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Aditya Kumar
Research Scholar, Department of Mathematics, J.P. University, Chapra, Bihar, India

Brijendra Kumar Singh
Retired Associate Professor, Department of Mathematics, J.P. University, Chapra, Chapra, Bihar, India

Corresponding Author:
Aditya Kumar
Research Scholar, Department of Mathematics, J.P. University, Chapra, Bihar, India

Finite single integral representation for the polynomial Set $A_n\{(xm), y\}$

Aditya Kumar and Brijendra Kumar Singh

Abstract
In the present paper, an attempt has been made to express a Finite Single Integral Representation for the Polynomial Set $A_n\{(x_m),y\}$. Many interesting new results may be obtained as particular cases out of these particular results some of them stand for well-known and some of them are believed to be new. These are of at most important for mathematician scientists, engineers and physical sciences because these occur in the solution of differential equation Integral equation etc.
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Introduction

We define the generalized hypergeometric polynomial set $A_n\{(x_m), y\}$ by means of generating functions.

$$(1 - \nu t^h)^{-\mu}(1 - \lambda_1 y^{-h_1} t^{h_1})^{\mu_1}$$

$$F\left[\begin{matrix}(E_r); (G_p); (\alpha_{u_m}) \\ (F_s); (H_q); (\beta_{v_m})\end{matrix} \lambda x_1^e t, \lambda_2 x_2^{h_2} y^{-h_2} \dots\dots \lambda_m x_m^{h_m} t^{h_m}\right]$$

$$= \sum_{n=0}^{\infty} R_{n,e,h;h_1;h_2;\dots\dots h_m;(F_s);(H_q);(\beta_{v_m})}^{\nu;\mu;\mu_1;\lambda;\lambda_1;\lambda_2\dots\dots\lambda_m;(E_r);(G_p);(\alpha_{u_m})} \{(x_m), y\} t^n$$

Where ν, μ, μ_1, λ, λ_1, λ_2, ……λ_m are real and e, h, h_1, …… h_m are non-negative integer.

The left hand side of (1.1) contains the product of generalized Appell function [1] of several variables in the notation of Burchanall and Chaundy [3] associated with Lauricella functions. The generalized hypergeometric polynomial set contains a number of parameters for simplicity the polynomial set shall be denote

$$A_{n,e;h;h_1;h_2;\dots\dots h_m;(F_s);(H_q);(\beta_{v_m})}^{\nu;\mu;\mu_1;\lambda;\lambda_1;\lambda_2;\dots\dots\lambda_m;(E_r);(G_p);(\alpha_{u_m})} \{(x_m), y\} by\ A_n\{(x_m), y\}.$$

where n denotes the order of the polynomial set.
After little simplification (1.1) gives

$$A_n\{(x_m), y\} = \sum_{k=0}^{\left[\frac{n}{e_1}\right]} \sum_{k_1=0}^{\left[\frac{n-hk}{h_1}\right]} \sum_{k_2=0}^{\left[\frac{n-hk-h_1k_1}{h_2}\right]} \dots\dots \sum_{k_m=0}^{\left[\frac{n-hk-h_1k_1-h_2k_2\dots\dots h_{m-1}k_{m-1}}{h_m}\right]}$$

$$\times \frac{[(E_r)]_{n-hk-h_1k_1-(h_2-1)k_2-\dots\dots-(h_m-1)k_m}[(G_p)]_{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}}{[(F_s)]_{n-hk-h_1k_1-(h_2-1)k_2-\dots\dots-(h_m-1)k_m}[(H_q)]_{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}}$$

$$\times \frac{[(\alpha_{u_1})]_{k_1}[(\alpha_{u_2})]_{k_2}\ldots\ldots[(\alpha_{u_m})]_{k_m}(\mu)_k(\mu_1)_{k_1}\nu^k\lambda_1^{k_1}x_2^{h_2k_2}}{[(\beta_{v_1})]_{k_1}[(\beta_{v_2})]_{k_2}\ldots\ldots[(\beta_{v_m})]_{k_m}k!\,k_1!\,y^{h_1k_1+h_2k_2}}$$

$$\times \frac{\lambda_2^{k_2}\ldots\ldots(\lambda_m x_m^{h_m})^{k_m}(\lambda x_1^e)^{n-hk-h_1k_1-h_2k_2\ldots\ldots h_mk_m}}{k_2!\,(k_m)!\,(n-hk-h_1k_1-h_2k_2\ldots\ldots h_mk_m)!} \tag{1.2}$$

