^4+120}\int_0^\infty x^{r+2}(\alpha\theta + x^3)e^{-\theta x}dx$$

$$E(X^r) = \frac{\theta^6}{2\alpha\theta^4+120}\left(\alpha\theta\int_0^\infty x^{(r+3)-1}e^{-\theta x}dx + \int_0^\infty x^{(r+6)-1}e^{-\theta x}dx\right) \tag{8}$$

After simplification of equation (8), we obtain

$$E(X^r) = \mu_r' = \frac{\alpha\theta^4\Gamma(r+3)+\Gamma(r+6)}{\theta^r(2\alpha\theta^4+120)} \tag{9}$$

Putting r = 1, 2, 3 and 4 in equation (9), we will obtain the first four moments of Area biased two parameter Pranav distribution.

$$E(X) = \mu_1' = \frac{6\alpha\theta^4+720}{\theta(2\alpha\theta^4+120)}$$

$$E(X^2) = \mu_2' = \frac{24\alpha\theta^4+5040}{\theta^2(2\alpha\theta^4+120)}$$

$$E(X^3) = \mu_3' = \frac{120\alpha\theta^4+40320}{\theta^3(2\alpha\theta^4+120)}$$

$$E(X^4) = \mu_4' = \frac{720\alpha\theta^4+362880}{\theta^4(2\alpha\theta^4+120)}$$

$$\text{Variance} = \frac{(24\alpha\theta^4+5040)(2\alpha\theta^4+120)-(6\alpha\theta^4+720)^2}{\theta^2(2\alpha\theta^4+120)^2}$$

$$S.D(\sigma) = \frac{\sqrt{(24\alpha\theta^4+5040)(2\alpha\theta^4+120)-(6\alpha\theta^4+720)^2}}{\theta(2\alpha\theta^4+120)}$$

4.2 Harmonic mean

The harmonic mean for the proposed model can be obtained as

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{1}{x} f_a(x)dx$$

$$H.M = \int_0^\infty \frac{\theta^6}{2\alpha\theta^4+120} x(\alpha\theta + x^3)e^{-\theta x}dx$$

$$H.M = \frac{\theta^6}{2\alpha\theta^4+120}\left(\alpha\theta \int_0^\infty xe^{-\theta x}dx + \int_0^\infty x^4 e^{-\theta x}dx\right)$$

$$H.M = \frac{\theta^6}{2\alpha\theta^4+120}\left(\alpha\theta \int_0^\infty x^{3-2}e^{-\theta x}dx + \int_0^\infty x^{5-1}e^{-\theta x}dx\right)$$

After simplification, we obtain

$$H.M = \frac{\theta^6}{2\alpha\theta^4+120}(\alpha\theta\gamma(3,\theta x) + \gamma(5,\theta x))$$

4.3 Moment Generating Function and Characteristic Function

Suppose the random variable X follows Area biased two parameter Pranav distribution with parameters θ and α, then the MGF of X can be obtained as:

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f_a(x)dx$$

Using Taylor's series, we obtain

$$M_X(t) = \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + \ldots\right) f_a(x)dx$$

$$M_X(t) = \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_a(x)dx$$

$$M_X(t) = \sum_{j=0}^\infty \frac{t^j}{j!} \mu_j{}'$$

$$M_X(t) = \sum_{j=0}^\infty \frac{t^j}{j!}\left(\frac{\alpha\theta^4\Gamma(j+3)+\Gamma(j+6)}{\theta^j(2\alpha\theta^4+120)}\right)$$

$$M_X(t) = \frac{1}{2\alpha\theta^4+120}\sum_{j=0}^\infty \frac{t^j}{j!\theta^j}(\alpha\theta^4\Gamma(j+3) + \Gamma(j+6))$$

