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Shock model approach to change points with ERLANG threshold

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Abstract
Cumulative damage process is related to shock models in reliability theory. The threshold or withstanding capacity of the system can be considered to be any one of the constant level, cumulative shock random threshold level, maximum shock random threshold etc. In this paper, cumulative shock random threshold level is considered wherein a system undergoes a shock and encounters random amount of damage but the system survives with the damages. Successive shocks at random epochs lead to the cumulative damages and when the cumulative damage crosses the threshold level of the system, the system fails. Assuming threshold level undergoes a change in the form of distribution after the change points are taken to be ERLANG random variable, the distribution function for the threshold level is obtained by taking exponential distribution as the threshold levels are before, in between and after the change points respectively. Using this distribution function for the threshold level, the expected time to the breakdown of the system and its variance are obtained by using shock model and cumulative damage process approach.

Keywords: Shock model, cumulative damage process, threshold, change points, Laplace transforms

1. Introduction
In deriving the expression for the expected time to recruitment after change points using exponential distribution of E (T) and its variance V (T) is discussed in chapter IV, if the manpower takes place at the decision epochs. Under the assumption that threshold random variable under goes a changes of distribution from exponential and ERLANG after two change points. In doing so it has been assumed that the inter arrival times between the decisions epochs are identically independently distributed (I.I.D). In this chapter it is assumed that the random variables denoting the inter arrival times between decision epoch of Mean and Variance are derived. Numerical illustrations are also provided.

2. Assumptions of the model
1. Wastage of manpower occurs at the policy decision epochs.
2. The threshold is a random variable whose distribution undergoes a change after a change point.
3. The process that generates the wastages and the interarrival times are mutually independent.
4. The breakdown of the activities occurs as and when the cumulative wastage crosses a threes hold level. This in other words is the time to recruitment.

3. Notations
Xi = A random variable denoting the amount of manpower wastage of the i^{th} decision epoch, i = 1, 2, k with P.D.F g (x) and C.F.F G (x).
Y = A random variable denoting the threshold with P.D.F. h (y) and C.F.F H (y).
Ui = A random variable denoting the interarrival times between depletions of manpower, i = 1, 2, k with P.D.F. f (.) and C.F.F F (.)
T = A random variable denoting the time to recruitment with P.D.F and L (.)
Gk (x), Fk (x) = are the k convolution of G (.) and F (.) respectively
L* (s) = Laplace transform of and L (.) respectively

$$h(y) = \begin{cases} \theta_1 e^{-\theta_1 y} & if, y \le \tau_1 \\ e^{-\theta_1 \tau_1}(y-\tau_1) \quad \theta_2^2 e^{-\theta_2 (y-\tau_1)} & if, \tau_1 \le y \le \tau_2 \\ \theta_3^2 (y-\tau_2) \quad e^{-\theta_3 (y-\tau_2)} & i \\ e^{-\theta_1 \tau_1} \quad e^{-\theta_2(\tau_2-\tau_1)} [1+\theta_2(\tau_2-\tau_1)] & if, y > \tau_2 \end{cases}$$

It is represented by

$$P\left[\sum_{i=1}^{k} x_i < Y\right] = \int_0^\infty g_k(x)\overline{H}(x)dx \tag{6.1}$$

$$P\left[\sum_{i=1}^{k} x_i < Y\right] = \int_0^\infty g_k(x)[1-H(x)]dx$$

$$S(t) = \sum_{k=0}^{\infty} [F_x(t) - F_{k+1}(t)] \times P\left[\sum_{i=1}^{k} x_i < y\right] \tag{6.2}$$

$$\begin{aligned} P\left(\sum x_i < y\right) &= [g^*(\theta_1)]^k - e^{-(\theta_1-\theta_2)\tau_1}\theta_2 \frac{d}{d\theta_2}[g^*(\theta_2)]^k + e^{-(\theta_1-\theta_2)\tau_1}[g^*(\theta_2)]^k \\ &\quad - e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3\tau_2)}\theta_3 \frac{d}{d\theta_3}[g^*(\theta_3)]^k \\ &+ e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}[g^*(\theta_3)]^k - e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_2(\tau_2-\tau_1)\theta_3 \frac{d}{d\theta_3}[g^*(\theta_3)]^k \\ &\quad + e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_2(\tau_2-\tau_1)[g^*(\theta_3)]^k \end{aligned} \tag{6.3}$$

$$S(t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+i}(t)] P[\sum X_i < Y]$$

$$= \left\{ \begin{aligned} & g_k^*(\theta_1)]^K - e^{-(\theta_1-\theta_2)\tau_1} - \theta_2 \frac{d}{d\theta_2}[g_k^*(\theta_2)]^K + e^{(\theta_1-\theta_2)\tau_1}[g_k^*(\theta_3)]^K - e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_3 \frac{d}{d\theta_3}[g_k^*(\theta_3)]^K + \\ & e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_3[g_k^*(\theta_3)]^K - e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_2(\tau_2-\tau_1)\theta_3 \frac{d}{d\theta_3}[g_k^*(\theta_3)]^K + e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_2(\tau_2-\tau_1)[g_k^*(\theta_3] \end{aligned} \right\} \tag{6.4}$$

