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The implications of taxation on the investment strategy of insurance company under constant elasticity of variance model and different utility preferences

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Abstract
This work investigates the impact of taxation on an investment strategy of an insurance company considering three utility functions; exponential, power, and logarithmic utility preferences. The risky asset is assumed to be driven by Constant Elasticity of Variance (CEV) model. The investment problem considered was insurance company that trades two assets; a risky stock with an investment behavior in the presence of a stochastic cash flow or a risk process and the money market account (bond), continuously in the economy. The Hamilton-Jacobi-Bellman (HJB) equation associated with the optimization problem is obtained using the Ito's lemma. Among others, it was found that in the case of power utility preference, the investment strategy, when there was taxation was more than the investment strategy when there was no taxation. Further found was that when exponential utility performance was adopted, the investment strategy when there was taxation was less than the strategy when there was no taxation. The impact of taxation on the investment strategy and choice of utility preference should be considered when making investment decisions.

Keywords: Implications of taxation; investment strategy; insurance company; constant elasticity; variance model; utility preferences

Introduction

Optimal Portfolio problems are considered in finance and insurance mathematics, for their relevance and being practical importance. Nowadays most insurance companies invest both in the stocks and money market, in continuous time. Proper planning for risks involved in investment plans, risk control and most especially in the stock market are resulting to great advantages. Most of the studies in insurance and mathematics finance have focused on finding optimal investment strategies that minimized the probability of wreck when the risk process of an insurance company goes in line with the Crammer-Lundeberg model. However, this does not come easily as it expresses difficult numerical computations for obtaining the probability ruin.

Due to the high risks involved in the stock market investment strategies and risk management are becoming important and advantageous, hence this work that is to obtain the optimized investment strategy and investigate the implication of taxation when the risky asset is transacted. It considers an insurance company that trades two assets; a risky stock and a riskless money market account (bond) under exponential, power and logarithmic utility preferences and constant elasticity of variance (CEV) model, where taxation is involved. The issue of portfolio selection in financial institutions has drawn a lot of attention and researches are being done. Studying this particular problem will contribute in guiding insurance companies in making investment decisions, which are optimal in a sense of optimizing their investment returns.

In order to achieve this objective some works done in this area are reviewed and the contributions penciled down in the sequel [1]. Examined the optimal investment strategy for a defined contribution (DC) pension scheme that was modeled such that the fund was invested partly in riskless assets and partly in risky assets and that the market has a constant investment rate and a stochastic volatility that follows the Heston model.

The salary was assumed to be constant over the entire career of the Pension Plan Participant (PPP) and the contribution was a constant proportion of the salary. They used CRRA Utility function to obtain the Hamilton–Jacobi– Bellman (HJB) equation which was solved by the Prandtl Asymptotic Matching method to get the required investment strategy [2]. Studied the effect of correlation of Brownian motions on an investor's optimal investment and consumption decision under Ornstein–Uhlenbeck model; they applied the maximum principle to obtain the HJB equation for the value function. The H-J-B equation derived was transformed into an ordinary differential equation; specifically the Bernoulli equation. Using the elimination of dependency on the variables they tackled the problem [3]. Worked on optimal investment problem under the Constant Elasticity of Variance (CEV) model for utility maximization where taxes, dividends and transaction are involved [4]. Studied the optimal investment strategies for a plan contributor in a defined pension scheme, with stochastic salary and extra contributions, under the affine interest rate model. They considered two cases where the extra contribution rates were stochastic and constant respectively. The work considered three different assets namely risk free asset (cash), zero coupon bonds and the risky asset (stock) were considered and using Legendre transformation and dual theory where exponential utility function for two of the cases they obtained the optimal investment strategies for the three investment that showed that the strategies for this respective investments when there was no extra contribution rate was constant but could not be used when it was stochastic which clearly gave the member and the fund manager good insight on how to invest to obtain a maximum profit with minimal risk [5].

Studied the optimal reinsurance contract for the insurer and arrived at the fact that the result may not be optimal and unacceptable for the reinsurer. This means that the reinsurance contract should be designed to take into account the interests of both the insurer and the reinsurer [6].

