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Mathematical modeling of some perishable items in inventory

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Abstract
Consider a particular (or group of) perishable item(s) for example: meat, fish, fruit, snacks etc., the product (item) spends $t \in (0, T]$ number of days on the shelf with a starting quantity m, at time t. The item has a cost price per unit and selling price per unit at time t (t varies with time per day). A discount d_t is introduced at time t. After expiration date, it must be disposed of at a cost; hence it is optimal to give a discount. We formulate a mathematical model to determine the profit and the optimal discount to be given at time t so as to minimize loss. Sensitivity analysis was carried out to determine the best time to sell the item under a discount regime.

Keywords: Mathematical modeling, perishable goods, inventory. 2010 mathematics subject classification: 37N40, 78M50, 80M50, 90C31

Introduction

Preliminaries

Many retail shops, grocery stores, and big supermarkets have faced series of challenges in keeping and disposing of expired products at costs incurred by the owners. The need to lessen wastages and possible loss in revenue has inspired business administrators to seek ways of minimizing loss, thus maximizing profit. One way of accomplishing this objective is to introduce discounts to these perishable goods and allow the discounts to run over time until the product(s) is(are) removed from inventory, either by selling it(them) off or by disposing it(them) off as bad product(s). We shall, in this paper, refer to the following terminologies which have their usual meaning in literature: *Ordering or set up cost*, *Holding or carrying cost*, *Price, etc.* (see [3] and references there in). We consider fresh tomatoes as an example of a perishable product and it is used as a regular household item in Nigeria.

Review of related literature

In a departmental store for example, goods are categorized into perishables and non-erishables. Supplies that have fixed useful life are said to be perishable commodities [11]. Common examples include decomposing organic products such as vegetables, fruits, bread, milk, meat etc. Perishable items may also have a fixed life time, after which they are no longer useful or fit for use, such as medicines, packaged juices (useful time is different when opened, depending on type and packaging) etc. In this paper we consider fresh Tomatoes out of many perishable products since it is used as a regular consumable item in almost all households in Nigeria, and have a useful or lifetime of 5 to 10 days. This is similar to the analysis of [22] and [21].

Inventory management of perishable goods has a long history in the operations literature [18]. Provided a review of early work. Recent reviews by [13] and [19] indicate a considerable renewed interest in the area. Numerous models have been developed in literature so that inventory management of perishable products can be easily understood and it can be useful in accepting complex situations of perishability. Some complex models include returns policies [24, 12], ordering policies for cyclic commodities [11], and demand with time modification, production and decline rates [10]. Problem associated to discounting and stocking decisions for short shelf-life perishable items has not achieved the due scope in literature [21, 27].

With the same intent of making the most of net profit, show that dynamic decision-making is superior to fixed decision-making in terms of retail price and promotional effort. In practice, it is common to put forward deteriorated items at a discounted price. In many stores, items that are close to their expiration date are marked down by a fixed percentage to manipulate the consumer's buying behavior. However, it is reported by [17] that 'irregular demand due to weather and promotions are reasons contributing to food waste. In practice, customers may demand a supply of items that will not expire within a specified time-window but which need not be freshest ones. Therefore, evaluating customer demands with different product shelf-life requirements can be an efficient management practice'. Most of the traditional tools of modeling are crisp, deterministic, and precise in character. But for many practical problems, there are incompleteness and unreliability of input information. This enforces some authors to use fuzzy optimization method with fuzzy parameters. Crisp mathematical programming approaches provide no such mechanism to quantify these uncertainties. Fuzzy optimization is a flexible approach that permits more adequate solutions of real problems in the presence of vague information, providing well defined mechanisms to quantify the uncertainties directly.

Most rational customers are more particular about shelf life, sense of taste and nourishing value of items when it comes to perishables, as compared to non-perishables, especially food items. Decisions related to stock size, price discounts, and after shelf- life, uses of such goods, are relatively difficult to take. Many wholesalers of perishables largely manage their inventories and income on know-how basis rather than using any objective criteria. Monte Carlo simulation approach has been used in literature to find out the economic stock size, amount and period of price discounts, and expected profit for wholesalers supplying items to some local shops [14]. Examination of data has shown that goods requiring a pre-determined situation have a shelf life equal to or less than 30 days [29].