4. On simplification

$$\begin{aligned} &= \sum_{k=0}^{\infty}[F_k(t) - F_{k+i}(t)] \ \ g_k^*(\theta_1)]^K - \sum_{k=0}^{\infty}[F_k(t) - F_{k+i}(t)]e^{-(\theta_1-\theta_2)\tau_1} - \theta_2 \frac{d}{d\theta_2}[g_k^*(\theta_2)]^K + \sum_{k=0}^{\infty}[F_k(t) - F_{k+i}(t)]e^{(\theta_1-\theta_2)\tau_1}[g_k^*(\theta_3)]^K \\ &- \sum_{k=0}^{\infty}[F_k(t) - F_{k+i}(t)]e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_3 \frac{d}{d\theta_3}[g_k^*(\theta_3)]^K + \sum_{k=0}^{\infty}[F_k(t) - F_{k+i}(t)]e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_3[g_k^*(\theta_3)]^K \\ &- \sum_{k=0}^{\infty}[F_k(t) - F_{k+i}(t)]e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_2(\tau_2-\tau_1)\theta_3 \frac{d}{d\theta_3}[g_k^*(\theta_3)]^K + \sum_{k=0}^{\infty}[F_k(t) - F_{k+i}(t)]e^{-(\theta_1-\theta_2)\tau_1} e^{-(\theta_2-\theta_3)\tau_2}\theta_2(\tau_2-\tau_1)[g_k^*(\theta_3]^k \end{aligned}$$

Using the equation (6.4) then,

$H(t) = 1 - S(t)$

$$= \left\{ \begin{array}{l} 1-1+(1-g^{*}(\theta_1))\sum_{k=1}^{\infty}[F_k(t)g^{*}(\theta_1)]^{k-1}+\theta_2 e^{-(\theta_1-\theta_2)\tau_1}\sum_{0}^{\infty}F_{k+i}(t)]\theta_2\frac{d}{d\theta_2}[g_k^{*}(\theta_2)]^{K} \\ -\theta_2 e^{-(\theta_1-\theta_2)\tau_1}\sum_{0}^{\infty}F_{k+i}(t)]\frac{d}{d\theta_2}[g_k^{*}(\theta_2)]^{K}-e^{-(\theta_1-\theta_2)\tau_1}[1-(1-g_k^{*}(\theta_2)]\sum_{k=1}^{\infty}[F_k(t)[g_k^{*}(\theta_3)]^{K-1} \\ +e^{-(\theta_1-\theta_2)\tau_1}e^{-(\theta_2-\theta_3)\tau_2}\theta_3\sum_{0}^{\infty}F_k(t)\frac{d}{d\theta_3}[g_k^{*}(\theta_3)]^{k}- \\ \theta_3 e^{-(\theta_1-\theta_2)\tau_2}e^{-(\theta_2-\theta_3)\tau_2}\sum_{0}^{\infty}F_{k+1}(t)\frac{d}{d\theta_3}\left[g^{*}(\theta_3)\right]^{k}+ \\ e^{-(\theta_1-\theta_2)\tau_2}e^{-(\theta_2-\theta_3)\tau_2}[1-(1-g^{k}(\theta_3)]\sum_{k=1}^{\infty}F_K(t)[g^{*}(\theta_3)]^{k-1}-e^{-(\theta_1-\theta_2)\tau_1}e^{-(\theta_2-\theta_3)\tau_2}\theta_3(\tau_2-\tau_1)\sum_{k=1}^{\infty}[g^{*}(\theta_3)]^{k-1} \end{array} \right\} \quad (6.5)$$

$E(T) = \frac{dH^{*}(s)}{d(s)}$ /s=0

$$E(T) = \left(\frac{(\omega+\theta_1)}{d\theta_1}\right) - \left(e^{-(\theta_1-\theta_2)\tau_1}\right)\left(\frac{d\omega^2}{(d\theta_2)^2}\right) - \left(e^{-(\theta_1-\theta_2)\tau_1-(\theta_2-\theta_3)\tau_2}\right)\left(\frac{d\omega^2}{(d\theta_3)^2}\right) - \left(e^{-(\theta_1-\theta_2)\tau_1-(\theta_2-\theta_3)\tau_2}\right)(\theta_2(\tau_2-\tau_1))\left(\frac{d\omega^2}{(d\theta_3)^2}\right)$$

5. Mathematical Illustration

Table 1: shows when d changes E(T) decreases

d	E(T)
1	3
2	1.5
3	1
4	0.75
5	0.6
6	0.5
7	0.4
8	0.37
9	0.3
10	0.3

Table 2: shows that when □ increases then E(T) decreases.