Investigated the optimal investment policy for insurers under the constant elasticity of variance model and solved for the explicit solution of the optimal portfolio choice for an insurer with negative exponential utility over terminal wealth under the constant elasticity of variance (CEV) price model. The surplus process is assumed to follow a Brownian motion with drift and whose risk is correlated with the Brownian motion driving the risky assets. Firstly they derive the corresponding Hamilton-Jacob-Bellman (HJB) equation, and then simplify it into two parabolic partial differential equations (PDEs) via the variable change techniques. Finally, they used the Feynman-Kac formula to solve the two PDEs and obtained the explicit solution of value function as well as the optimal investment policy [7]. Worked on a problem on how to take the risk reserve of an insurance company, to follow Brownian motion with drift and tackle an optimal portfolio selection problem of the company. The investment case considered was insurance company that trades two assets, the money market account (bond) growing at a rate "r" and a risky stock with investment behavior in the pressure of a stochastic cash flow or a risk process continuously in the economy. He focused on obtaining investment strategies that are optimal in the sense of optimizing the returns of the company. He established among others that the optimized investment in the risky asset and optimal value function were dependent on horizon and the wealth available for investment [8]. Work on optimal investment problem under modified constant elasticity of variance (M-CEV) model for the assets price and power utility over the final wealth for a finite horizon agent. The effects of the changes in the various factors such as the stock volatility on optimal investment strategies have been investigated [9]. Considered an optimal asset allocation problem of an insurance company. In their model, an insurance company is represented by a compound-Poison risk process, which is perturbed by diffusion. The investments are in both risky and risk-free types of assets similar to stocks/real estates and bonds respectively. The problem was framed such that the insurance company can borrow at a constant interest rate in the event of a negative surplus. They gave a numerical analysis which appeared to show that an optimal asset allocation range can be estimated for certain parameters and can be compared with using insurance data. They were able to show that a reasonable optimal asset allocation range for a typical insurance is about 4.5 to 8 percent invested in risky stock/real estate assets [6]. Considered a constrained investment problem with the objective of minimizing the investment return. In their paper, they formulated the cash reserve and investment model for the insurance company and analyzed the Value-at-Risk in a short time horizon imposing risk regulation as a risk constraint dynamically making the problem become a minimization problem of the investment return. By solving the corresponding Hamilton-Jacobi-Bellman equations, they derived analytic expressions for the optimal value function and the corresponding optimal strategies. Looking at the Value-at-Risk alone, they were able to show that it was possible to reduce the overall risk by an increased exposure of the risky assets with the stochastic of the fundamental insurance business. Studying the optimal strategies, they found that a different investment strategy would be in place depending on the sharp ratio of the risky asset [10]. Worked on optimal proportional reinsurance and investment for stochastic factor models. In this work they sort to find the optimal proportional reinsurance- investment strategy of an insurance company which wishes to maximize the expected exponential utility of its terminal wealth in a finite horizon and extend the classical Cramer-Lundberg model by introducing a stochastic factor, which affects the intensity of the claims arrival process, described by a Cox process, as well as the insurance and re-insurance premier. Following the classical stochastic control approach based on the Hamilton-Jacobi-Bellman equation they characterize the optimal strategy and provide a verification result for the value function through classical solutions of two backward partial differential equations [11]. Also researched the optimal investment and excess–of–loss reinsurance strategies. Furthermore [12], researched the optimal reinsurance and investment to maximize the expected exponential utility of terminal wealth based on the Constant Elasticity of Variance (CEV) model. All the reviewed works had no content on the impact of taxation on the investment strategies, hence the goal of this work.

The model formulation and the model

In this section we give brief notes on, Constant Elasticity of Variance Model, Ito's Lemma, and formulate the Model.

Constant Elasticity of Variance Model

The constant elasticity of variance model is a one dimensional diffusion model with the instantaneous volatility specified to be a power function of the underlying spot price $(s) - aS^{\beta}$.

Ito's Lemma

Ito's Lemma is used to determine the derivative of a time dependent function of a stochastic process. It performs the role of chain rule in a stochastic setting, analogous to the chain rule in ordinary differential calculus. Ito's lemma is a cornerstone of quantitative finance and it is intrinsic to the derivation of the Black Schole's equation for contingent claims (option) pricing.

The Hamilton-Jacobi-Bellman (HJB) Equation

The Hamilton-Jacobi-Bellman (HJB) equation is a partial differential equation which is central to optimal control theory. The solution of the H.IB equation is the value function which gives the minimum cost for a given dynamic system with an associated cost function.

The Model

To obtain the model we here explain that the Constant Elasticity of Variance [CEV] Model is an extension of the Geometric Brownian Motion (GBM) which is used to describe the price dynamics of the risky asset [13].