2. Notations

I. (i) $(m) = 1, 2, 3, \ldots\ldots m-1, m$.
(ii) $(a_p) = a_1, a_2, \ldots\ldots a_p$.
(iii) $(a_p; i) = a_1, a_2, \ldots\ldots a_{i-1}\, a_{i+1} \ldots\ldots a_p$.

II. (i) $[(a_p)] = a_1\, a_2 \ldots\ldots a_p$.
(ii) $[(a_p)]_n = \prod_{i=1}^{n}(a_i)_n = (a_1)_n(a_2)_n \ldots\ldots (a_p)_n$
(iii) $[(a_p)]_n = \prod_{i=1}^{n}(a_i)_n = (a_1)_n(a_2)_n \ldots\ldots (a_p)_n$

III. (i) $\Delta[a;b] = \frac{b}{a}, \frac{b+1}{a} \ldots \frac{b+a-1}{a}$
(ii) $\Delta[a(i);b] = \frac{b}{a}, \frac{b+1}{a} \ldots \frac{b+a-2}{a}$
(iii) $\Delta\{m;(a_p)\} = \left(\frac{a_i+r-1}{m}\right) \begin{matrix} r = 1,2,\ldots\ldots m \\ i = 1,2,\ldots\ldots p \end{matrix}$
(iv) $\Delta(a; b \pm c \pm d) = \Delta(a; b+c+d), \Delta(a; b+c-d), \Delta(a; b-c+d), \Delta(a; b-c-d)$

IV. (i) $\Delta_k[a;b] = \prod_{r=1}^{a}\left(\frac{a+r-1}{a}\right)_k = \left(\frac{b}{a}\right)_k \left(\frac{b+1}{a}\right)_k \ldots \left(\frac{b+a-1}{a}\right)_k$
(ii) $\Delta_k[a(i);b] = \left(\frac{b}{a}\right)_k \left(\frac{b+1}{a}\right)_k \ldots \left(\frac{b+a-z}{a}\right)_k$
(iii) $\Delta_k[m;(a_m)] = \prod_{i=1}^{p}\prod_{r=1}^{m}\left(\frac{a_i+r-1}{m}\right)_k$

V. (i) $\Gamma[(a_p)] = \prod_{i=1}^{p}(a_i)$
(ii) $\Gamma\left[a + \frac{(m)}{m}\right] = \prod_{r=1}^{m}\Gamma\left(a + \left(\frac{r}{m}\right)\right)$
(iii) $\Gamma[(a_p);(a)] = \prod_{i=a+1}^{p}\Gamma(a_i)$
(iv) $\Gamma[(a,b)] = \prod_{r=1}^{a}\Gamma\left(\frac{b+r-1}{a}\right)$
(v) $\Gamma\left[\Delta\left(m;(a_p)\right)\right]\prod_{i=1}^{p}\prod_{r=1}^{m}\Gamma\left(\frac{a_i+r-1}{m}\right)$

