International Journal of Statistics and Applied Mathematics 2022; 7(5): 31-35

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(5): 31-35

www.mathsjournal.com
Received: 26-06-2022
Accepted: 31-07-2022

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An economic model based on SkSP-2 with double sampling plan as the reference plan

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DOI: https://doi.org/10.22271/maths.2022.v7.i5a.879

Abstract
Skip-lot sampling plans have been widely used in industries to reduce inspection efforts when products have excellent quality records. This paper provides an economic model based on SkSP-2 with the double sampling plan as a reference plan. The various combinations of producer quality level and consumer quality level by minimizing both the producer's and the consumer's risk have been discussed. Numerical examples are provided to confirm the application of the proposed concept.

Keywords: Acceptance sampling, skip-lot sampling plan-2, double sampling plan, consumer's risk, producer's risk

Introduction

The main objective of inspection at the producer's end is to control the quality of the produced items by examining them at strategic points. Inspection ensures that the quality of the lot accepted is in accordance with the specifications of the consumer. Thus, the sampling inspection, if meticulously designed, puts more effective pressure on quality improvement and, therefore, results in the submission of a better-quality product for inspection. In statistical quality control, samples of items are inspected, and the results are used to control their quality. Such a procedure, besides ensuring the quality of the product, keeps down the cost of production. Acceptance sampling plays a vital role in quality control. In acceptance sampling, items are grouped into lots, a sample is taken, and based on the observed quality of the sample, the entire lot is either accepted or rejected. The first point to decide, in acceptance sampling, is to define lots. It is to ensure that all items in the lot are produced under the same condition. As far as possible, items manufactured under different conditions should not be mixed into one lot. Attempts should be made to maintain the identity of different lots. It is further essential that a sample be drawn from each lot based on which the decision to accept or reject the lot is to be taken.

Mohammad Saber Fallah Nezhad, Ahmad Ahmadi Yazdi, Parvin Abdollahi, and Muhammad Aslam (2015) [1] developed a mathematical model to design single stage and double stage sampling plan that can determine the optimal tolerance limits and the sample size. They found that the two-stage sampling plan showed better robustness than the single stage plan. Lie-Fern Hsu, and Jia-Tzer Hsu (2012) [2] designed an economic model to devise a sampling plan to minimize the total quality cost both for the producer and customer. The plan was found to be dependent on inspection, internal failure, and post-sale failure. S. Balamurali and J. Subramani (2012) [3] proposed an optimized design to establish the parameters that affect the skip-lot sampling plan of type SkSP-2 plan with double sampling plan as the reference plan. The plan has been illustrated numerically using various combinations of acceptable and limiting quality levels. The proposed plan was found to be advantageous over conventional sampling plans.

Ferrell, W.G., and Chhoker A (2002) [4] developed the design of economically optimal acceptance sampling plans using a unique approach by employing a continuous loss function to quantify the deviations. They also discussed inspection error. The proposed models were found to be easy and user friendly. S. Balamurali and J. Subramani (2010) [5] designed a skip-lot sampling plan, SkSP-3 to reduce the inspection cost using Markov chain formulation.

Dodge and Romig (1998) [6] provided double sampling plans with a minimum average total inspection The construction and evaluation of matched sets of single, double, and multiple sampling plans were developed by Shilling and Johnson (1978) [7] Dodge and Romig (1998) [6] provided double sampling plans with a minimum average total inspection. R. Vijayaraghavan & V. Soundararajan (1998) [8] developed the concepts of design and evaluation of skip-lot sampling inspection plans with a double-sampling plan as the reference plan. Their work aimed at reducing the sample size without compromising the efficiency. Robert L. Perry (1973) [9] presented a system of skip-lot sampling plans for lot inspection designated as type SkSP-2. This system allows for skipping lots when the quality of the submitted product is good.

Operating procedure for double sampling plan and skip-lot sampling plan-2

In the single sampling plan, the decision to accept or reject a lot is taken based on one sample drawn from the lot. It could happen that the quality of the sample may not be truly representative of the quality of the lot, and the entire lot may be rejected even though the quality might be as per specification, or the entire lot is accepted even though the quality of the lot is not as per specifications. While such a risk cannot be eliminated, it can be reduced by resorting to taking a second sample from the lot whenever the first sample does not provide ample evidence of the exceptionally good or inferior quality of the lot. A double sampling inspection plan can be described as follows.

A sample of n_1 items is drawn from a given lot. The lot is accepted, if the number of defective items in the sample is less than or equal to c_1. If the sample contains more than c_2 defective items ($c_2 > c_1$), the lot is immediately rejected. If the number of defective items is greater than c_1 but is less than or equal to c_2, a second sample of size n_2 is drawn. If the total number of defectives in the combined sample size is less than the combined sample size, the lot is accepted. However, if it is greater than c_2, the lot is rejected. Thus, the double sampling plan is described by four numbers n_1, n_2, c_1 and c_2.