The market structure being investigated consists of a risk free asset and a single risky asset which follows the Constant Elasticity of Variance [CEV] model. We denote the price of the risk-free asset (the bank account) at time t by $B(t)$, which satisfies the following formula;

$$dB(t) = rB(t)dt \quad (1)$$

The single risky asset (the stock) for investment is assumed to have at any time t the price $S(t)$ which is governed by the Constant Elasticity of Variance (CEV) of model, given as

$$dS(t) = S(t)\left(\mu dt + KS^{\gamma}(t)dZ^{(1)}(t)\right) \quad (2)$$

Where μ is the rate of appreciation of the risky asset. The elasticity parameter of the local volatility is given as γ, and K the local volatility scale parameter. The Brownian motion parameter is $dZ(t).Z(t), t \geq 0$ is a Standard Brownian Motion and $KS(t)^{\gamma}$ is the instantaneous volatility. Generally, the elasticity parameter satisfies the condition $\gamma \leq 0$. Further if the elasticity parameter $\gamma = 0$ in equation (2) then the [CEV] model reduces to a [GBM], given by

$$dS(t) = S(t)\left(\mu dt + KdZ^{(1)}(t)\right). \quad (3)$$

We concern ourselves with the case where the investment behavior has a random cash flow or a risk process denoted by $R(t): t > 0$ that satisfies the stochastic differential equation

$$dR(t) = \alpha dt + \beta dZ^{(2)}(t), \quad (4)$$

Where α and β are constant with $\beta \geq 0$. $Z^{(2)}(t)$,is another Standard Brownian motion correlates $Z^{(1)}(t)$, such that their correlation coefficient given by ρ. That is, the expectation of $\left(Z^{(1)}(t)Z^{(2)}(t)\right)$ is given by

$$E\left(Z^{(1)}(t)Z^{(2)}(t)\right) = \rho t. \quad (5)$$

We assume that the total wealth of the insurance company is $W(t)$ and she allocates her wealth such that $\pi(t)$ is the total amount the company wealth invests in risky asset and remaining balance $\left(W(t) - \pi(t)\right)$ is invested in a risk-less asset and the policy is that $\pi(t)$ the total wealth process of an insurance company evolves according to the stochastic differential equation

$$dW^{\pi} = \pi(t)\frac{d(t)}{S(t)} + \left(1 - C(t)\right)(W - \pi)\frac{dB(t)}{B(t)} + dR(t) \quad (6)$$

Using the equations (1), (2) and (4), the stochastic differential equation (6) for the wealth process becomes

$$dW^{\pi}(t) = \mu\pi(t)dt + K\pi(t)S^{\gamma}(t)dZ^{(1)}(t) + rW(t)dt - r\pi(t)dt -$$

$$rC(t)W(t)dt + rC(t)\pi(t)dt + \alpha dt + \beta dZ^{(2)}(t), \quad (7)$$

Which further simplifies to

$$dW^{\pi}(t) = \{\pi(t)(\mu - (1 - C(t)r) + W(t)[1 - C(t)]r + \alpha\}dt +$$

$$\pi(t)KS^{\gamma}(t)dZ^{(1)}(t) + \beta dZ^{(2)}(t). \quad (8)$$

Also using the property that $Z^{(1)}(t)$ and $Z^{(2)}(t)$ are correlated standard Brownian motions with correlation coefficient ρ, the quadratic variation of wealth process is obtained as

$$(dW(t))^2 = \left(\pi^2(t)K^2\left(S^{\gamma}(t)\right)^2 + \beta^2 + 2K\rho\beta\pi(t)S^{\gamma}(t)\right)dt \quad (9)$$

Where

$$\left.\begin{array}{c} dZ^{(1)}(t)dZ^{(1)}(t) = dZ^{(2)}(t)dZ^{(2)}(t) = dt \\ and \\ dtdZ^{(1)}(t) = dtdZ^{(2)}(t) = dtdt = 0 \end{array}\right\} \quad (10)$$

The insurance company's problem can therefore be written as

$$V(t, W) = sup_{\pi}E^{(t,w)}[u(w_{\tau}^{\pi})] \quad (11)$$

Subject to (8):

$$dW^{\pi}(t) = \{\pi(t)(\mu - (1 - C(t)r) + W(t)[1 - C(t)]r + \alpha\}dt + \pi(t)KS^{\gamma}(t)dZ^{(1)}(t) + \beta dZ^{(2)}(t).$$

The optimization of the investment strategy

In this section, we present the optimization of the insurance company's investment strategy.

