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An inflated probability model for the adult male migrants

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Abstract
The objective of this paper is to develop an Inflated Probability Model on the basis of Himanshu Distribution for the Adult Male Migrants. Estimation technique based on Method of Maximum likelihood have been proposed to estimate the parameters of the Model. The Model fitted to observed data found a better approximation at the survey area and draw some conclusion.

Keywords: Inflated distributions, Himanshu distribution, method of Maximum likelihood, migration pattern

Introduction
Inflated Models have been developed to analyze real life count data that exhibits many zeros. The skew nature of the resulting distribution makes it difficult to transform the data to a Normal distribution. In cases the model based on regular distributions are not able to handle a no. of zeros, they are extended to inflated form. The Inflated Poisson and Inflated Negative Binomial are the most popular models and have been used in many literatures.
Agarwal and Pandey (2022) [1] has been introduced a discrete type Himanshu Distribution with probability mass function

$$P(X=x)=p^n(1-p^n)^x;\quad \left.\begin{matrix} x=0,1,2,\ldots \\ 0<p<1 \\ n\in I^+ \end{matrix}\right\} \quad (1.1)$$

$$\text{with Mean}=\frac{1-p^n}{p^n} \text{ and Variance}=\frac{1-p^n}{p^{2n}}$$

Again the probability mass function of zero truncated Himanshu distribution (ZTHD) (Agarwal and Pandey; 2022) [1] is given as

$$P(X=x)=p^n(1-p^n)^{x-1};\quad \left.\begin{matrix} x=1,2,3,\ldots \\ 0<p<1 \\ n\in I^+ \end{matrix}\right\} \quad (1.2)$$

$$\text{with Mean}=\frac{1}{p^n} \text{ and Variance}=\frac{1-p^n}{p^{2n}}$$

Which is suitable for the count data in different field of real life problems

Proposed Inflated Probability Model
Let X denotes the number of Male Migrants from a household at the survey point then the distribution of X is derived under the following assumptions-

a) Let α be the probability that household is exposed to the risk of Migration at the survey point and (1-α) be the probability that a household is not exposed to risk of Migration.

b) The probability of x males migrating from a household is more than the probability of (x+1) males migrating (1,2,…) thus probability is decreasing function of x therefore number of adult male migrants from a households follows a zero truncated Himanshu distribution.

Under these two assumptions the proposed inflated probability model for the number of adult Migrants X is given by

$$P(X=x)=\begin{cases} 1-\alpha & ; \quad x=0 \\ \alpha\, p^n(1-p^n)^{x-1} & ; \quad \begin{matrix} x=1,2,3\ldots \\ 0<p<1 \\ n\in I^+ \end{matrix} \end{cases} \qquad (2.1)$$

where α and p are parameters with

$$\text{Mean}=\alpha\left(\frac{1}{p^n}\right)=\frac{\alpha}{p^n}$$

$$\text{Variance}=\frac{2\alpha-\alpha p^n-\alpha^2}{p^{2n}}$$

Estimation of the parameters

The parameter of the proposed inflated probability Model are estimated by the method of maximum likelihood which is explained under the following-

$$L=(1-\alpha)^{f_0}\,(\alpha p^n)^{f_1}[1-\{(1-\alpha)+\alpha p^n\}]^{f-f_0-f_1}$$

$$\log \mathrm{L}=f_0\,\log(1-\alpha)+f_1\log(\alpha p^n)+(f-f_0-f_1)\log(\alpha-\alpha p^n) \quad (3.1)$$

Now partially differentiate (3.1) with respect to α and equating to zero we get-

$$\hat{\alpha}=1-\frac{f_0}{f}$$

And partially differentiate (3.1) with respected to p and equating to zero we get

$$\hat{p}=\left(\frac{f_1}{f-f_0}\right)^{\frac{1}{n}}$$

Where $\hat{\alpha},\hat{p}$ are maximum likelihood estimates of α and p respectively. Now

$$\frac{E\left(\frac{-\partial^2}{\partial\alpha^2}\log L\right)}{f}=\left[\frac{1}{1-\alpha}+\frac{1}{\alpha}\right]$$

And

$$\frac{E\left(\frac{-\partial^2}{\partial p^2}\log \mathrm{L}\right)}{f}=\frac{n^2\alpha p^{n-2}}{1-p^n}$$

Ultimately we get-

$$E(f_0)=f(1-\alpha)$$

$$E(f_1)=f\alpha p^n$$

After estimating all the parameters involved in the proposed model by MLE, We are interested in calculating the variances of each estimated parameters.

