International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(5): 50-53
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www.mathsjournal.com
Received: 10-07-2022
Accepted: 15-08-2022

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Reliability estimation for mean of normal population with known coefficient of variation

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DOI: https://doi.org/10.22271/maths.2022.v7.i5a.882

Abstract
An attempt has been made to estimate reliability for mean of normal population with known CV. The reliability function is derived and curves have been shown for this reliability function. It is seen that effect of skewness is considerable but the effect of kurtosis is not much.

Keywords: Reliability, normal distribution, coefficient of variation

Introduction

Reliability estimation is carried out at all stages of product life cycle from the beginning of their creation to guarantee and post-guarantee operation. It's especially important to provide high reliability of items at a design stage, in particular at a stage of product completion. Thus, the qualitative system cannot be created without authentic knowledge of reliability parameters for completing products. Such knowledge is based on the data, which are gained during exploitation analogous products of concrete firms-manufacturers. A reliability of completing products is characterizing by duration of a time between failures, duration of restoration or a complex parameter. Some important contributions in this field are by Searls (1964) [4], Pandey and Srivastava (1985) [5], Khan (1968) [2], Govindarajulu and Sahai (1972) [1], Singh *et al.* (1973) [1], Singh and Pandey (1974) [8], Zellner and Park (1979) [10], Pandey (1983) [3] and Pandey and Singh (1977) [4].

Reliability Function for Normal Distribution with Known Coefficient of Variation

Suppose $x_1, x_2, \ldots x_n$ be a random sample of size *n* drawn from a population. Assuming that the population is infinite and the c.v. is known (i.e.- c.v.=c). Let $\overline{X}^*$ denote the estimate of the mean of a characteristic based on the sample size *n*. After mathematical manipulation of Sen (1978) estimator, we have the mean squared error (MSE) of $\overline{X}^*$ is as,

$$MSE(\overline{X}^*) = \frac{(\beta_2 - \beta_1 - 1)}{(\beta_2 - \beta_1 - 1) + \left(\sqrt{\beta_1} - 2c\right)^2} \cdot \frac{\sigma^2}{n}$$

$$= \frac{\sigma^2}{n} . T , \qquad (2.1)$$

where $T = \dfrac{(\beta_2 - \beta_1 - 1)}{(\beta_2 - \beta_1 - 1) + \left(\sqrt{\beta_1} - 2c\right)^2}$.

The maximum likelihood estimator of reliability function is

$$\hat{R}(t) = 1 - \Phi\left(\frac{t - \bar{x}^*}{\sigma}\right), \tag{2.2}$$

And $\tilde{R}(t)$ is defined as

$$\tilde{R}(t) = E\left[D(x) \mid \bar{x}^*\right],$$

where *D(x)* is any estimator of *R(t)*. Take

$$D(x_1) = \begin{cases} 1 & if \quad x_1 \geq t \\ 0 & otherwise. \end{cases} \tag{2.3}$$

Following Zacks and Even (1966), we have obtained the reliability estimation with known CV as

$$\tilde{R}(t) = P\left[\frac{x_1 - \bar{x}^*}{\sqrt{\sigma^2\left(1 - \frac{T}{n}\right)}} \geq \frac{t - \bar{x}^*}{\sqrt{\sigma^2\left(1 - \frac{T}{n}\right)}}\right]$$

$$= 1 - \Phi\left[\frac{t - \bar{x}^*}{\sqrt{\sigma^2\left(1 - \frac{T}{n}\right)}}\right], \tag{2.4}$$

where $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$.

Illustration and Conclusions

To discuss the problem a simple example, is cited below. It will be seen from equation (2.1) that proposed estimator depends on the population moment ratios $\sqrt{\beta_1}$ *and* β_2.

Table 1: Reliability Estimation for normal population with known coefficient of variation, C=0