We shall consider the cases of exponential, power and logarithmic utility preferences.

The Hamilton-Jacobi-Bellman (HJB) partial differential equation is obtained starting with the Bellman equation given as

$$V(W, t; T) = Sup_{\pi}E[V(w, t + \Delta t; T)], \quad (12)$$

Where W denotes the wealth of the company at time $t + \Delta t$. Equation (12) modifies to

$$Sup_{\pi}\frac{1}{dt}E(dV) = 0. \quad (13)$$

Rearranging equation (12) and dividing both sides by Δt and taking limit to zero. The result is the required Bellman equation.

Applying the Ito's lemma which states that

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial W}dw + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}(dW)^2 \quad (14)$$

And making use (8) and (9) in (14), we obtain

$$dV = \frac{\partial V}{\partial w}\left[\pi(t)\left[\mu - \left(1 - C(t)\right)r\right] + W(t)[1 - C(t)]r + \alpha\right]dt + \frac{\partial V^2}{2\partial w^2}\left(\pi^2(t)K^2\left(S^\gamma(t)\right)^2 + \beta^2 + 2K\rho\beta\pi(t)S^\gamma(t)\right)dt + \pi(t)KS^\gamma(t)dz^{(1)}(t) + \beta dz^{(2)}(t) + \frac{\partial V}{\partial t}dt. \quad (15)$$

Applying (52) to bellman equation (13) we get the HJB equation

$$V_t + \left[\pi(t)\left(\mu - \left(1 - C(t)\right)r\right) + W(t)\left(1 - C(t)\right)r + \alpha\right]V_W + \frac{1}{2}\left(\pi^2(t)K^2\left(S^\gamma(t)\right)^2 + \beta^2 + 2K\rho\beta\pi(t)S^\gamma(t)\right)V_{WW} = 0 \quad (16)$$

Now we consider the special cases of utility preferences;
The case of exponential utility function is of the form

$$V(W) = \frac{-e^{-\varphi W(t)}}{\varphi}, \varphi > 0. \quad (17)$$

Considering the homogeneity of the objective function and the restriction and the terminal condition, we conjecture that the value function V must be linear to $V(W) = \frac{-e^{-\varphi W(t)}}{\varphi}, \varphi > 0$, we let

$$V(W, t{:}T) = g(t,T)\left(-\frac{e^{-\varphi w(t)}}{\varphi}\right), \quad (18)$$

From which we obtain

$$\left.\begin{array}{l} V_t = -\frac{e^{-\varphi w(t)}}{\varphi}g_t;\ V_w = e^{-\varphi w(t)}g; \\ V_{ww} = -\varphi e^{-\varphi w(t)}g \end{array}\right\} \quad (19)$$

Equation (16) becomes

$$\left\{\left[\pi(t)\left(\mu - \left(1 - C(t)\right)r\right) + W(t)\left(1 - C(t)\right)r + \alpha\right]e^{-\varphi w(t)}g - \frac{-\varphi}{2}\left(\pi^2(t)K^2\left(S^\gamma(t)\right)^2 + \beta^2 + 2K\rho\beta\pi(t)\right)e^{-\varphi w(t)}g\right\} - \frac{e^{-\varphi w(t)}}{\varphi}g_t. \quad (20)$$

Equation (20) is a modified form of the Hamilton-Jacobi-Bellman (HJB) equation.
The optimal investment strategy in the risky asset $\pi^*(t)$ of $\pi(t)$ is obtained by differentiating (20) with respect to $\pi(t)$ to obtain

$$\left(\mu - \left(1 - C(t)\right)r\right) - \frac{\varphi}{2}\left(2\pi(t)K^2\left(S^\gamma(t)\right)^2 + 2K\rho\beta S^\gamma(t)\right) = 0. \quad (21)$$

This simplifies to

$$\pi^*(t) = \frac{\mu - \left(1 - C(t)\right)r}{\varphi K^2\left(S^\gamma(t)\right)^2} - \frac{\rho\beta}{KS^\gamma(t)} \quad (22)$$

This is optimal investment strategy is not dependent on the wealth but on the time of investment.