Similarly, the characteristic function of Area biased two parameter Pranav distribution can be obtained as:

$$\phi_X(t) = M_X(it)$$

$$M_X(it) = \frac{1}{2\alpha\theta^4+120}\sum_{j=0}^\infty \frac{it^j}{j!\theta^j}(\alpha\theta^4\Gamma(j+3) + \Gamma(j+6))$$

5. Order Statistics

Order statistics have large applications in the field of applied and statistical sciences especially in reliability and life testing. Let $X_{(1)}$, $X_{(2)}$,…, $X_{(n)}$ denotes the order statistics of a random sample X_1, X_2,…, X_n drawn from a continuous population with

probability density function $f_x(x)$ and cumulative distribution function $F_X(x)$, then the probability density function of rth order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)(F_X(x))^{r-1}(1-F_X(x))^{n-r} \tag{10}$$

Using equations (5) and (7) in equation (10), we will get the probability density function of rth order statistics of Area biased two parameter Pranav distribution.

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{\theta^6}{2\alpha\theta^4+120} x^2(\alpha\theta + x^3)e^{-\theta x}\right)$$

$$\times\left(\frac{1}{2\alpha\theta^4+120}(\alpha\theta^4\gamma(3,\theta x) + \gamma(6,\theta x))\right)^{r-1}$$

$$\times\left(1 - \frac{1}{2\alpha\theta^4+120}(\alpha\theta^4\gamma(3,\theta x) + \gamma(6,\theta x))\right)^{n-r}$$

Therefore, the probability density function of first order statistic $X_{(1)}$ of Area biased two parameter Pranav distribution can be obtained as

$$f_{x(1)}(x) = \frac{n\theta^6}{2\alpha\theta^4+120} x^2(\alpha\theta + x^3)e^{-\theta x}\left(1 - \frac{1}{2\alpha\theta^4+120}(\alpha\theta^4\gamma(3,\theta x) + \gamma(6,\theta x))\right)^{n-1}$$

and the probability density function of higher order statistic $X_{(n)}$ of Area biased two parameter Pranav distribution can be obtained as.

$$f_{x(n)}(x) = \frac{n\theta^6}{2\alpha\theta^4+120} x^2(\alpha\theta + x^3)e^{-\theta x}\left(\frac{1}{2\alpha\theta^4+120}(\alpha\theta^4\gamma(3,\theta x) + \gamma(6,\theta x))\right)^{n-1}$$

6. Likelihood Ratio Test

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from the Area biased two parameter Pranav distribution. The hypothesis is to be tested.

$H_o: f(x) = f(x;\theta,\alpha)$ against $H_1: f(x) = f_a(x;\theta,\alpha)$

In order to test, whether the random sample of size n comes from the two parameter Pranav distribution or Area biased two parameter Pranav distribution, the following test statistic is used.

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n} \frac{f_a(x\,;\theta,\alpha)}{f(x\,;\theta,\alpha)}$$

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\left(\frac{{x_i}^2\theta^2(\alpha\theta^4+6)}{2\alpha\theta^4+120}\right)$$

$$\Delta = \frac{L_1}{L_o} = \left(\frac{\theta^2(\alpha\theta^4+6)}{2\alpha\theta^4+120}\right)^n \prod_{i=1}^{n} {x_i}^2$$

We should reject the null hypothesis, if

$$\Delta = \left(\frac{\theta^2(\alpha\theta^4+6)}{2\alpha\theta^4+120}\right)^n \prod_{i=1}^{n} {x_i}^2 > k$$

or Equivalently, we should reject the null hypothesis, if

$$\Delta^* = \prod_{i=1}^{n} x_i^2 > k\left(\frac{2\alpha\theta^4+120}{\theta^2(\alpha\theta^4+6)}\right)^n$$

$$\Delta^* = \prod_{i=1}^{n} x_i^2 > k^*, \text{Where } k^* = k\left(\frac{2\alpha\theta^4+120}{\theta^2(\alpha\theta^4+6)}\right)^n$$

For large sample of size *n*, *2log Δ* is distributed as chi-square distribution with one degree of freedom and also *p* value is obtained from the chi-square distribution. Also, we reject the null hypothesis, when the probability value is given by

$p(\Delta^* > \lambda^*)$, Where $\lambda^* = \prod_{i=1}^{n} x_i^2$ is less than a specified level of significance and $\prod_{i=1}^{n} x_i^2$ isthe observed value of the statistic Δ^*.