Thus, if the incoming quality is exceptionally good, the lot is accepted based on the first sample itself and rejected if the quality is bad. However, if the quality is of an average type, a second sample may be required. Psychologically, a double sampling plan has the advantage that it gives a good chance to lots of doubtful quality. Another advantage of double sampling plan over a single sampling plan is that, for a given amount of risk, it needs lesser items to be inspected, thus reducing costs involved.

The double sampling plan can be summarized as follows:

1. Take a first sample of size n_1
   - If the number of defectives (d_1) is $\le c_1$ - Accept the lot
   - If the number of defectives (d_1) is $> c_2$ - Reject the lot
2. Take a second sample of size n_2. Let the number of defectives be d_2.
   - If $d_1 + d_2 \le c_2$ - Accept the lot
   - If $d_1 + d_2 > c_2$ - Reject the lot

Recently, Balamurali and J. Subramani (2012) [3] determined the parameters of a skip-lot sampling plan of type SkSP-2 plan with double sampling plan as the reference plan. The skip-lot sampling plan is extremely useful, and it has economic advantages in the design of sampling plans.

The operating procedure for skip-lot sampling plan-2 with double sampling plan as a reference plan is stated as follows:

a) To begin with, inspect every lot using a double sampling plan as the reference plan. At the time of inspection, lots are inspected one by one in the order of production
b) When *i* consecutive lots on the double sampling plan are accepted, discontinue normal inspection and switch to skipping inspection.
c) During skipping inspection, inspect only a fraction *f* of the lots using the double sampling plan as the reference plan. Skipping inspection is continued until a sampled lot is rejected.
d) If a lot is rejected, resume normal inspection immediately as per step (i).

2.1 Notations and assumptions

n_1= First sample size of the double sampling plan
n_2= Second sample size of the double sampling plan
n = Sample size of the single sampling plan
d_1= Number of nonconforming items observed in the first sample
d_2= Number of nonconforming items observed in the second sample
c_1= First sample acceptance number in double sampling plan
c_2= Cumulative acceptance number based on two samples in double sampling plan
c= Acceptance number of the single sampling plan
p_1= Acceptable quality level
p_2= Limiting quality level
α = Producer's risk
β= Consumer's risk
p= Probability of accepting the lot
f= Fraction of lots
I = *Pre*-specified number of lots
D_d = Number of defective items detected
D_n = Number of defective items not detected
C_f = Internal failure cost
C_i = Inspection cost per item
C_O= Cost of an outgoing defective unit

Economic model for SkSP-2 with double sampling plan as a reference plan

Although there are a handful of works that highlight the advantages of skip-lot sampling plans, the present work proposes a robust skip-lot sampling plan-2 with double sampling plan as a reference plan. The model can be applied readily in quality control routines to reduce the total cost. The challenges in designing a sampling plan are two-fold, *viz*., the number of lots to be assessed and the efficiency of the sampling technique in terms of outcome. Moreover, sampling needs to consider the risk of both the producer and the consumer, and a proven plan must as far as possible, reduce this key factor. The proposed model first requires the determination of P_a (p) value using SkSP-2. Followed by this, a mathematical model has been designed such that it finally reduces the total production cost (TC). This was implemented by calculating D_d and D_n values, and then arriving at the total cost for production using Acceptable Quality Level (AQL) and Lot Tolerance Percent Defective (LTPD). In accordance with Perry (1973) [9], the operating characteristic function of SkSP-2 plan is given by,

$$P_a(p) = \frac{f\,P + (1-f)P^i}{f + (1-f)P^i} \quad \text{-------------------- (1)}$$

$$P = R(c_1;m_1) + \sum_{k=c_1+1}^{c_2} q(k_1;m_1)\, R(c_2-k;m_2)$$

Where

$$R(c_1;m_1) = \sum_{k=0}^{c_1} q(k;m)$$

$$q(k;m) = \frac{e^{-m}m^k}{k!}\ ;\ m_1 = n_1 p;\ \ m_2 = n_2 p$$

$$D_d = np + (1-p_a(p))(N-n)p \quad \text{------------------ (2)}$$

$$D_n = P_a(p)(N-n)p \quad \text{---------------------------- (3)}$$

The mathematical model for an economic sampling plan is:

$$Minimize \quad TC = C_i\ ATI + C_f\ D_d + C_0\, D_n$$

$$Subject \quad to\ condition$$

$$1 - P_a(AQL) \le \alpha$$

$$P_a(LTPD) \le \beta$$

Finally, tables can be constructed using the above model, which can be used a ready reckoner in quality control processes.