VI. (i) $\Gamma(a \pm b) = \Gamma(a+b)\Gamma(a-b)$
(ii) $\Gamma(a+b) = \Gamma(a+b)\Gamma(a+b)$

$$M = \frac{[(E_r)]_n[(G_p)]_n(\lambda x_1^e)^n}{[(F_s)]_n[(H_q)]_n n!}$$

3. Theorem

For $h_2 > 1, \ldots\ldots h_m > 1$

$$A_n\{(x_m),y\} = \frac{\Gamma(1+a)\Gamma(1+a-n-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)}$$

$$\times \int_0^{'} \xi^{d-l}(1-\xi)^{a-2d}\, {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \xi \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right] \times d\xi$$

$$\times F^{s+q+1\,:\,u_1;u_2;\ldots\ldots u_m}_{r+p\,:\,v_1;v_2;\ldots\ldots v_m}\left[\begin{matrix}(-n):h,h_1,h_2;\ldots\ldots h_m] \\ \overline{\qquad\qquad}, \end{matrix}\right.$$

$$[(1-(F_s)-n): h, h_1, h_2-1 \ldots h_m-1], [(1-(H_q)-n): h, h_1, h_2; \ldots\ldots h_m]$$
$$[(1-(E_r)-n): h, h_1, h_2-1 \ldots h_m-1], [(1-(G_p)-n): h, h_1, h_2; \ldots\ldots h_m]$$

$$[(\alpha_{u_1}):1],[(\alpha_{u_2}):1]\dots\dots[(\alpha_{u_m}):1][\mu:1],[\mu_1:1],[1+a-b-s-d;1],[(1+a-2d):1]$$
$$[(\beta_{v_1}):1],[(\beta_{v_2}):1]\dots\dots[(\beta_{v_m}):1],[d:1][(1+a-b-d):1],[(1+a-c-d):1]$$

$$\frac{\nu(-1)^{h(r+s+p+q+1)h}}{(\lambda x_1^e)^h},\frac{\lambda_1(-1)^{h_1(r+s+p+q+1)}}{(\lambda x_1^e y)^{h_1}},\dots\dots\frac{\lambda_2 x_2^{h_2}(-1)^{h_2(r+s+p+q+1)+r+s}}{(\lambda x_1^e y)^{h_2}}$$

$$\dots\dots\frac{\lambda_m x_m^{r_m}(-1)^{h_m(r+s+p+q+1)+r+s}}{(\lambda x_1^e)^{h_m}}\Bigg] \qquad \dots(3.1)$$

Provided that $Re(d)$, $Re(a-z-d)>-1$ and $Re(b+c+d-a)>-1$.

Proof: we have from (1.2)

$$I=\int_0^{'}\xi^{d-l}(1-\xi)^{a-2d}\ {}_4F_3\left[\begin{matrix}a,1+\frac{a}{2},b,c;\\ x\\ \frac{a}{2},1+a-b,1+a-c;\end{matrix}\right]$$

$$=\sum_{k=0}^{\left[\frac{n}{h}\right]}\sum_{k_1=0}^{\left[\frac{n-hk}{h_1}\right]}\sum_{k_2=0}^{\left[\frac{n-hk-h_1k_1}{h_2}\right]}\dots\dots\sum_{k_m=0}^{\left[\frac{n-hk-h_1k_1\dots\dots-h_{m-1}k_{m-1}}{h_m}\right]}$$

$$\times\frac{[(E_r)]_{n-hk-h_1k_1-(h_2-1)k_2-\dots\dots-(h_m-1)k_m}[(G_p)]_{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}}{[(F_s)]_{n-hk-h_1k_1-(h_2-1)k_2-\dots\dots-(h_m-1)k_m}[(H_q)]_{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}}$$

$$\times\frac{[(\alpha_{u_1})]_{k_1}[(\alpha_{u_2})]_{k_2}\dots\dots[(\alpha_{u_m})]_{k_m}(\mu)_k\nu^k\lambda_1^{k_1}\left(x_2^{h_2k_2}\right)^{k_2}}{[(\beta_{v_1})]_{k_1}[(\beta_{v_2})]_{k_2}\dots\dots[(\beta_{v_m})]_{k_m}k!\,k_1!\,y^{h_1k_1+h_2k_2}\,k_2!}$$