7. Bonferroni and Lorenz curves

The bonferroni and Lorenz curves are applied in different fields like reliability, medicine, insurance and demography. The proposed curves are oldest classical curves and are also known as income distribution curves. The bonferroni and Lorenz curves are defined as:

$$B(p) = \frac{1}{p\mu_1'}\int_0^q xf(x)dx$$

and

$$L(p) = \frac{1}{\mu_1'}\int_0^q xf(x)dx$$

Where

$$\mu_1' = E(X) = \frac{6\alpha\theta^4+720}{\theta(2\alpha\theta^4+120)} \text{ and } q = F^{-1}(p)$$

$$B(p) = \frac{\theta(2\alpha\theta^4+120)}{p(6\alpha\theta^4+720)}\int_0^q \frac{\theta^6}{2\alpha\theta^4+120}x^3(\alpha\theta + x^3)e^{-\theta x}dx$$

$$B(p) = \frac{\theta^7}{p(6\alpha\theta^4+720)}\int_0^q x^3(\alpha\theta + x^3)e^{-\theta x}dx$$

$$B(p) = \frac{\theta^7}{p(6\alpha\theta^4+720)}\left(\alpha\theta\int_0^q x^{4-1}e^{-\theta x}dx + \int_0^q x^{7-1}e^{-\theta x}dx\right)$$

After simplification, we get

$$B(p) = \frac{\theta^7}{p(6\alpha\theta^4+720)}(\alpha\theta\gamma(4,\theta q) + \gamma(7,\theta q))$$

$$L(p) = \frac{\theta^7}{(6\alpha\theta^4+720)}(\alpha\theta\gamma(4,\theta q) + \gamma(7,\theta q))$$

8. Entropy

The concept of entropy is used in various fields such as probability and statistics, physics, communication theory and economics. Entropy discover the diversity, uncertainty or randomness of a system. Entropy of a random variable *X* is a measure of variation of uncertainty.

a) Renyi Entropy

The Renyi entropy is important in ecology and statistics as index of diversity. The entropy is named after Alfred Renyi. For a given probability distribution, Renyi entropy is given by

$$e(\beta) = \frac{1}{1-\beta} log\left(\int {f_a}^{\beta}(x)\, dx\right)$$

Where $\beta > 0$ and $\beta \neq 1$

$$e(\beta) = \frac{1}{1-\beta} log \int_0^{\infty} \left(\frac{\theta^6}{2\alpha\theta^4+120} x^2(\alpha\theta + x^3)e^{-\theta x}\right)^{\beta} dx$$

$$e(\beta) = \frac{1}{1-\beta} log \left(\left(\frac{\theta^6}{2\alpha\theta^4+120}\right)^{\beta} \int_0^{\infty} x^{2\beta} e^{-\theta\beta x}(\alpha\theta + x^3)^{\beta} dx\right)$$

Using binomial expansion in above equation, we obtain

$$e(\beta) = \frac{1}{1-\beta} log \left(\left(\frac{\theta^6}{2\alpha\theta^4+120}\right)^{\beta} \sum_{j=0}^{\infty} \binom{\beta}{j} (\alpha\theta)^{\beta-j} x^{3j} \int_0^{\infty} x^{2\beta} e^{-\theta\beta x} dx\right)$$

$$e(\beta) = \frac{1}{1-\beta} log \left(\left(\frac{\theta^6}{2\alpha\theta^4+120}\right)^{\beta} \sum_{j=0}^{\infty} \binom{\beta}{j} (\alpha\theta)^{\beta-j} \int_0^{\infty} x^{(2\beta+3j+1)-1} e^{-\theta\beta x}\, dx\right)$$