More than a few authors over time have studied different pricing stratagems for deteriorating inventory when shortages are allowed and somewhat backordered dependent on the its waiting time [2, 1, 6, 5, 7, 8, 9, 15, 16, 25, 26, 23, 28], also inventory schemes dealing with time value of money and shortage [5, 4, 28, 30, 31]. Most of the studies with exception of a few [5, 4, 28] do not attempt to combine pricing decisions and deterioration study streams with studies where shortages are tolerable, and/or the time value of money is considered [4]. considers the existence of price-sensitive renewal demand methods to make the most of the discounted profit by determining the selling prices dynamically for items with fixed lifetime, but shortages are not considered. An inventory model under inflation for deteriorating items with price and stock-dependent demand and limited capacity was developed by [5]. Meanwhile [28], present a production lot-sizing model for deteriorating item under inflation by considering a coordinated pricing and production decisions as well as price decision disregarding the production cost. This paper is presents a simple model to minimize the cost of keeping an item (in this case, Tomatoes) in inventory and determine the best time to sell off the product after the introduction of a discount.

Model Formulation

Model Parameters

t = time (seconds)
q = quantity of goods (kg)
H_c= Holding cost
S_p= Selling price per unit
C_p = Cost price of the goods
d_t= discount for sale of the goods
R = Revenue generated from selling of the goods
P = Profit generated from selling of the goods
m = initial quantity of the goods (kg)
r = rate of fall in price per day
w = rate of fall in quantity per day
k = cost of keeping a unit of tomatoes per day

Assumptions and Model

$$H_c = kqt \tag{1}$$

$$d_t = rt \tag{2}$$

$$S_p = C_p + H_c - rt \tag{3}$$

$$q = m - wt \tag{4}$$

$$R = S_p * q \tag{5}$$

$$P = R - C_p m \tag{6}$$

Solution: Maximize P

$$P = \begin{array}{c}(C_p m + kqt - rt)(m - wt) - C_p m \\ = (C_p m + k(m - wt)t - rt)(m - wt) - C_p m\end{array} \tag{7}$$

Application

We consider a perishable product (fresh tomatoes) that sells for N300 per kg. Tomatoes are assumed to have a shelf life of between 7-10 days. If the cost of keeping the product in inventory is 1 Kobo per kg. Assume that the initial quantity (opening stock) in inventory is 200kg and the rate of fall in price is 0.05 per day. Also, the rate of fall in quantity is 0.005 per day.

We wish to determine the optimal time at which it is best to sell the product.

Let

m = initial quantity of goods = 200kg

C_p = cost price of 1kg of tomatoes = 300

r = rate of fall in price per day = 0.05

w = rate of fall in quantity per day = 0.005

k = cost of keeping 1kg of tomatoes per day = 0.01

Substituting the above parameters in equation (7), we have:

$$P = (300 + 0.01(200 - 0.005t)t - 0.05t)(200 - 0.005t) - 300$$

$$= 59700 + 388.5t - 0.01975t^2 + 0.00000025t^3 \quad (8)$$

Differentiating equation (8), gives:

$$\frac{dP}{dt} = 388.5 - 0.0395t + 0.00000075t^2 \quad (9)$$

Setting $\frac{dP}{dt} = 0$, we have:

$t_1 = 13088, t_2 = 39579$

Substituting $t_1 = 13088$, and $t_2 = 39579$ into equation (8)

$P(t_1) = 2321776.73$

$P(t_2) = -2080.43$

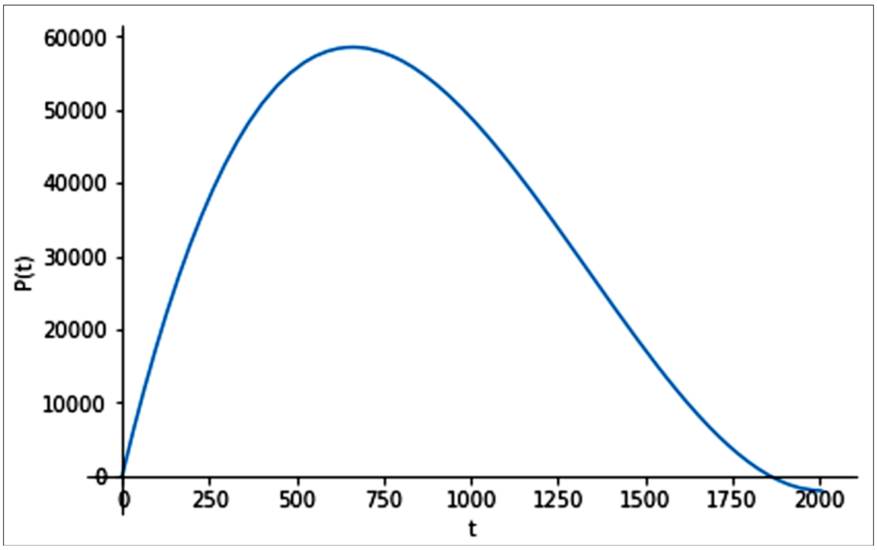

Fig 1: Graph of optimal time to sell (r=w=0.01, k=0.5; 11hrs)