Numerical Illustrations

Two conditions, *viz*., $n_1=n_2$ and $n_2=2\times n_1$, were considered to exemplify the model numerically. Tables 1 and 3 show the total production cost and sensitivity analysis of the Skip- Lot Sampling plan-2 with different level of p values when $n_1=n_2$ and $n_2=2\times n_1$, respectively. If the values presented in the table are observed closely, it is evident that the probability of accepting the lot has a direct effect on AOQ. As the probability of accepting the lots increases, the average outgoing quality values are found to decrease. If p values are greater than 0.1, the AOQ value approaches zero, resulting in 100% inspection of the entire lot.

Table 1: The total production cost and sensitivity analysis of the Skip-Lot Sampling plan-2 for $n_1=n_2$

P	TC	AOQ	ASN	ATI	D_d	D_n	$P_a(p)$
0	207.0	0.8620	4830.00	207.0	0.000	0.0000	1.0000
0.01	319.1	0.8258	4676.22	232.6	1.556	8.3416	0.9677
0.02	529.2	0.6549	3760.20	385.2	5.178	13.3662	0.7753
0.03	770.9	0.3977	2333.33	622.8	12.509	12.2990	0.4756
0.04	940.3	0.1904	1164.51	817.5	21.719	7.9338	0.2301
0.05	1024.7	0.0813	541.77	921.3	30.318	4.2798	0.0993
0.06	1064.6	0.0328	261.82	967.9	37.871	2.0947	0.0405
0.07	1086.3	0.0128	145.18	987.3	44.651	0.9654	0.0160
0.08	1101.2	0.0050	98.99	995.0	50.947	0.4344	0.0063
0.09	1113.9	0.0020	80.90	998.0	56.948	0.1940	0.0025
0.1	1125.6	0.0008	73.76	999.2	62.751	0.0862	0.0010

Similarly, the relationships of inspection cost C_i and the failure cost C_f with total production cost when n1=n2 and n2=2×n1, respectively, are summarized in Table 2 and Table 4. When C_f increases, the total production cost also increases

Table 2: The inspection cost Ci and the failure cost Cf with total production cost for $n_1=n_2$

TC	C_O	C_i	C_f	AOQ	ATI	D_d	D_n
20.7	10	0.1	0.5	0.8620	207.00	0.0000	0.0000
146.5	10	0.2	1	0.8258	232.61	1.6584	8.3416
315.6	10	0.3	1.5	0.6549	385.19	6.6338	13.3662
549.1	10	0.4	2	0.3977	622.85	17.7010	12.2990
808.8	10	0.5	2.5	0.1904	817.53	32.0662	7.9338
1052.8	10	0.6	3	0.0813	921.26	45.7202	4.2798
1277.5	10	0.7	3.5	0.0328	967.88	57.9053	2.0947
1489.8	10	0.8	4	0.0128	987.31	69.0346	0.9654
1695.5	10	0.9	4.5	0.0050	995.00	79.5656	0.4344
1898.0	10	1	5	0.0020	998.02	89.8061	0.1940
2099.1	10	1.1	5.5	0.0008	999.21	99.9138	0.0862

Figures 1 and 2 show the plot between the probability of accepting the lot and average outgoing quality. The curves show that when p value increases, the AOQ value decreases gradually upto p = 0.04 and then starts approaching zero. This means after this point; the sampling plan requires 100% inspection of the entire lot.

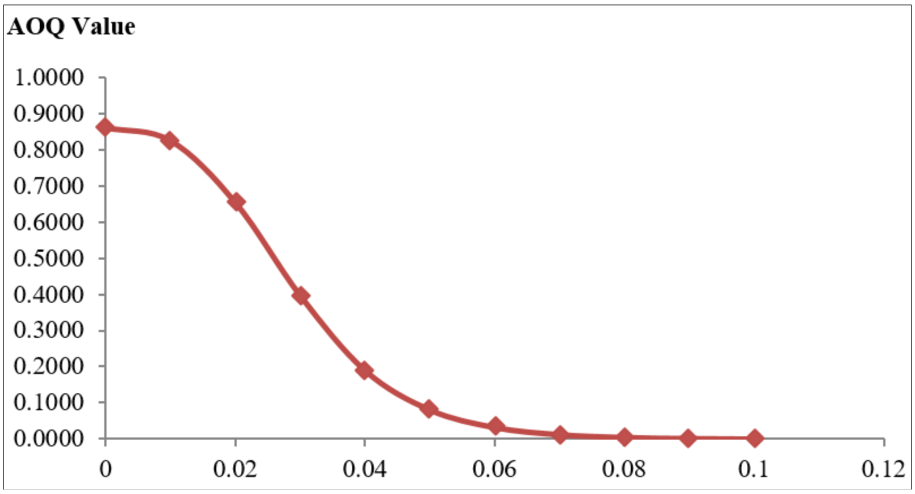

Fig 1: AOQ Curve for $n_1=n_2$

Table 3: The total production cost and sensitivity analysis of the Skip-Lot Sampling plan-2 for $n_2=2\times n_1$