$$e(\beta) = \frac{1}{1-\beta} log \left(\left(\frac{\theta^6}{2\alpha\theta^4+120}\right)^{\beta} \sum_{j=0}^{\infty} \binom{\beta}{j} (\alpha\theta)^{\beta-j} \frac{\Gamma(2\beta+3j+1)}{(\theta\beta)^{2\beta+3j+1}}\right)$$

b) Tsallis Entropy

The generalization of Boltzmann-Gibbs (B.G) statistical properties initiated by Tsallis has focused a great deal to attention. This generalization of B-G statistics was proposed firstly by introducing the mathematical expression of Tsallis entropy (Tsallis, 1988) for a continuous random variable is defined as follows:

$$S_{\lambda} = \frac{1}{\lambda-1}\left(1 - \int_0^{\infty} {f_a}^{\lambda}(x)\, dx\right)$$

$$S_{\lambda} = \frac{1}{\lambda-1}\left(1 - \int_0^{\infty} \left(\frac{\theta^6}{2\alpha\theta^4+120} x^2(\alpha\theta + x^3)e^{-\theta x}\right)^{\lambda} dx\right)$$

$$S_{\lambda} = \frac{1}{\lambda-1}\left(1 - \left(\frac{\theta^6}{2\alpha\theta^4+120}\right)^{\lambda} \int_0^{\infty} x^{2\lambda} e^{-\lambda\theta x}(\alpha\theta + x^3)^{\lambda} dx\right)$$

Using binomial expansion in above equation, we get

$$S_{\lambda} = \frac{1}{\lambda-1}\left(1 - \left(\frac{\theta^6}{2\alpha\theta^4+120}\right)^{\lambda} \sum_{j=0}^{\infty} \binom{\lambda}{j} (\alpha\theta)^{\lambda-j}\, x^{3j} \int_0^{\infty} x^{2\lambda} e^{-\lambda\theta x}\, dx\right)$$

$$S_{\lambda} = \frac{1}{\lambda-1}\left(1 - \left(\frac{\theta^6}{2\alpha\theta^4+120}\right)^{\lambda} \sum_{j=0}^{\infty} \binom{\lambda}{j} (\alpha\theta)^{\lambda-j} \int_0^{\infty} x^{(2\lambda+3j+1)-1}\, e^{-\lambda\theta x} dx\right)$$

$$S_{\lambda} = \frac{1}{\lambda-1}\left(1 - \left(\frac{\theta^6}{2\alpha\theta^4+120}\right)^{\lambda} \sum_{j=0}^{\infty} \binom{\lambda}{j} (\alpha\theta)^{\lambda-j} \frac{\Gamma(2\lambda+3j+1)}{(\lambda\theta)^{2\lambda+3j+1}}\right)$$

9. Maximum Likelihood Estimation and Fisher's Information Matrix

In this section, we will estimate the parameters of Area biased two parameter Pranav distribution by using the method of maximum likelihood estimation and also derive its Fisher's information matrix. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the Area biased two parameter Pranav distribution, then the likelihood function is given by

$$L(x) = \prod_{i=1}^{n} f_a(x)$$

$$L(x) = \prod_{i=1}^{n} \left(\frac{\theta^6}{2\alpha\theta^4+120} x_i^2 (\alpha\theta + x_i^3) e^{-\theta x_i} \right)$$

$$L(x) = \frac{\theta^{6n}}{(2\alpha\theta^4+120)^n} \prod_{i=1}^{n} \left(x_i^2 (\alpha\theta + x_i^3) e^{-\theta x_i} \right)$$

The log likelihood function is given by

$$\log L = 6n \log \theta - n \log(2\alpha\theta^4 + 120) + 2\sum_{i=1}^{n} \log x_i + \sum_{i=1}^{n} \log(\alpha\theta + x_i^3) - \theta \sum_{i=1}^{n} x_i \quad (11)$$