□1	E(T)
0.5	9.01
1.0	6.58
1.5	4.68
2.0	4.12
2.5	3.74

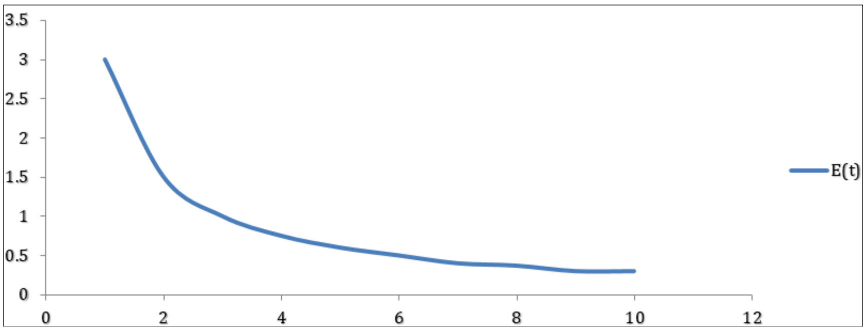

Fig 1: E (T)

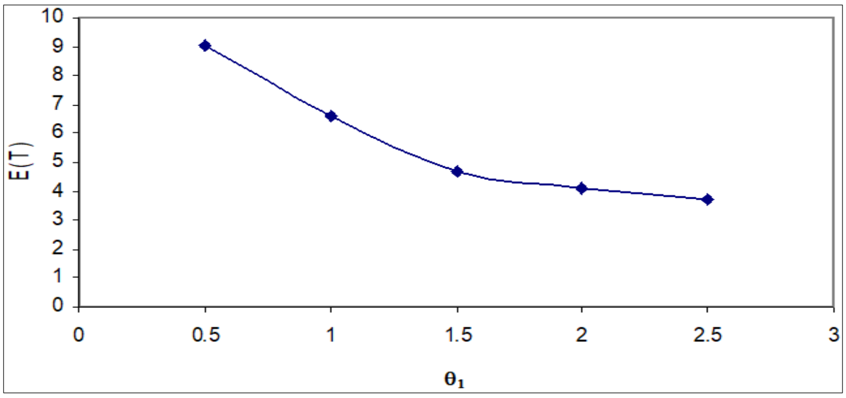

Fig 2:

Table 3: Variation in E (T) for Changes τ_1

τ_1	E(T)
0.5	4.51
1.0	4.68
1.5	7.07
2.0	12.92
2.5	34.20

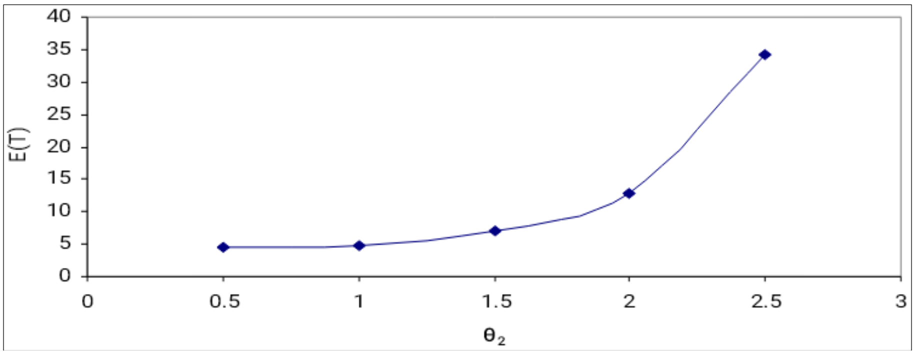

Fig 3:

6. Conclusion

The following conclusions can be drawn on the basis of the numerical illustration taken up in this model.

Case (i): As the value of d which is the parameter of the random variable denoting the magnitude the damages namely X increases then E (T) increases. This is due to the fact that the average level of depletion decreases and hence E(T) increases.

Case (ii): As the value $\square_1$ which is the parameter of the random variable denoting the threshold increases, E(T) decreases. This is due to the fact that the threshold random variable is exponentially distributed and hence its mean value decreases when the parameter increases. Hence E(T) becomes smaller

Case (ii): As the value of τ_1 which is the parameter of the threshold after the first change point increases then E(T) also increases. This is due to the fact that in this model the threshold follows ERLANG 2 distribution after the truncation point. The threshold level becomes smaller, as τ_1 increases. Only after the truncation point but prior to the truncation point it is higher, so due to the combined value of threshold E(T) increases.

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