International Journal of Statistics and Applied Mathematics 2022; 7(6): 139-142

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(6): 139-142

https://www.mathsjournal.com
Received: 20-09-2022
Accepted: 22-10-2022

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New Homeomorphism in topological spaces

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DOI: https://doi.org/10.22271/maths.2022.v7.i6b.909

Abstract
In this paper a new class of homeomorphism called minimal Regular weakly homeomorphism and maximal Regular weakly homeomorphism are introduced and investigated and during this process some properties of the new concepts have been studied.

Keywords: Minimal homeomorphism, maximal homeomorphism, minimal weakly homeomorphism, Maximal weakly homeomorphism, Minimal weakly open set, Maximal weakly open set.

Introduction

In the year 2001 and 2003, F.Nakaoka and N.oda [1, 2, 3] introduced and studied minimal open [resp. minimal closed] sets which are subclass of open [resp.closed sets]. The family of all minimal open [minimal closed] in a topological space X is denoted by $m_io(X)$ $[m_ic(X)]$. Similarly the family of all maximal open [maximal closed] sets in a topological space X is denoted by $M_aO(X)[M_aC(X)]$.The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2000, M.Sheik john [4] introduced and studied weakly homeomorphism in topological spaces and in the year 2008 B.M.Ittanagi [5] introduced and studied minimal open sets and maps in topological spaces and minimal homeomorphism and maximal homeomorphism in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre [6, 7] introduced and studied minimal weakly open sets and maximal weakly closed sets and maximal weakly open sets and minimal weakly closed sets in topological spaces.In the year 2014 [8] Vivekananda Dembre, Manjunath Gowda and Jeetendra Gurjar introduced and studied minimal weakly and maximal weakly continuous functions in topological spaces. In the year 2014 [9] Vivekananda Dembre and Jeetendra Gurjar introduced and studied minimal weakly and maximal weakly open maps in topological spaces.

Definition 1.1 [1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is φ or U.

Definition 1.2 [2]: A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is X or U.

Definition 1.3 [3]: A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is φ or F.

Definition 1.4 [3]: A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is X or F.

Definition 1.5 [4]: Let X and Y be the topological spaces. A bijective function $f : X \longrightarrow Y$ is called weakly homeomorphism if both f and f^{-1} are weakly continuous.

Definition 1.6 [5]: Let X and Y be the topological spaces. A bijective function $f : X \longrightarrow Y$ is called

I) Minimal homeomorphism if both f and f^{-1} are minimal continuous maps.
II) Maximal homeomorphism if both f and f^{-1} are maximal continuous maps.

Definition 1.7 [5]**:** Let X and Y be the topological spaces. A map $f : X \longrightarrow Y$ is called

I) Minimal continuous function if for every minimal open set N in Y, $f^{-1}(N)$ is an open set in X.
II) Maximal continuous function if for every maximal open set N in Y, $f^{-1}(N)$ is an open set in X.
III) Minimal open map for every open set U of X, f(U) is minimal open set in Y.
IV) Maximal open map for every open set U of X, f(U) is maximal open set in Y.
V) Minimal closed map for every closed set F of X, f(F) is minimal closed set in Y.
VI) Maximal closed map for every closed set F of X, f(F) is maximal closed set in Y.
VII) Minimal-Maximal open map for every minimal open set N of X, f(N) is maximal open set in Y.
VIII) Maximal-Minimal openmap for every maximal open set N of X,f(N) is minimal open set in Y.
IX) Minimal-Maximal continuous if for every minimal open set N in Y, $f^{-1}(N)$ is a maximal open set in X.
X) Maximal-Minimal continuous if for every maximal open set N in Y, $f^{-1}(N)$ is a minimal open set in X.

Definition 1.8 [6]**:** Let X and Y be the topological spaces. A map $f : X \longrightarrow Y$ is called

(i). Minimal weakly continuous if for every minimal weakly open set N in Y,$f^{-1}(N)$ is an open set in X.
(ii). Maximal weakly continuous if for every maximal weakly open set N in Y,$f^{-1}(N)$ is an open set in X.