Now differentiating the log likelihood equation (11) with respect to parameters θ and α, we must satisfy the normal equations as

$$\frac{\partial \log L}{\partial \theta} = \frac{6n}{\theta} - n\left(\frac{8\alpha\theta^3}{2\alpha\theta^4+120}\right) + n\left(\frac{\alpha}{(\alpha\theta+x_i^3)}\right) - \sum_{i=1}^{n} x_i = 0$$

$$\frac{\partial \log L}{\partial \alpha} = -n\left(\frac{2\theta^4}{2\alpha\theta^4+120}\right) + n \rightleftarrows \left(\frac{\theta}{(\alpha\theta+x_i^3)}\right) = 0$$

The above likelihood equations are too complicated to solve it algebraically. Therefore, we use R and wolfram mathematics for estimating the required parameters of the proposed distribution.

For the purpose of obtaining the confidence interval, we use the asymptotic normality results. We have that if $\hat{\lambda} = (\hat{\theta}, \hat{\alpha})$ denotes the MLE of $\lambda = (\theta, \alpha)$. We can state the result as follows:

$$\sqrt{n}(\hat{\lambda} - \lambda) \to N_2(0, I^{-1}(\lambda))$$

Where $I(\lambda)$ is Fisher's Information matrix. *i.e.*

$$I(\lambda) = -\frac{1}{n} \begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial \theta \partial \alpha}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \alpha \partial \theta}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) \end{pmatrix}$$

Here we see

$$E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) = -\frac{6n}{\theta^2} - n\left(\frac{(2\alpha\theta^4+120)(24\alpha\theta^2)-(64\alpha^2\theta^6)}{(2\alpha\theta^4+120)^2}\right) - n\left(\frac{\alpha^2}{(\alpha\theta+x_i^3)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = n\left(\frac{(4\theta^8)}{(2\alpha\theta^4+120)^2}\right) - n\left(\frac{\theta^2}{(\alpha\theta+x_i^3)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \theta \partial \alpha}\right) = -n\left(\frac{(2\alpha\theta^4+120)(8\theta^3)-(8\alpha\theta^3)(2\theta^4)}{(2\alpha\theta^4+120)^2}\right) - n\left(\frac{\alpha\theta}{(\alpha\theta+x_i^3)^2}\right)$$

Since λbeing unknown, we estimate $I^{-1}(\lambda)$ by $I^{-1}(\hat{\lambda})$ and this can be used to obtain asymptotic confidence intervals for θ andα.

10. Data Analysis

In this section, we have fitted two real data sets in Area biased two parameter Pranav distribution to discuss it goodness of fit and the fit of Area biased two parameter Pranav distribution has been compared over two parameter Pranav, Pranav, exponential and Lindley distributions. The two real data sets are as under.

Data set I**:** The following data set represents the survival times (in months) of patients of melanoma studied by Susarla and Vanryzin (1978) [18]. The data set is given below in table 1 as

Table 1: Data regarding Survival times (in months) studied by Susarla and Vanryzin

3.25	3.50	4.75	4.75	5.00	5.25	5.75	5.75	6.25
6.50	6.50	6.75	6.75	7.78	8.00	8.50	8.50	9.25
9.50	9.50	10.00	11.50	12.50	13.25	13.50	14.25	14.50
14.75	15.00	16.25	16.25	16.50	17.50	21.75	22.50	24.50
25.50	25.75	27.50	29.50	31.00	32.50	34.00	34.50	35.25
58.50								

Data set II: The following data set represents the relief times (in minutes) of 20 patients receiving an analgesic studied by Gross and Clarke (1995) [5] and the data set is given in table 2 as

Table 2: Data regarding relief times (in minutes) of 20 patients receiving an analgesic

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7
4.1	1.8	1.5	1.2	1.4	3.0	1.7	2.3	1.6	2.0