$$\times\frac{(\lambda x_1^e)^{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}(1-a-2d)_{2k_1}(1+a-b-c-d)_{k_1}}{(n-hk-h_1k_1-h_2k_2\dots\dots h_mk_m)!\,(d)_k(1+a-b-d)_{k_1}(1+a-c-d)_{k_1}}d\xi$$

$$=\int_0^{'}\xi^{d+k_1-l}(1-\xi)^{a-2d-2k_1}\ {}_4F_3\left[\begin{matrix}a,1+\frac{a}{2},b,c;\\ x\\ \frac{a}{2},1+a-b,1+a-c;\end{matrix}\right]$$

$$\times\sum_{k=0}^{\left[\frac{n}{h}\right]}\sum_{k_1=0}^{\left[\frac{n-hk}{h_1}\right]}\sum_{k_2=0}^{\left[\frac{n-hk-h_1k_1}{h_2}\right]}\dots\dots\sum_{k_m=0}^{\left[\frac{n-hk-h_1k_1-h_2k_2\dots\dots-h_{m-1}k_{m-1}}{h_m}\right]}$$

$$\times\frac{[(E_r)]_{n-hk-h_1k_1-(h_2-1)k_2-\dots\dots-(h_m-1)k_m}[(G_p)]_{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}}{[(F_s)]_{n-hk-h_1k_1-(h_2-1)k_2-\dots\dots-(h_m-1)k_m}[(H_q)]_{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}}$$

$$\times\frac{[(\alpha_{u_1})]_{k_1}[(\alpha_{u_2})]_{k_2}\dots\dots[(\alpha_{u_m})]_{k_m}(\mu)_k\nu^k\lambda_1^{k_1}\left(x_2^{h_2k_2}\right)^{k_2}}{[(\beta_{v_1})]_{k_1}[(\beta_{v_2})]_{k_2}\dots\dots[(\beta_{v_m})]_{k_m}k!\,k_1!\,y^{h_1k_1+h_2k_2}\,k_2!}$$

$$\times\frac{\left(\lambda_m x_m^{h_m}\right)^{k_m}(\lambda x_1^e)^{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}(1-a-2d)_{2k_1}(1+a-b-c-d)_{k_1}}{k_m!(n-hk-h_1k_1-h_2k_2\dots\dots h_mk_m)!\,(d)_{k'}(1+a-b-d)_{k_1}(1+a-c-d)_{k_1}}d\xi$$

$$\times\sum_{k=0}^{\left[\frac{n}{h}\right]}\sum_{k_1=0}^{\left[\frac{n-hk}{h_1}\right]}\sum_{k_2=0}^{\left[\frac{n-hk-h_1k_1}{h_2}\right]}\dots\dots\sum_{k_m=0}^{\left[\frac{n-hk-h_1k_1-h_2k_2\dots\dots-h_{m-1}k_{m-1}}{h_m}\right]}$$

$$\times\frac{[(E_r)]_{n-hk-h_1k_1-(h_2-1)k_2-\dots\dots-(h_m-1)k_m}[(G_p)]_{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}}{[(F_s)]_{n-hk-h_1k_1-(h_2-1)k_2-\dots\dots-(h_m-1)k_m}[(H_q)]_{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}}$$

$$\times\frac{[(\alpha_{u_1})]_{k_1}[(\alpha_{u_2})]_{k_2}\dots\dots[(\alpha_{u_m})]_{k_m}(\mu)_k\nu^k\lambda_1^{k_1}\left(x_2^{h_2k_2}\right)^{k_2}\dots\left(\lambda_m x_m^{h_m}\right)^{k_m}}{[(\beta_{v_1})]_{k_1}[(\beta_{v_2})]_{k_2}\dots\dots[(\beta_{v_m})]_{k_m}k!\,k_1!\,y^{h_1k_1+h_2k_2}\,k_2!\dots k_m!}$$

