International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(5): 08-09

www.mathsjournal.com
Received: 08-06-2022
Accepted: 12-07-2022

SK Pandey
Department of Mathematics, Sardar Patel University of Police, Security and Criminal Justice, Vigyan Nagar, Jodhpur, Rajasthan, India

Corresponding Author:
SK Pandey
Department of Mathematics, Sardar Patel University of Police, Security and Criminal Justice, Vigyan Nagar, Jodhpur, Rajasthan, India

A note on locally invo-regular rings

SK Pandey

DOI: https://doi.org/10.22271/maths.2022.v7.i5a.873

Abstract
In this paper we mainly exhibit that if R is a locally invo-regular ring of characteristic three with no central nilpotent elements then R need not be isomorphic to the field of order three.

Keywords: Invo-regular ring, locally invo-regular ring, quasi invo-regular ring

Introduction

Locally invo-regular rings, invo-regular unital rings and quasi invo-regular rings have been studied recently [1-3].

Recall that a ring R is called an invo-regular ring if $r = rvr$ for each $r \in R$. Here $v \in R$ is an element satisfying $v^2 = 1$ [1-3]. One may note that as per [1] a ring R is called locally invo-regular if $r = rvr$ or $1 - r = (1-r)v(1-r)$ for each $r \in R$. Thus one may easily find that each invo-regular ring is a locally invo-regular ring.

In this paper we provide some results related to locally invo-regular rings. We prove that a locally invo-regular ring R of characteristic three with no central nilpotent elements is not necessarily isomorphic to the field of order three.

We produce an example of a ring of order nine and characteristic three which is a locally invo-regular ring with no central nilpotent element however it is not isomorphic to the field of order three. This example works as a counterexample for the following result. Thus the present work improves the results of existing mathematical literature.

In [1, theorem 3] it has been noted that if R is a locally invo-regular ring of characteristic three with no central nilpotent elements then R is isomorphic to the field of order three.

In addition we exhibit that a ring without central nilpotent elements need not be a ring without non-trivial idempotent element.

It may be noted that a locally invo-regular ring without non-trivial idempotents is a weakly tripotent ring [1]. In the next section we exhibit that the converse of this result is not true. A weakly tripotent ring is a ring in which for each $a \in R$ we have $a^3 = a$ or $(1+a)^3 = 1 + a$ [4-5].

In this note R is a unital and associative ring. In the next section we produce desired results.

Main Results

Proposition 1: A ring without central nilpotent elements need not be a ring without non-trivial idempotent element.

Proof: One can easily note that the field Z_3 of order three does not contain any non-zero nilpotent element and it has been considered as a ring without central nilpotent element in [1]. In addition Z_3 does not contain any non trivial idempotent element. Now we shall prove the result of this proposition by means of the following example.

Let $R = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$. One may verify that R is a ring under component wise addition and multiplication modulo three.

One may easily find that R does not have central nilpotent elements as it does not have any non-zero nilpotent elements. Clearly $(1,0)$ and $(0,1)$ are non-trivial idempotent elements of R. Therefore a ring without central nilpotent elements need not be a ring without non-trivial idempotent element.

Proposition 2: A weakly tripotent invo-regular ring need not be a ring without non-trivial idempotent elements.

Proof: Let us consider the above example. One may easily verify that each element a of R satisfies $a^3 = a$ or $(1+a)^3 = 1+a$. Therefore R is a weakly tripotent ring. Further we have

$$(0,0) = (0,0)(1,1)(0,0)$$

$$(0,1) = (0,1)(1,1)(0,1)$$

$$(0,2) = (0,2)(2,2)(0,2)$$

$$(1,0) = (1,0)(1,1)(1,0)$$

$$(1,1) = (1,1)(1,1)(1,1)$$

$$(1,2) = (1,2)(1,2)(1,2)$$

$$(2,0) = (2,0)(2,2)(2,0)$$

$$(2,1) = (2,1)(2,1)(2,1)$$

$$(2,2) = (2,2)(2,2)(2,2)$$

Thus R is an invo-regular ring and hence a locally invo-regular ring. Also, $(1,0)$ and $(0,1)$ are non-trivial idempotent elements of R. Therefore R is a weakly tripotent ring which is a locally invo-regular ring with non-trivial idempotent elements.

Proposition 3: A locally invo-regular ring R of characteristic three with no central nilpotent elements is not necessarily isomorphic to the field of order three.

Proof; In order to prove this result we consider the same example given in the proof of proposition 1.

Let $R = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$.

We have already noted that R is a ring under component wise addition and multiplication modulo three. Also, it does not have central nilpotent elements and the characteristic of R is three.

It has been seen that R is an invo-regular ring. And each invo-regular ring is a locally invo-regular ring. Hence R is a locally invo-regular ring.

Since R is a ring of order nine as well as it is a ring with zero divisors therefore it cannot be isomorphic to the field of order three. Thus if R is a locally invo-regular ring of characteristic three with no central nilpotent elements then R need not be isomorphic to the field of order three.

References

1. Danchev PV. Locally Invo-Regular rings, Azerbaijan Journal of Mathematics. 2021;11(1):28-44
2. Danchev PV. Invo-Regular Unital Rings, Ann. Univ. Mariae Curie-Sklodowska, Sect. A-Math. 2018;72:45-53.
3. Danchev PV. Quasi invo-clean rings, Bulletin of the Transilvania University of Brasov, Series III: Mathematics, Informatics, Physics. 2021 Jul 8:71-80.
4. Breaz S, Cimpean A. Weakly Tripotent Rings, Bull. Korean Math. Soc. 2018;55(4):1179-1187,
5. Pandey SK. Some Counterexamples in Ring Theory, arXiv:2203.02274 [math.RA], 2022 Feb 17.