In order to estimate the model comparison criterion values, the unknown parameters are also estimated through the technique of R software. In order to compare the Area biased two parameter Pranav distribution with two parameter Pranav, Pranav, exponential and Lindley distributions, we are using the criterion values *AIC* (Akaike Information Criterion), *BIC* (Bayesian Information Criterion), *AICC* (Akaike Information Criterion Corrected) and *-2logL*. The better distribution is which corresponds to lower values of *AIC*, *BIC*, *AICC* and *-2logL*. For the calculation of criterion values, following formulas are used.

$$AIC = 2k - 2\log L\,, BIC = k\log n - 2\log L \text{ and } AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

Where *k* is the number of parameters in the statistical model, *n* is the sample size and *–2logL* is the maximized value of the log-likelihood function under the considered model.

Table 3: Performance of Fitted Distributions

Data sets	Distribution	MLE	S.E	-2logL	AIC	BIC	AICC
1	Area Biased Two Parameter Pranav	$\hat{\alpha} = 6.226400$ $\hat{\theta} = 2.366357$	$\hat{\alpha} = 2.344597$ $\hat{\theta} = 0.000000$	330.9685	334.9685	338.6258	335.2475
	Two Parameter Pranav	$\hat{\alpha} = 0.14490543$ $\hat{\theta} = 0.25543800$	$\hat{\alpha} = 150.52523540$ $\hat{\theta} = 0.02767165$	342.2436	346.2436	349.9008	346.5226
	Pranav	$\hat{\theta} = 0.25532302$	$\hat{\theta} = 0.01880244$	342.2422	344.2422	346.0708	344.3331
	Exponential	$\hat{\theta} = 0.063865343$	$\hat{\theta} = 0.00941412$	345.0919	347.0919	348.9205	347.1828
	Lindley	$\hat{\theta} = 0.12084543$	$\hat{\theta} = 0.01263491$	333.6992	335.6992	339.5279	335.7901
2	Area Biased Two Parameter Pranav	$\hat{\alpha} = 0.0010000$ $\hat{\theta} = 3.1548229$	$\hat{\alpha} = 0.1344569$ $\hat{\theta} = 0.1418558$	37.70726	41.70726	43.69872	42.4131
	Two Parameter Pranav	$\hat{\alpha} = 0.0010000$ $\hat{\theta} = 2.0987426$	$\hat{\alpha} = 0.1665749$ $\hat{\theta} = 0.1068157$	41.95912	45.95912	47.95059	46.6650
	Pranav	$\hat{\theta} = 1.4014009$	$\hat{\theta} = 0.1247081$	62.38652	64.38652	65.38225	64.6087
	Exponential	$\hat{\theta} = 0.5263164$	$\hat{\theta} = 0.1176875$	65.67416	67.67416	68.66989	67.8963
	Lindley	$\hat{\theta} = 0.8161188$	$\hat{\theta} = 0.1360929$	60.4991	62.4991	63.49483	62.7213

From table 3 given above, it has been observed from the results that the Area biased two parameter Pranav distribution have the lesser *AIC*, *BIC*, *AICC* and *–2logL* values as compared to two parameter Pranav, Pranav, exponential and Lindley distributions. Hence, it can be concluded that the Area biased two parameter Pranav distribution fits better over two parameter Pranav, Pranav, exponential and Lindley distributions.

11. Conclusion

In the present study a new distribution namely Area biased two parameter Pranav distribution has been discussed. The subject distribution is generated by using the Area biased technique and taking the two parameter Pranav distribution as the base distribution. Its various statistical properties including its mean, variance, MGF, characteristic function, survival function, hazard rate function, order statistics, Renyi entropy, bonferroni and Lorenz curves etc. have been derived and discussed. Its parameters have also been estimated through the maximum likelihood estimation technique. Finally, the two real data sets have been fitted in newly proposed distribution to discuss its superiority and flexibility.

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