$\sqrt{\beta_1}$ =	0	0	0	0	0	0	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
β_2 =	0	2	3	4	5	6	0	2	3	4	5	6	0	2	3	4	5	6
t																		
12	0.9514	0.9514	0.9514	0.9514	0.9514	0.9514	0.9574	0.9454	0.9484	0.9494	0.9499	0.9502	0.9466	0.9454	0.9484	0.9494	0.9499	0.9502
21	0.8835	0.8835	0.8835	0.8835	0.8835	0.8835	0.8920	0.8753	0.8793	0.8807	0.8814	0.8818	0.8769	0.8753	0.8793	0.8807	0.8814	0.8818
30	0.7663	0.7663	0.7663	0.7663	0.7663	0.7663	0.7746	0.7587	0.7624	0.7637	0.7643	0.7647	0.7601	0.7587	0.7624	0.7637	0.7643	0.7647
39	0.6029	0.6029	0.6029	0.6029	0.6029	0.6029	0.6067	0.5995	0.6011	0.6017	0.6020	0.6022	0.6001	0.5995	0.6011	0.6017	0.6020	0.6022
48	0.4188	0.4188	0.4188	0.4188	0.4188	0.4188	0.4158	0.4215	0.4202	0.4197	0.4195	0.4194	0.4210	0.4215	0.4202	0.4197	0.4195	0.4194
57	0.2512	0.2512	0.2512	0.2512	0.2512	0.2512	0.2432	0.2585	0.2549	0.2537	0.2531	0.2527	0.2571	0.2585	0.2549	0.2537	0.2531	0.2527
66	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1191	0.1361	0.1320	0.1307	0.1300	0.1296	0.1345	0.1361	0.1320	0.1307	0.1300	0.1296

Table 2: Reliability Estimation for normal population with known coefficient of variation, C=02

$\sqrt{\beta_1}$ =	0	0	0	0	0	0	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
β_2 =	0	2	3	4	5	6	0	2	3	4	5	6	0	2	3	4	5	6
t																		
12	0.9525	0.9505	0.9509	0.9511	0.9511	0.9512	0.9527	0.9454	0.9498	0.9505	0.9507	0.9509	0.9527	0.9454	0.9498	0.9505	0.9507	0.9509
21	0.8851	0.8823	0.8828	0.8830	0.8831	0.8832	0.8853	0.8753	0.8813	0.8822	0.8826	0.8828	0.8853	0.8753	0.8813	0.8822	0.8826	0.8828
30	0.7678	0.7652	0.7657	0.7659	0.7660	0.7660	0.7680	0.7587	0.7642	0.7651	0.7654	0.7656	0.7680	0.7587	0.7642	0.7651	0.7654	0.7656
39	0.6036	0.6024	0.6026	0.6027	0.6028	0.6028	0.6037	0.5995	0.6020	0.6024	0.6025	0.6026	0.6037	0.5995	0.6020	0.6024	0.6025	0.6026
48	0.4183	0.4192	0.4190	0.4190	0.4189	0.4189	0.4182	0.4215	0.4196	0.4192	0.4191	0.4190	0.4182	0.4215	0.4196	0.4192	0.4191	0.4190
57	0.2497	0.2522	0.2518	0.2516	0.2515	0.2514	0.2495	0.2585	0.2532	0.2523	0.2520	0.2518	0.2495	0.2585	0.2532	0.2523	0.2520	0.2518
66	0.1262	0.1290	0.1285	0.1283	0.1282	0.1281	0.1260	0.1361	0.1301	0.1292	0.1288	0.1286	0.1260	0.1361	0.1301	0.1292	0.1288	0.1286

Table 3: Reliability estimation for normal population with known coefficient of variation, C=0.3

$\sqrt{\beta_1}$ =	0	0	0	0	0	0	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
β_2 =	0	2	3	4	5	6	0	2	3	4	5	6	0	2	3	4	5	6
t																		
12	0.9547	0.9498	0.9505	0.9507	0.9509	0.9510	0.9519	0.9454	0.9505	0.9509	0.9511	0.9511	0.9519	0.9454	0.9505	0.9509	0.9510	0.9511
21	0.8882	0.8813	0.8822	0.8826	0.8828	0.8829	0.8842	0.8753	0.8823	0.8828	0.8830	0.8831	0.8842	0.8753	0.8823	0.8828	0.8830	0.8831
30	0.7709	0.7642	0.7651	0.7654	0.7656	0.7657	0.7670	0.7587	0.7652	0.7657	0.7659	0.7660	0.7670	0.7587	0.7652	0.7657	0.7658	0.7660
39	0.6050	0.6020	0.6024	0.6025	0.6026	0.6027	0.6032	0.5995	0.6024	0.6026	0.6027	0.6028	0.6032	0.5995	0.6024	0.6026	0.6027	0.6028
48	0.4171	0.4196	0.4192	0.4191	0.4190	0.4190	0.4186	0.4215	0.4192	0.4190	0.4190	0.4189	0.4186	0.4215	0.4192	0.4190	0.4190	0.4189
57	0.2468	0.2532	0.2523	0.2520	0.2518	0.2517	0.2505	0.2585	0.2522	0.2518	0.2516	0.2515	0.2505	0.2585	0.2522	0.2518	0.2516	0.2515
66	0.1230	0.1301	0.1292	0.1288	0.1286	0.1284	0.1271	0.1361	0.1290	0.1285	0.1283	0.1282	0.1271	0.1361	0.1290	0.1285	0.1284	0.1282