p	TC	AOQ	ASN	ATI	D_d	D_n	$P_a(p)$
0	528.0	0.6040	34980.00	528.0	0.000	0.0000	1.0000
0.01	609.5	0.5720	33467.60	548.5	1.616	5.7779	0.9566
0.02	741.8	0.4530	26801.17	638.8	5.285	9.2448	0.7653
0.03	854.4	0.3372	20190.51	728.3	10.913	10.4299	0.5756
0.04	973.8	0.1688	10279.74	862.6	20.433	7.0354	0.2912
0.05	1046.2	0.0506	3205.59	958.4	30.607	2.6636	0.0882
0.06	1073.0	0.0173	1194.86	985.6	38.179	1.1053	0.0305
0.07	1088.8	0.0079	619.87	993.4	44.722	0.5919	0.0140
0.08	1102.0	0.0031	327.15	997.4	50.975	0.2706	0.0056
0.09	1114.1	0.0008	184.27	999.3	56.993	0.0815	0.0015
0.1	1125.7	0.0011	201.70	999.1	62.702	0.1208	0.0020

Table 4: The inspection cost Ci and the failure cost Cf with total production cost for $n_2=2\times n_1$

TC	C_O	C_i	C_f	AOQ	ATI	D_d	D_n
528.0	10	0.1	0.5	0.6040	528.00	0.0000	0.0000
609.5	10	0.2	1	0.5720	548.48	1.6161	5.7779
741.8	10	0.3	1.5	0.4530	638.78	5.2854	9.2448
854.4	10	0.4	2	0.3372	728.32	10.9134	10.4299
973.8	10	0.5	2.5	0.1688	862.55	20.4332	7.0354
1046.2	10	0.6	3	0.0506	958.37	30.6068	2.6636
1073.0	10	0.7	3.5	0.0173	985.60	38.1794	1.1053
1088.8	10	0.8	4	0.0079	993.39	44.7222	0.5919
1102.0	10	0.9	4.5	0.0031	997.36	50.9751	0.2706
1114.1	10	1	5	0.0008	999.29	56.9930	0.0815
1125.7	10	1.1	5.5	0.0011	999.06	62.7020	0.1208

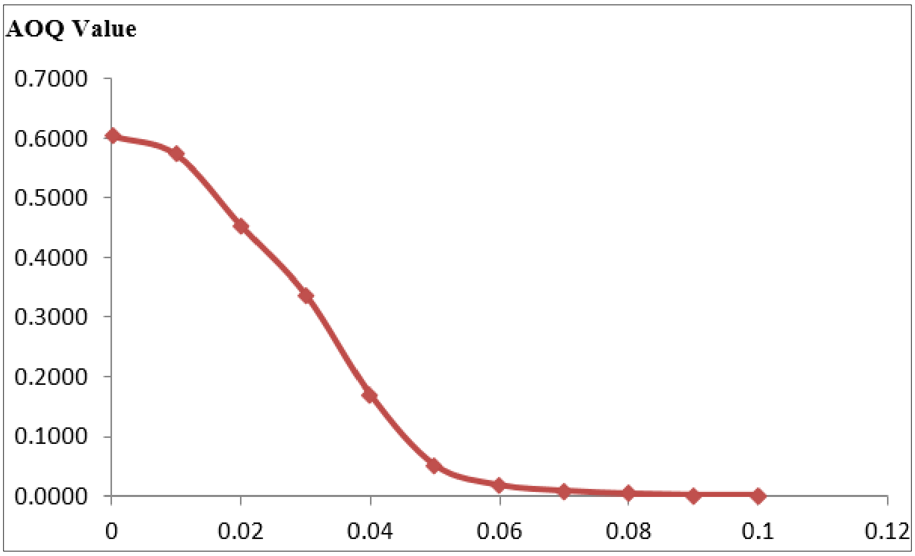

Fig 2: AOQ Curve for $n_2=2\times n_1$

Conclusion

In this paper, we have designed an economic model based on SkSP-2 with double sampling plan as a reference plan. Based on this model, it is found that the proposed plan needs a small number of sample units for the inspection than the conventional plans. So, this method is helpful in reducing the cost and time of inspection at the time of production. The application of the model has been illustrated numerically. Also, AOQ curves were constructed to check the efficiency of the proposed plan.

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