It is clear that when there is tax the optimal investment strategy is given by

$$\pi^*{}_{no\ tax} = \frac{(\mu - r)}{\varphi K^2\left(S^\gamma(t)\right)^2} - \frac{\rho\beta}{KS^\gamma(t)}, \quad (23)$$

And where taxation is involved we have

$$\pi^*{}_{tax} = \frac{\mu - \left(1 - C(t)\right)r}{\varphi K^2\left(S^\gamma(t)\right)^2} - \frac{\rho\beta}{KS^\gamma(t)} \quad (24)$$

Comparing (23) and (24), we have

$$\pi^*{}_{tax} = \pi^*{}_{no\ tax} - \frac{C(t)r}{\varphi K^2\left(S^\gamma(t)\right)^2}. \quad (25)$$

In the case of Power utility preference of the form

$$V(W, t; T = \frac{W^{1-\varphi}}{1-\varphi}, \varphi \neq 1. \quad (26)$$

Equation (18) becomes

$$V(W, t{:}T) = g(t,T)\frac{W^{1-\varphi}}{1-\varphi}, \varphi \neq 1 \ (27)$$

We have

$$V_t = \frac{W^{1-\varphi}}{1-\varphi}g_t;\ V_w = w^{1-\varphi}g;\ V_{ww} = (1-\varphi)\,w^{-\varphi}g \quad (28)$$

Using equation (28) in (16) we obtain

$$\frac{W^{1-\varphi}}{1-\varphi}g_t + \left[\pi(t)\left(\mu - \left(1 - C(t)\right)r\right) + W(t)\left(1 - C(t)\right)r + \alpha\right]w^{1-\varphi}g + \frac{1}{2}\left(\pi^2(t)K^2\left(S^\gamma(t)\right)^2 + \beta^2 + 2K\rho\beta\pi(t)S^\gamma(t)\right)(1-\varphi)\,w^{-\varphi}g = 0. \quad (29)$$

This is the new HJB equation in the case of power utility function.
The differentiation of (29) with respect to $\pi(t)$ and simplifying we got the optimal investment strategy to be

$$\pi^*_{\text{tax}} = -\frac{W(t)[\mu - r + c(t)]}{(1-\varphi)K^2(S^\gamma(t))^2} - \frac{\beta\rho}{KS^\gamma(t)}. \quad (30)$$

This is the optimal investment strategy with taxation of the investor where she has a power utility function preference. It depends on both wealth and horizon.
Clearly we have that when there is no taxation the optimal strategy is

$$\pi^*_{\text{no tax}} = -\frac{W(t)[\mu - r]}{(1-\varphi)K^2(S^\gamma(t))^2} - \frac{\beta\rho}{KS^\gamma(t)}, \quad (31)$$

The comparison of equations (30) and (31) gives

$$\pi^*_{\text{no tax}} = \pi^*_{\text{tax}} - \frac{c(t)W}{(1-\varphi)K^2(S^\gamma(t))^2} \quad (32)$$

Equation (32) shows that charging tax will warrant an increment in the amount required to be invested in the risky asset investment by the fraction -$\frac{c(t)}{(1-\varphi)K^2(S^\gamma(t))^2}$ of the total amount for in investment.

In the case of Logarithmic Utility preference, we have

$$V(W,t;T) = \ln W(t), \quad (33)$$

And letting

$V(w,t;\ T) = g(t{:}\ T)lnW\ (t)$, (34) we obtain the following

$$V_t = \ln W\ g_t; V_w = \frac{1}{W}g; V_{ww} = \frac{1}{W^2}g, \quad (35)$$

From which with equation (16) we get

$$\ln W\ g_t + [\pi(t)(\mu-(1-C(t))r) + W(t)(1-C(t))r + \alpha]\frac{1}{W}g + \frac{1}{2}[\pi^2(t)K^2(S^\gamma(t))^2 +$$

$$\beta^2 + 2K\rho\beta\pi(t)S^\gamma(t)]\frac{1}{W^2}g = 0. \quad (36)$$

Equation (36) is the resulting new HJB equation when the investor prefers the logarithmic utility function which when differentiate d with respect to $\pi(t)$ and simplified the optimal investment strategy when there is taxation is obtained as

$$\pi^*_{tax}(t) = \frac{-w(\mu-r+C(t))}{2K^2S^{2\gamma}(t)} - \frac{\beta\rho}{KS^\gamma(t)}. \quad (37)$$

Therefore, when tax is not charged, the optimal investment strategy becomes

$$\pi^*_{no\ tax}(t) = \frac{-w(\mu-r)}{2K^2S^{2\gamma}(t)} - \frac{\beta\rho}{KS^\gamma(t)} \quad (38)$$

Comparing equations (37) and (38) we have

$$\pi^*_{tax}(t) = \pi^*_{no\ tax}(t) - \frac{w(C(t))}{2K^2S^{2\gamma}(t)}. \quad (39)$$

Equation (39) shows that the optimal investment strategy when tax is paid is more than the optimal investment strategy when tax is not paid by $\frac{(C(t))}{2K^2S^{2\gamma}(t)}$, a fraction of the wealth available for investment.