$$FIM=\begin{bmatrix}\frac{1}{1-\alpha}+\frac{1}{\alpha} & 0\\ 0 & \frac{n^2\alpha p^{n-2}}{1-p^n}\end{bmatrix}=\begin{bmatrix}\varphi_{11} & \varphi_{12}\\ \varphi_{21} & \varphi_{22}\end{bmatrix}$$

$$V(\hat{\alpha})=\frac{1}{f}\left[\frac{\varphi_{22}}{\varphi_{11}\varphi_{22}}\right]=\frac{\alpha(1-\alpha)}{f}$$

$$V(\hat{p})=\frac{1}{f}\left[\frac{\varphi_{11}}{\varphi_{11}\varphi_{22}}\right]=\frac{1}{f}\left(\frac{1-p^n}{n^2\alpha p^{n-2}}\right)$$

Application

The proposed inflated Model at n=2 has been applied to the data for adult Male Migrants at household level from the four surveys namely “RDPG-Survey 1978, BHU”, “A case study of North Eastern Bihar 2009-10”, “Sample survey of the Rupandehi and Palpa district in Nepal-2011” and “Impact of Migration on fertility in Bangladesh: A study of Comilla district.

Table 1: Observed and Expected distribution of the number of households according to single Male Migrants from a household in Semi Urban type villages.

No. of Male migrants aged 15 and over	Semi Urban type villages	
	Observed No. of households	Expected No. of households
0	1042	1042
1	95	94.94
2	19	25.02
3	10	6.59
4	2	2.45
5	2	
6	0	
7	1	
8	0	
	5	
Total	1171	1171
Mean=0.1605 Variance=0.23425 $\hat{p}=0.8581$ $\hat{\alpha}=0.1101$ $V(\hat{\alpha})$ =0.00008 $V(\hat{p})$ =0.00051	$\chi^2=5.85$ (after pooling) p-value=0.0536 $\chi^2_{(2)}=5.99$ at 5% level of significance	

Table 2: Observed and Expected distribution of the number of households according to single Male Migrants from a household in Remote type of villages

No. of Male migrants aged 15 and over	Remote type villages	
	Observed No. of households	Expected No. of households
0	872	872.00
1	176	176.00
2	59	58.19
3	18	19.24
4	6 4 0 0 0 } 10	9.56
5		
6		
7		
8		
Total	1135	1135
Mean=0.3453 Variance=0.56863 $\hat{p} = 0.8181$ $\hat{\alpha} = 0.2317$ $V(\hat{\alpha})$ =0.00015 $V(\hat{p})$ =0.00031	$\chi^2 = 0.1$ (after pooling) p-value=0.9512 $\chi^2_{(2)} = 5.99$ at 5% level of significance	

Table 3: Observed and Expected distribution of the number of households according to single Male Migrants from a household Growth centre type of villages

No. of Male migrants aged 15 and over	Growth centre type villages	
	Observed No. of households	Expected No. of households
0	978	978
1	154	154
2	47	50.85
3	18	16.80
4	9 1 0 1 0 } 11	8.34
5		
6		
7		
8		
Total	1208	1208
Mean=0.2897 Variance=0.48402 $\hat{p} = 0.8182$ $\hat{\alpha} = 0.1903$ $V(\hat{\alpha})$ =0.00012 $V(\hat{p})$ =0.00035	$\chi^2 = 1.21$ (after pooling) p-value=0.5460 $\chi^2_{(2)} = 5.99$ at 5% level of significance	

Table 4: Observed and expected frequency of the number of households according to the North Eastern Bihar.

Number of migrants	Observed No. of households	Expected No. of households
0	401	401.00
1	147	147.00
2	57	64.80
3	29	28.58
4	16	12.60
5	8 5 1 } 14	10.00
6		
7		
Total	664	664
Mean=0.7365 Variance=1.3249 $\hat{\alpha} = 0.3960$ $\hat{p} = 0.7475$ $V(\hat{\alpha})$ =0.00036 $V(\hat{p})$ =0.00041	$\chi^2 = 3.44$ (after pooling) p-value=0.3286 $\chi^2_{(3)} = 7.815$ at 5% level of significance	

Table 5: Observed and Expected frequency of the number of households according to migrants in Nepal.