Definition 1.9 [7]**:** A proper non-empty weakly open subset U of X is said to be minimal weakly open set if any weakly open set which is contained in U is ∅ or U.

Definition 1.10 [8]**:** A proper non-empty weakly closed subset U of X is said to be maximal weakly open set if any weakly open set which is contained in U is X or U.

Definition 1.11 [9]**:** Let X and Y be topological spaces.

(i) A map $f : X \longrightarrow Y$ is called minimal weakly open map for every open set U of X , f(U) is minimal weakly open set in Y.
(ii) A map $f : X \longrightarrow Y$ is called maximal weakly open map for every open set U of X , f(U) is maximal weakly open set in Y.
(iii) A map $f : X \longrightarrow Y$ is called minimal weakly closed map for every closed set F of X , f(F) is minimal weakly closed set in Y.
(iv) A map $f : X \longrightarrow Y$ is called maximal weakly closed map for every closed set F of X , f(F) is maximal weakly closed set in Y.

Minimal regular weakly and maximal regular weakly homeomorphism.

Definition 2.1: A bijection function $f : X \longrightarrow Y$ is called

I) Minimal regular weakly homeomorphism if both f and f^{-1} are minimal regular weaklycontinuous maps.
II) Maximal regular weakly homeomorphism if both f and f^{-1} are maximal regular weakly continuous maps.

Theorem 2.2: Every homeomorphism is minimal regular weakly homeomorphism but not conversely.

Proof: Let $f : X \longrightarrow Y$ be a homeomorphism. Now f and f^{-1} are continuous maps then f and f^{-1} are minimal regular weakly continuous as every continuous map is minimal regular weakly continuous. Hence f is a minimal regular weakly homeomorphism.

Example 2.3: Let X=Y={a,b,c} be with τ = { X, φ, {a},{a,b}} and μ = { Y, φ, {a},{b},{a,b} } then $f : X \longrightarrow Y$ be a function defined by f(a)=a, f(b)=a and f(c)=c, then f and f^{-1} are minimal regular weakly continuous maps then f is a minimal regular weakly homeomorphism but it is not a homeomorphism. Since f is not continuous map for the open set {b} in Y ; f^{-1}({b}) = b which is not open set in X.

Theorem 2.4: Every homeomorphism is maximal regular weakly homeomorphism but not conversely.

Proof: Let $f : X \longrightarrow Y$ be a homeomorphism. Now f and f^{-1} are continuous maps then f and f^{-1} are maximal regular weakly continuous as every continuous map is minimal regular weakly continuous. Hence f is a minimal regular weakly homeomorphism.

Example 2.5: Let X = Y = {a,b,c} be with τ = { X, φ, {a,b}} and μ = { Y, φ, {a},{b},{a,b} } then $f : X \longrightarrow Y$ be a identity function then f and f^{-1} are maximal regular weakly continuous maps then f is a maximal regular weakly homeomorphism but it is not a homeomorphism. Since f is not a continuous map for the open set {b} in Y ; f^{-1}({b}) = b which is not open set in X.

Remark 2.6: Minimal regular weakly homeomorphism and maximal regular weakly homeomorphism are independent of each other.

Example 2.7: In example 2.3 f is minimal regular weakly homeomorphism but it is not maximal regular weakly homeomorphism. In example 2.5 f is maximal regular weakly homeomorphism but it is not minimal regular weakly homeomorphism.

Theorem 2.8: Let $f : X \longrightarrow Y$ be a bijective and minimal regular weakly continuous then the following statements are equivalent.
(i) $f : X \longrightarrow Y$ is minimal regular weakly homeomorphism.
(ii) f is minimal regular weakly open map.
(iii) f is maximal regular weakly closed map.

Proof:
(i) $\longrightarrow$ (ii) : Let N be any minimal regular weakly open set in X ; by assumption $(f^{-1})^{-1}$ (N) is an open set in Y. But $(f^{-1})^{-1}$ (N) = f (N) is an open set in Y ; therefore f is a minimal regular weakly open map.