$$\times\frac{(\lambda x_1^e)^{n-hk-h_1k_1-h_2k_2-\dots\dots-h_mk_m}(1+a-2d)_{2k_1}(1+a-b-c-d)_{k_1}}{(n-hk-h_1k_1-h_2k_2\dots\dots h_mk_m)!\,(d)_{k'}(1+a-b-d)_{k_1}(1+a-c-d)_{k_1}}$$

$$\times\frac{\Gamma(d+k_1)\Gamma(1+a-2d-2k_1)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-b-c-d-k_1)}{\Gamma(1+a)\Gamma(1+a-b-c)\Gamma(1+a-b-d-k_1)}$$

$$\times \frac{\Gamma(1+a-b-c-d-k_1)}{\Gamma(1+a-d-c-k_1)}$$

$$= \frac{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)}{\Gamma(1+a)\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}$$

$$\times\ M \sum_{k,k_1,k_2\ldots k_m=0}^{\infty} \frac{[1-(F_s)-n]_{hk+h_1k_1+(h_2-1)k_2+\cdots\ldots+(h_m-1)k_m}}{[1-(E_r)-n]_{hk+h_1k_1+(h_2-1)k_2+\cdots\ldots+(h_m-1)k_m}}$$

$$\times \frac{[1-(H_q)-n]_{hk+h_1k_1+h_2k_2+\cdots\ldots+h_mk_m}[(\alpha_{u_1})]_{k_1}[(\alpha_{u_2})]_{k_2}\ldots\ldots}{[1-(G_p)-n]_{hk+h_1k_1+h_2k_2+\cdots\ldots+h_mk_m}[(\beta_{v_1})]_{k_1}[(\beta_{v_2})]_{k_2}\ldots\ldots}$$

$$\times \frac{(-n)_{n+hk+h_1k_1+h_2k_2+\cdots\ldots+h_mk_m}(-1)^{(r+s+p+q+1)(hk+h_m+\cdots+h_mk_m)s}}{(x_1^e\lambda)^{hk+h_1k_1+h_2k_2+\cdots\ldots+h_mk_m}} \quad \ldots (3.2)$$

The single terminating factor $(-n)_{n+hk+h_1k_1+h_2k_2+\cdots\ldots+h_mk_m}$ makes all summation in (3.2) runs upto □. Then we finally achieve.

$$= \frac{\Gamma(1+a-b)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)\Gamma(1+a-c)}{\Gamma(1+a)\Gamma(1-a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)} A_n\{(x_m), y\}$$

Hence the proof.
We have from [4]

$$\int_0^{'} x^{dl}\,(1-x)^{a-2d}\ {}_4F_3\begin{bmatrix} a, 1+\frac{a}{2}, b, c; \\ x \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{bmatrix} dx$$

$$= \frac{\Gamma(d)\Gamma(1+a-2d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-b-c-d)}{\Gamma(1+a)\Gamma(1-a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}$$

Provided that $Re(d)$, $Re(a-2d)>-1$, $Re\,(b+c+d-a)>-1$.

Particular Cases of (3.1)

(i) On making the substitution $r = 0 = s = p = q = u_1$; $h = h_1$ = □□= $v_1 = 1 = e = y$ = □□= □□= $□_1$; $□_1$ = 1 + □, $□_1$ = – 1 and $x_1 = \frac{1}{y}$ in (3.1), we get

$$L_n(x) = \frac{\Gamma(1+\alpha)_n\Gamma(1+a)\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{n!\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-c-d)}$$

$$\times \int_0^{'} \xi^{d-l}\,(1-\xi)^{a-2d}\ {}_4F_3\begin{bmatrix} a, 1+\frac{a}{2}, b, c; \\ \xi \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{bmatrix}$$

$$\times F\begin{bmatrix} -n, 1+a-2d, 1+a-c-d \\ y \\ 1+\alpha, d, 1+a-b-d, -1+a-c-d; \end{bmatrix} d\xi$$

where $L_n(x)$ are the Laguerre Polynomial.