Table 4. Reliability estimation for normal population with known coefficient of variation, C=0.4

$\sqrt{\beta_1}$ =	0	0	0	0	0	0	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
β_2 =	0	2	3	4	5	6	0	2	3	4	5	6	0	2	3	4	5	6
t																		
12	0.9620	0.9490	0.9499	0.9503	0.9505	0.9507	0.9515	0.9454	0.9511	0.9512	0.9513	0.9513	0.9515	0.9454	0.9511	0.9512	0.9513	0.9513
21	0.8991	0.8802	0.8814	0.8820	0.8823	0.8825	0.8836	0.8753	0.8831	0.8833	0.8833	0.8834	0.8836	0.8753	0.8831	0.8833	0.8833	0.8834
30	0.7816	0.7632	0.7644	0.7649	0.7652	0.7654	0.7664	0.7587	0.7660	0.7661	0.7662	0.7662	0.7664	0.7587	0.7660	0.7661	0.7662	0.7662
39	0.6099	0.6015	0.6020	0.6023	0.6024	0.6025	0.6030	0.5995	0.6028	0.6028	0.6028	0.6029	0.6030	0.5995	0.6028	0.6028	0.6028	0.6029
48	0.4132	0.4199	0.4195	0.4193	0.4192	0.4191	0.4187	0.4215	0.4189	0.4189	0.4188	0.4188	0.4187	0.4215	0.4189	0.4189	0.4188	0.4188
57	0.2364	0.2541	0.2530	0.2525	0.2522	0.2521	0.2510	0.2585	0.2515	0.2513	0.2513	0.2513	0.2510	0.2585	0.2515	0.2513	0.2513	0.2513
66	0.1119	0.1311	0.1299	0.1294	0.1290	0.1288	0.1277	0.1361	0.1282	0.1280	0.1280	0.1279	0.1277	0.1361	0.1282	0.1280	0.1280	0.1279

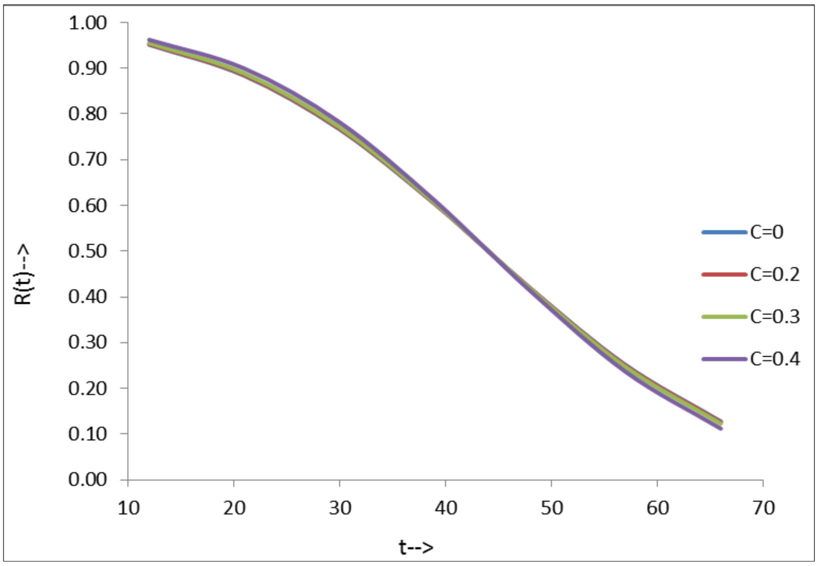

Fig 1: Reliability curve for mean of Normal distribution with known CV and $\sqrt{\beta_1}$ =0, β_2=0