(ii) $\longrightarrow$ (iii) : Let F be any maximal regular weakly closed set in X ; then X-F is a minimal regular weakly open set in X ; by assumption f (X-F) is an open set in Y. But f (X-F) = Y- f (F) is an open set in Y ; therefore f(F) is a closed set in Y. Hence f is a maximal regular weakly closed map.

(iii) $\longrightarrow$ (i): Let N be any minimal regular weakly open set in X ; then X-N is a maximal regular weakly closed set in X ; by assumption f (X-N) is an closed set in Y. But f (X-N) = (f^{-1}) (X - N) = Y - (f^{-1}) (N) is closed set in Y ; therefore (f^{-1}) (N) is an open set in Y. Hence
$f^{-1} : Y \longrightarrow X$ is a minimal regular weakly homeomorphism and similarly f is minimal regular weakly homeomorphism.

Theorem 2.9: Let f : X ⟶ Y be a bijective and maximal regular weakly continuous then the following statements are equivalent.

(i). f^{-1} : X ⟶ Y is maximal regular weakly homeomorphism.

(ii). f is maximal regular weakly open map.

(iii). f is minimal regular weakly closed map.

Proof : (i) ⟶ (ii) : Let N be any maximal regular weakly open set in X ; by assumption $(f^{-1)})^{-1}$ (N) is an open set in Y. But $(f^{-1)})^{-1}$ (N) = f (N) is an open set in Y ; therefore f is a minimal regular weakly open map.

(ii) ⟶ (iii): Let F be any minimal regular weakly closed set in X ; then X-F is a maximal regular weakly open set in X ; by assumption f (X-F) is an open set in Y. But f (X-F) = Y- f (F) is an open set in Y ; therefore f(F) is a closed set in Y. Hence f is a maximal regular weakly closed map.

(iii) ⟶(i): Let N be any maximal regular weakly open set in X ; then X-N is a minimal regular weakly closed set in X ; by assumption f (X-N) is an closed set in Y. But f (X-N) = $(f^{-1)})$ (X - N) = Y - $(f^{-1)})$ (N) is closed set in Y ; therefore $(f^{-1)})$ (N) is an open set in Y. Hence

f^{-1} : Y ⟶ X is a minimal regular weakly homeomorphism and similarly f is minimal regular weakly homeomorphism.

Definition 2.10: Let X and Y be the topological spaces. A map f : X ⟶ Y is called

(i). Minimal-Maximal regular weakly continuous if for every minimal regular weakly open set N in Y, f^{-1}(N) is a maximal regular weakly open set in X.

(ii). Maximal-Minimal regular weakly continuous if for every maximal regular weakly open set N in Y, f^{-1}(N) is a minimal regular weakly open set in X.

(iii). Minimal-Maximal regular weakly open map for every minimal regular weakly open set N of X, f(N) is maximal regular weakly open set in Y.

(iv). Maximal-Minimal regular weakly open map for every maximal regular weakly open set N of X, f(N) is minimal regular weakly open set in Y.

(v). Minimal-Maximal regular weakly closed map for every minimal regular weakly closed set F of X, f(F) is maximal regular weakly closed set in Y.

(vi). Maximal-Minimal regular weakly closed map for every maximal regular weaklyclosed set F of X, f(F) is minimal regular weakly closed set in Y.

Definition 2.11: A bijection f : X ⟶ Y is called

(i). min - max regular weakly homeomorphism if both f and f^{-1} are min - max regular weakly continuous maps.

(ii). max- min regular weakly homeomorphism if both f and f^{-1} are max - min regular weakly continuous maps.

Theorem 2.12: Every min-max regular weakly homeomorphism is minimal regular weakly homeomorphism but not conversely.