(ii) On taking $r = 0 = s = p = u_1$; $q = 1 = v_1 = e$ = □□= □□= $y = h$; $A = \frac{1}{2} = \lambda_1$; $H_1 = \frac{1}{2}\beta_1$ □and $x_1 = \frac{x-1}{x+1}$ in (3.1), we get

$$T_n(x) = \left(\frac{x-1}{2}\right)^n \frac{\Gamma 1+a\ \Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{n!\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-c-d)}$$

$$\times \int_0^{'} \xi^{d-l}\,(1-\xi)^{a-2d}\ {}_4F_3\begin{bmatrix} a, 1+\frac{a}{2}, b, c; \\ \xi \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{bmatrix}$$

$$\times F\begin{bmatrix} -n, -\frac{1}{2}-n, 1+a-2d, 1+a-c-d \\ \frac{x+1}{x-1} \\ \frac{1}{2}, d, 1+a-b-d, -1+a-c-d; \end{bmatrix} d\xi$$

Where $T_n(x)$ are the Tchebicheffe Polynomials of first kind.

(iii) On setting $r = 0 = s = p = u_1$; $q = 1 = v_1 = e =$ □□= □; $h = h_1$; $H_1 = \frac{3}{2} = \beta_1$; $\lambda = \frac{1}{2} = \lambda_1$ □and $\frac{x-1}{x+1}$ for x_1 in (3.1), we get

$$U_n(x) = \frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-c-d)}$$

$$\times \frac{(n+1)!}{n!}\left(\frac{x-1}{2}\right)^n \int_0^{'} \xi^{d-l}\,(1-\xi)^{a-2d}\;{}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ \xi \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times F\left[\begin{matrix} -n, -\frac{1}{2}-n, 1+a-2d, 1+a-c-d \\ \frac{x+1}{x-1} \\ \frac{3}{2}, d, 1+a-b-d, -1+a-c-d; \end{matrix}\right] d\xi$$

where $U_n(x)$ are the Tchebicheffe Polynomials of second kind.

(iv) On putting $r = 0 = s = p = u_1$; $q = 1 = v_1 = e = y =$ □□= □$_1$ = □□= $h = h_1$; H_1 = 1+□□ =□□; □$\lambda = \frac{1}{2} = \lambda_1$ and $\frac{x+1}{x-1}$ for x_1 in (3.1), we obtain

$$P_n^{(\alpha,\alpha)}(x) = \frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-c-d)}$$

$$\times \left(\frac{x-1}{2}\right)^n \frac{(1+\alpha)_n}{n!} \int_0^{'} \xi^{d-l}\,(1-\xi)^{a-2d}\;{}_4F_3\left[\begin{matrix} 1+\frac{a}{2}a, b, c; \\ \xi \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times F\left[\begin{matrix} -n, -\alpha-n, 1+a-2d, 1+a-c-d \\ \frac{x-1}{x+1} \\ 1+\alpha, \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right] d\xi$$

(v) If we put $r = 0 = s = p = q$; □□= 1 = □$_1$ = $e = y =$ □□= $h = h_1$; □$_1$ = –1, $x_1 = \frac{1}{x}$ and instead of $(\alpha_{u_1}) = (a_p)$ and $(\beta_{v_1}) = (b_q)$ in (3.1), then we achieve

$$1F_1(-n; b; x) = \frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-c-d)}$$

$$\times \frac{1}{\Gamma(1+a-b-c-d)} \int_0^{'} \xi^{d-l}\,(1-\xi)^{a-2d}\;{}_4F_3\left[\begin{matrix} \frac{a}{2}+1, a, b, c; \\ \xi \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times F\left[\begin{matrix} -n, (a_p), 1+a-2d, 1+a-c-d \\ x \\ (b_q), d, 1+a-b-d, 1+a-c-d; \end{matrix}\right] d\xi$$

Where $1F_1(-n; b; x)$ are the Abdul-Helim and Al-Salam Polynomials [2].

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