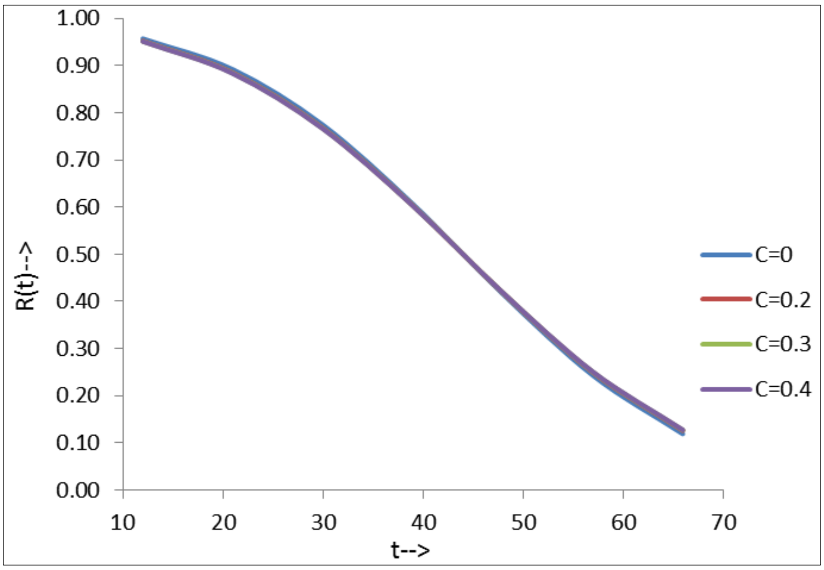

Fig 2: Reliability curve for mean of Normal distribution with known CV and $\sqrt{\beta_1}$ =1, β_2=0

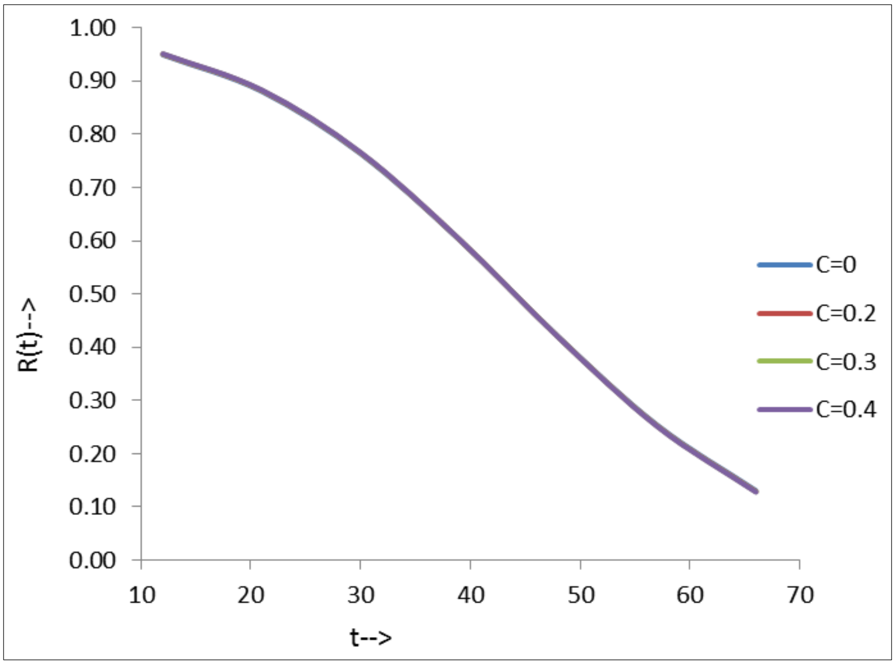

Fig 3: Reliability curve for mean of Normal distribution with known CV and $\sqrt{\beta_1}$=-1, β_2=4

If the form of the distribution is assumed or specified mostly β_1 *and* β_2 are also specified. However, where the form of the distribution is neither specified nor known, and hence β_1 , β_2 not known, it would be better to estimate these from past sample presumably of larger sizes.

Example

15 items were put on test and the failure times (in hours) were:
13.4, 14.2, 28.8, 29.0, 29.8, 33.0, 37.8, 39.6, 43.4, 49.8, 54.8, 58.2, 67.4, 70.2, 91.2.
The values of $\hat{R}(t)$ in Table-1 to Table-4 for different values of $c, \sqrt{\beta_1}$ *and* β_2 at different point of *t*. The reliability curve has drawn in Figure-7.1 to Figure-7.4 for different value known CV. It is evident from the Tables and Figures, the $\hat{R}(t)$ is increases as the values of *c* increases. The effect of skewness is increases as the values of *c* increases. As β_2 increases the values of $\hat{R}(t)$ is more or less constant for different values of *c*. This study of $\hat{R}(t)$ can be considered where the distribution is not normal or near normal. Overall gain in $\hat{R}(t)$ is seen for all the true values of *c* available from previous studies. The underlying assumption is that the CV remains unchanged over certain occasions in industry. The effect of non-normality is serious for estimating $\hat{R}(t)$.

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