No. of migrants	Observed No. of households	Expected No. of households
0	623	623
1	126	126
2	42	41.53
3	13	13.69
4	4, 2, 1 } 7	6.77
5		
6		
Total	811	811
Mean=0.3465 Variance=0.5666 $\hat{\alpha} = 0.2318$ $\hat{p} = 0.8186$ $V(\hat{\alpha})$ =0.00021 $V(\hat{p})$ =0.00043	$\chi^2 = 0.04$ (after pooling) p-value=0.9801 $\chi^2_{(2)} = 5.99$ at 5% level of significance	

Table 6: Observed and Expected frequency of the number of households according to the migrants in Comilla district of Bangladesh.

No. of migrants	Observed No. of households	Expected No. of households
0	1941	1941
1	542	542
2	124	140.61
3	48	36.48
4	13, 4, 1 } 18	12.90
5		
6		
Total	2673	2673
Mean=0.3786 Variance=0.49237 $\hat{\alpha} = 0.2738$ $\hat{p} = 0.8604$ $V(\hat{\alpha})$ =0.00007 $V(\hat{p})$ =0.00008	$\chi^2 = 7.60$ (after pooling) p-value=0.0223 $\chi^2_{(2)} = 9.21$ at 1% level of significance	

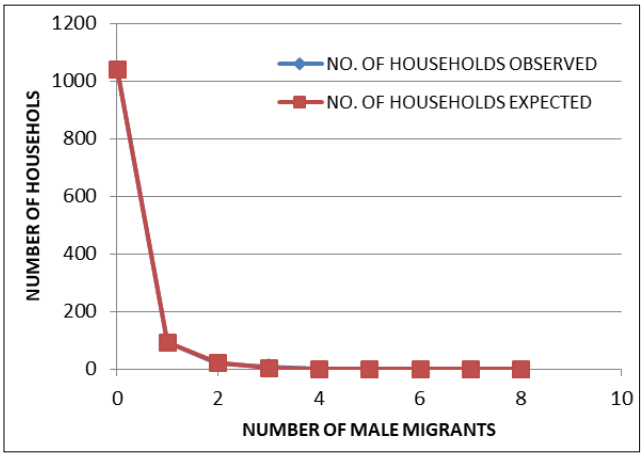

Fig 1: Graphical presentation showing Observed and Expected number of households having adult male migrants aged (15 years and above) in semi urban type villages.

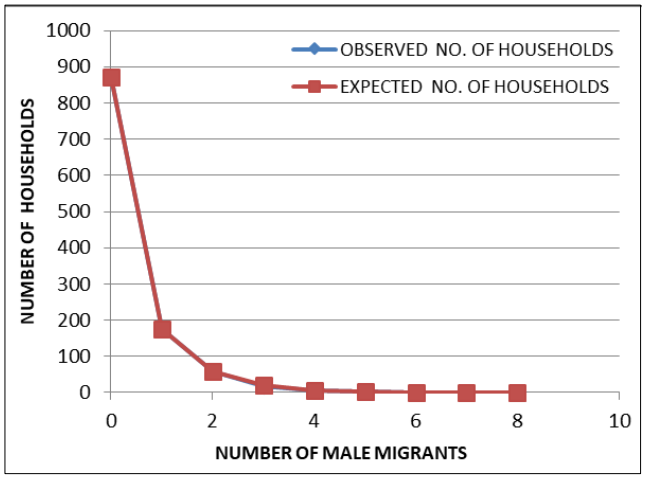

Fig 2: Graphical presentation showing Observed and Expected number of households having adult male migrants aged (15 years and above) in Remote type of villages.

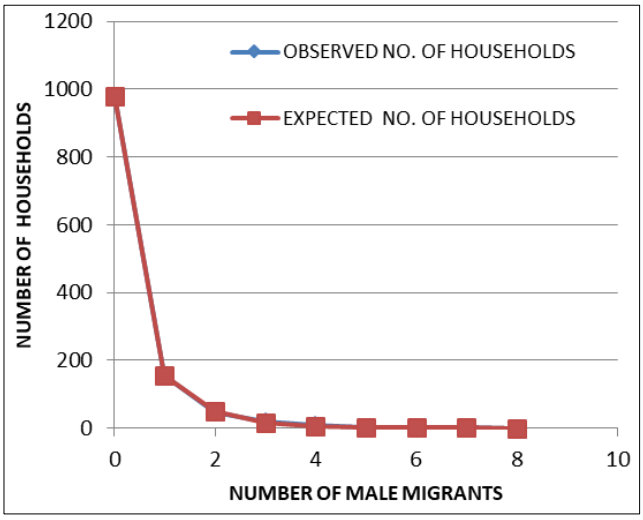

Fig 3: Graphical presentation showing Observed and Expected number of households having adult male migrants aged (15 years and above) in Growth centre type of villages