Proof : : Let f : X ⟶ Y be a homeomorphism. Now f and f^{-1} are continuous maps then f and

f^{-1} are min-max regular weakly continuous as every continuous map is min-max regular weakly continuous. Hence f is a min-max regular weakly homeomorphism.

Example 2.13: In example 2.3 f is a minimal regular weakly homeomorphism but it is not a min-max regular weakly homeomorphism. Since f is not a min-max regular weakly continuous map for the minimal regular weakly open set {b} in Y, f^{-1} (b) = {b} which is not a maximal regular weakly open set in X.

Theorem 2.14: Every max-min regular weakly homeomorphism is maximal regular weakly homeomorphism but not conversely.

Proof : Let f : X ⟶ Y be a homeomorphism. Now f and f^{-1} are continuous maps then f and

f^{-1} are max-min regular weakly continuous as every continuous map is max-min regular weakly continuous. Hence f is a min-max regular weakly homeomorphism.

Theorem 2.15: Let f : X ⟶ Y be a bijective and min-max regular weakly continuous map then the following statements are equivalent.

(i). f is minimal - maximal regular weakly homeomorphism.

(ii). f is minimal - maximal regular weakly open map.

(iii). f is maximal - minimal regular weakly closed map.

Proof :

(i) ⟶ (ii) : Let N be any minimal regular weakly open set in X ; by assumption $(f^{-1)})^{-1}$ (N) is an open set in Y. But $(f^{-1)})^{-1}$ (N) = f (N) is a maximal regular weakly open set in Y ; therefore f is a min-max regular weakly open map.

(ii) ⟶ (iii) : Let F be any maximal regular weakly closed set in X ; then X-F is a minimal regular weakly open set in X ; by assumption f (X-F) is a maximal regular weakly open set in Y. But f (X-F) = Y- f (F) is a maximal regular weakly open set in Y ; therefore f(F) is a minimal regular weakly closed set in Y. Hence f is a max-min regular weakly closed map.

(iii) ⟶ (i) : Let N be any minimal regular weakly open set in X ; then X-N is a maximal regular weakly closed set in X ; by assumption f (X-N) is a minimal regular weakly closed set in Y.

But f (X-N) = $(f^{-1)})$ (X - N) = Y - $(f^{-1)})$ (N) is a minimal regular weakly open set in Y ; therefore $(f^{-1)})$ (N) is a maximal regular weaklyopen set in Y. Hence f^{-1}:Y X is a minimal-maximal regular weakly homeomorphism and similarly f is min-max regular weakly homeomorphism.

Theorem 2.16: Let f : X ⟶ Y be a bijective and min-max regular weakly continuous map then the following statements are equivalent.

(i). f is max - min regular weakly homeomorphism.

(ii). f is max - min regular weakly open map.

(iii). f is min - max regular weakly closed map.

Proof : (i) ⟶ (ii) : Let N be any maximal regular weakly open set in X ; by assumption $(f^{-1)})^{-1}$ (N) is an open set in Y. But $(f^{-1)})^{-1}$ (N) = f (N) is a minimal regular weakly open set in Y ; therefore f is a max - min regular weakly open map.

(ii) ⟶ (iii) : Let F be any minimal regular weakly closed set in X ; then X-F is a maximal regular weakly open set in X ; by assumption f (X-F) is a minimal regular weakly open set in Y.

But f (X-F) = Y- f (F) is a minimal regular weakly open set in Y ; therefore f(F) is a minimal regular weakly closed set in Y. Hence f is a max-min regular weakly closed map.

(iii) ⟶ (i) : Let N be any minimal regular weakly open set in X ; then X-N is a maximal regular weakly closed set in X ; by

assumption f (X-N) is a minimal regular weakly closed set in Y.
But f (X-N) = (f^{-1}) (X - N) = Y - (f^{-1}) (N) is a minimal regular weakly open set in Y ; therefore (f^{-1}) (N) is a maximal regular weaklyopen set in Y. Hence f^{-1}:Y ⟶ X is a minimal-maximal regular weakly homeomorphism and similarly f is min-max regular weakly homeomorphism.

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