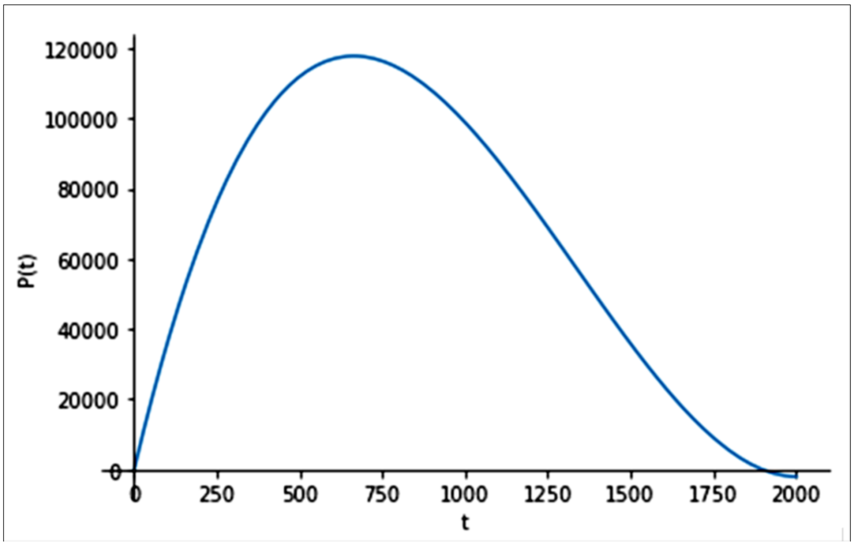

Fig 2: Graph of optimal time to sell (r=w=0.01, k=1.0; 11hrs)

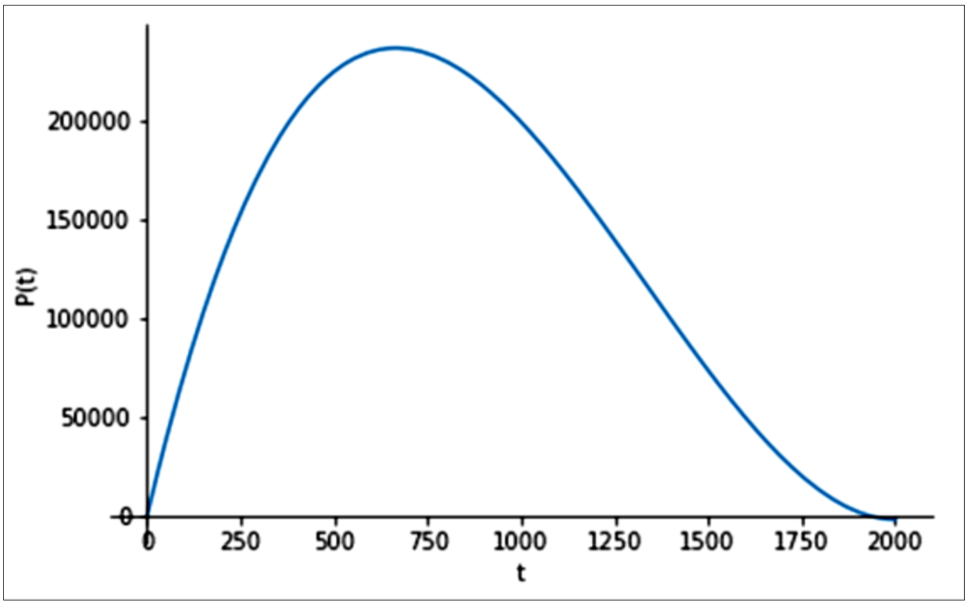

Fig 3: Graph of optimal time to sell (r=w=0.01, k=2.0; 11hrs)