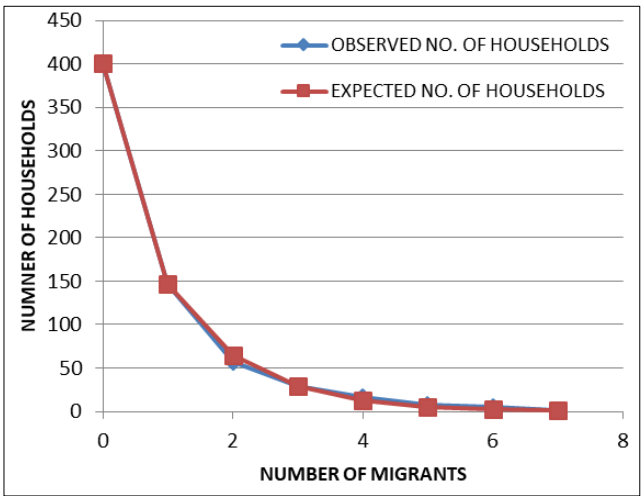

Fig 4: Graphical presentation showing Observed and Expected number of households having adult male migrants aged (15 years and above) in North Eastern Bihar

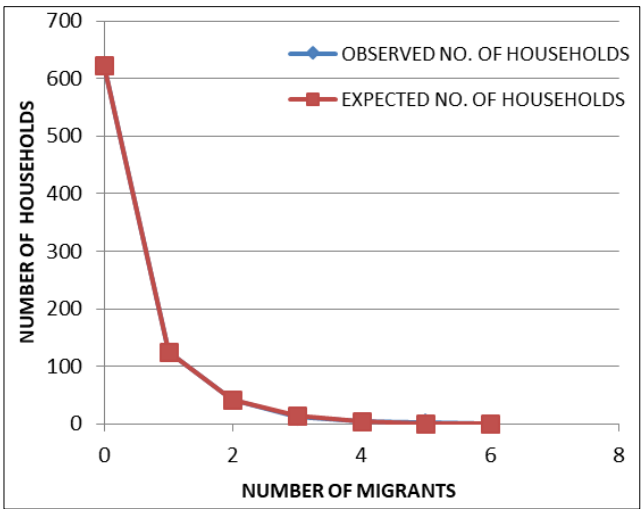

Fig 5: Graphical presentation showing Observed and Expected number of households having adult male migrants aged (15 years and above) in Nepal

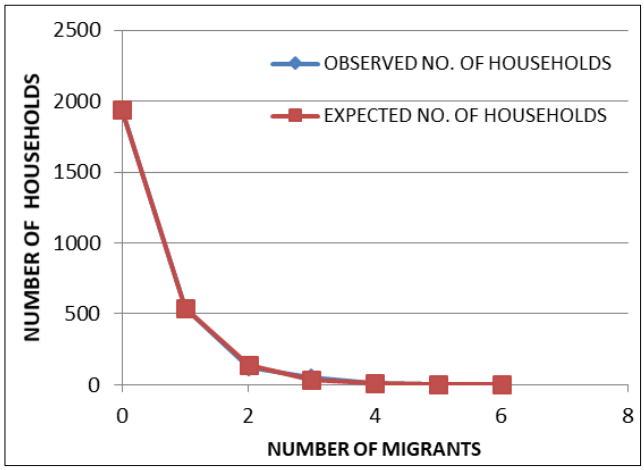

Fig 6: Graphical presentation showing Observed and Expected number of households having adult male migrants aged (15 years and above) in Comilla district of Bangladesh.

Conclusion

From table 1 to 5 it is found that the value of χ^2 is insignificant at 5% level of significance for all set of data while in table 6, value of χ^2 is insignificant at 1% level of significance.

Proposed Inflated probability Model suitably describe the pattern of Adult Male Migrants from the households at Micro level. The author believe that this model contains the information of Risk of Migration (α) which leads to provide very relevant information regarding the Male Migration patterns. Thus it will become more useful in calculating various probabilities of Migrants connected with the process of Migration from a household and also for predictions to the Migration Rates is a specified population. In the light of work done by Rao and Pandey (2021, 2022) [5, 6], It is also possible for the Bayesian Analysis of the proposed model.

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