Further comparisons show that when there is no taxation we get the following

1. From equations (23) and (31) the difference of

$$\frac{(\mu-r)[1+\varphi(W(t)-1)}{(1-\varphi)\varphi K^2S^{2\gamma}(t)}, \varphi \neq 1. \quad (40)$$

For exponential and power utility preferences.

2. For exponential and logarithmic utility preferences, equations (23) and (38), we have

$$\frac{[2+W(t)][\mu-r]}{2\varphi K^2S^{2\gamma}(t)}. \quad (41)$$

3. In the case of power and logarithmic utility preferences, equations (31) and (38) we obtain

$$\frac{(1+\varphi)W(t)(\mu-r)}{2(1-\varphi)K^2S^{2\gamma}(t)} - \frac{\beta\rho}{KS^\gamma(t)} \quad (42)$$

Also, when there is taxation following differences

1. For exponential and power utility preferences we have

$$\frac{(1-\varphi)[\mu-(1-C(t))r]+\varphi W(t)[\mu-r+C(t)]}{(1-\varphi)\varphi K^2S^{2\gamma}(t)} \quad (43)$$

2. For exponential and logarithmic utility preferences, the difference is

$$\frac{\varphi W(t)[\mu-r+c(t)]+[\mu-(1-c(t))r\}}{2\varphi K^2S^{2\gamma}(t)}. \quad (44)$$

3. The case of power and logarithmic utility preferences gave the difference

$$\frac{(1+\varphi)W(\mu-r+C(t))}{2(\varphi-1)K^2S^{2\gamma}(t)}. \quad (45)$$

It can easily be seen that all the differences are dependent on horizon and the amount of wealth available for investment.

Findings

It is found that the optimized investment strategy for the investor when he has both power and logarithmic utility preferences when there is taxation and there is no taxation are horizon and wealth dependent as shown in equations (30), (31), (37) and (38). While the investor's optimized investment strategy for exponential utility preference case, when there is taxation and there is no taxation are only horizons dependent as seen in equations (22) and (23).

Further findings include the following

1. That the investment strategy when there is taxation is less than the strategy when there is no taxation by the fraction, $\frac{C(t)r}{\varphi K^2(S^\gamma(t))^2}$, equation (25), when the investor adopts exponential utility function.
2. That Equation (32) shows that the investment strategy when there is taxation is more than the investment strategy when there is no taxation by the fraction,$\frac{c(t)W}{(1-\varphi)K^2(S^\gamma(t))^2}$, when the investor adopts power utility preference.
3. That adopting exponential utility preference, the investment strategy when there is taxation is less than the strategy when there is no taxation by the fraction, $\frac{w(C(t))}{2K^2S^{2\gamma}(t)}$, as shown ion equation (39).

Conclusion

This work investigated the impact of taxation on the optimal investment strategy of an insurance investor analyzing it under different utility preferences; exponential, power and logarithmic utility functions. It adopted the Constant Elasticity of Variance (CEV) model to describe the dynamic movements of the risky asset's (stock) price. Applying the Ito's lemma the Hamilton-Jacob-Bellman (HJB) equations; stochastic differential equations were obtained. The optimal investment strategies for the different utility functions were got by applying the first principle.

Among the findings is that the optimal investment strategies for power and logarithmic utility functions are horizon and wealth dependent. The case of exponential utility function shows only time dependency.

The investigation on the impact of taxation on the investment strategy shows that the investment strategy when taxation is involved is less than the strategy when there is no taxation when the investor adopts exponential utility function. It was also found that in the case of power utility preference the investment strategy when there is taxation is more than the investment strategy when there is no taxation. Further finding is that the investment strategy when there is taxation is less than the strategy when there is no taxation when exponential utility preference is adopted.

It is advised that the investor should consider the impact taxation will make and his choice of utility preference while making investment decisions.

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