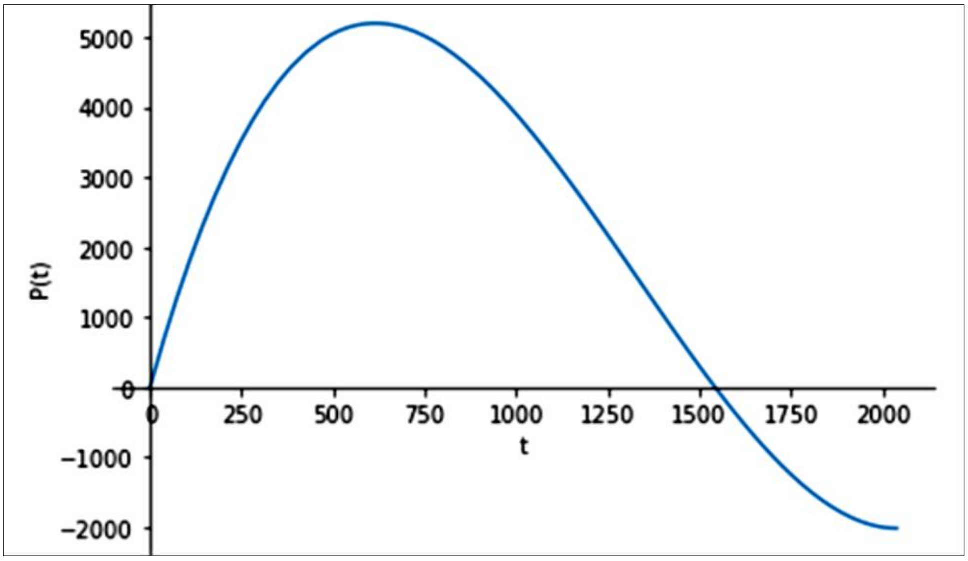

Fig 4: Graph of optimal time to sell (r=w=0.01, k=0.5; 11hrs)

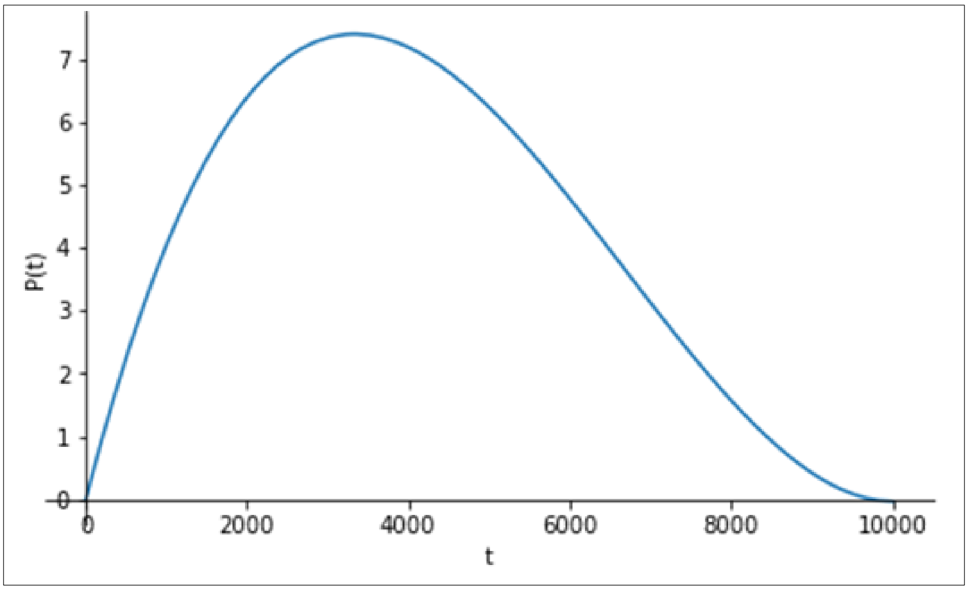

Fig 5: Graph of optimal time to sell (m=100kg, cp=300, r=w=0.01, k=0.5; 2.3 days)

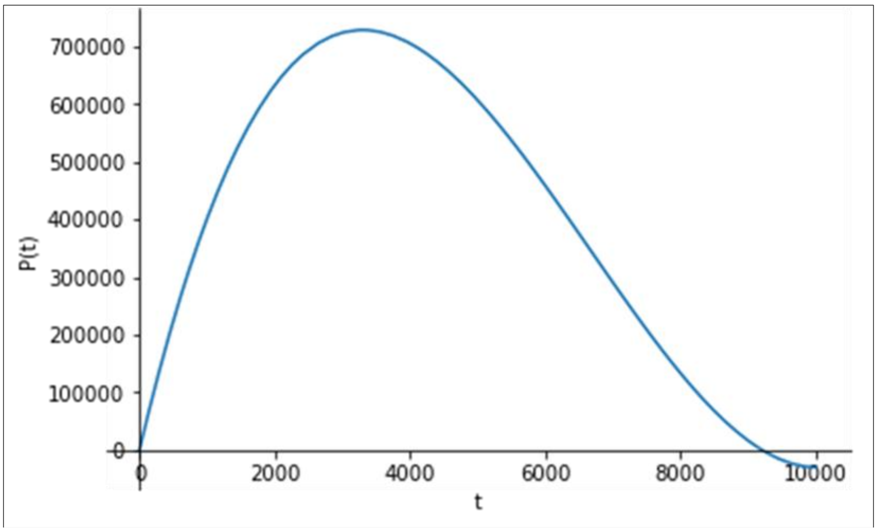

Fig 6: Graph of optimal time to sell (m=100kg, cp=300, r=w=0.01, k=0.05; 2.3 days)

The result obtained by the model show different times to sell the tomatoes, ranging from 11hours to about 3 days respectively so as to maximize the profit. The model shows that the curve of the optimal time to sell the product vary with different values of the parameters considered. The answer is valid as long as the assumptions made above holds. From the figures above, the optimal times to sell the product off were approximately within the shelf – life.

Sensitivity Analysis

We can see that the optimal time to sell the product is sensitive to the parameter r. constructing equation (7) in terms of t and r, we have:

$$P = 59700 + (398.5 - 200r)t - 0.015rt^2 + 0.00000025t^3 \tag{10}$$

Differentiating equation (10) with respect to t and r equating to zero, we have;

$$\frac{dp}{dt} = 398.5 - 200r - 0.03rt + 0.00000075t^2 \tag{11}$$

$$\frac{dp}{dr} = -(200t + 0.015t^2) \tag{12}$$

$$\frac{d^2p}{dt^2} = -0.03r + 0.0000015t \tag{13}$$

$$\frac{d^2p}{dr^2} = 0$$

For the system to be harmonic,

$$\frac{d^2p}{dt^2} + \frac{d^2p}{dr^2} = 0 \tag{14}$$

Thus

$$0.03r = 0.0000015t \tag{15}$$

From equation (10)

$$\frac{dp}{dt} \cong 398.5 - 200r - 0.03rt \tag{16}$$

$$t = \frac{398.5 - 200r}{0.03r} \tag{17}$$

$$\frac{dt}{dr} = \frac{-13283.33}{r^2} \tag{18}$$

See the graph of best time to sell versus the rate at which the price is falling.

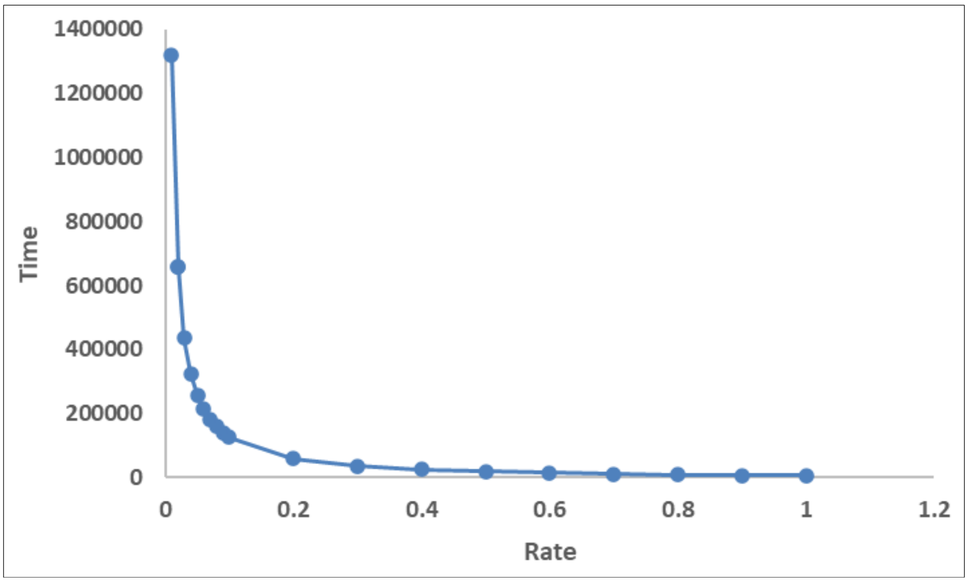

Fig 7: Graph of best time to sell

$$S(t,r) = \frac{dt}{t} \Big/ \frac{dr}{r} = \frac{dt}{dr} \cdot \frac{r}{t} \tag{19}$$

At $t = 8$ and $r = 1$,

$$S(t,r) = \frac{-13283}{8}$$

Discussion: Equation (19) shows the sensitivity of the time and the rate of fall in price of the tomatoes. For the 200kg pack of tomatoes costing ₦60,000.00 for example, the price depreciates by ₦132.83 in every 8 minutes

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