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Finite double integral representation for the classical polynomial Set $J_n\{(x_m), y\}$ of several variables

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Abstract
In the present paper, an attempt has been made to express a Finite Double Integral Representation for the Classical polynomial set $Jn\{(xm), y\}$ of Several Variables. Many interesting new results may be obtained as particular cases out of these particular results some of them stand for well-known and some of them are believed to be new. These are of at most importance for mathematician's scientists, engineers and physical sciences because these occur in the solution of differential equations, Integral equations, etc.
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Keywords: Hypergeometric function, generating function, integral representation

1. Introduction

We have derived a generalized hypergeometric polynomial set $J_n\{(x_m), y\}$ of several variables by means generating relation. An infinitely differentiable function f is said to be completely monotonic on an interval I if it satisfies $(-1)^k f^{(k)}(x) \geq 0$ on I for all $k \geq 0$ [3]. Evaluation of two-dimensional Gaussian integrals in the constructions of representation theory and related topics of differential geometry and analysis [4]. In the present paper. we obtain a finite integral involving general class of polynomials and I-function. Next with the application of this and the lamma due to Srivastava *et al.* [5]

$$(\xi + \lambda y^{-e} t^{e})^{-\sigma} F\left[\begin{matrix}\lambda_1; (a_g); \\ (b_h);\end{matrix} \mu_1 y^{-e_1} t^{e_1}\right]$$

$$\times F\left[\begin{matrix}(A_r); (C_p); (\alpha_{u_k}) \\ (B_s); (D_q); (\beta_{v_k})\end{matrix} \mu x_1^d t, \mu_2 x_2^{e_2} y^{-e_2} t^{e_2} \ldots\ldots \mu_k x_k^{e_k} t^{e_k}\right]$$

$$= \sum_{n=0}^{\infty} J_{n,e,e_1;e_2;e_3;\ldots\ldots e_m;(b_k);(B_s);(D_q);(\beta_{v_k})}^{\xi;\lambda;\lambda_1;\sigma;d;\mu;\mu_1;\mu_2\ldots\ldots\mu_m;(a_g);(A_r);(C_p);(\alpha_{u_k})} \{(x_m), y\} \qquad \ldots (1.1)$$

Where ξ, λ, λ_1, σ, δ; μ, μ_1, μ_2 …… μ_m are real and *e*, e_1, are non-negative integers and e_2; …… e_m are natural number.

The left hand side of (1.2) are the product of generalized hypergeometric function and Lauricella function in the notation of Burchnall and Chaundy [1]. The polynomial set contain number of parameters. For simplicity we shall denote

$$J_{n,e,e_1;e_2;e_3;\ldots\ldots e_m;(b_k);(B_s);(D_q);(\beta_{v_k})}^{\xi;\lambda;\lambda_1;\sigma;d;\mu;\mu_1;\mu_2\ldots\ldots\mu_m;(a_g);(A_r);(C_p);(\alpha_{u_k})} \{(x_m), y\} = J_n\{(x_m), y\}$$

Where *n* denotes the order of the generalized polynomial set. After little simplification (1.1) give

$$J_n\{(x_m),y\} = \xi^{\sigma}\sum_{m=0}^{\left[\frac{n}{e}\right]}\sum_{m_1=0}^{\left[\frac{n-em}{e_1}\right]}\sum_{m_2=0}^{\left[\frac{n-em-e_1m_1}{e_2}\right]}\ldots\ldots\sum_{m_k=0}^{\left[\frac{n-em-e_1m_1-e_2m_2\ldots\ldots e_{k-1}e_{k-1}}{e_k}\right]}$$

$$\times\frac{[(A_r)]_{n-em-e_1m_1-(e_2-1)m_2-\cdots\ldots-(e_k-1)m_k}[(C_p)]_{n-ek-e_1k_1-e_2k_2-\cdots\ldots-e_km_k}}{[(B_s)]_{n-em-e_1m_1-(e_2-1)m_2-\cdots\ldots-(e_k-1)m_k}[(D_q)]_{n-ek-e_1k_1-e_2k_2-\cdots\ldots-e_km_k}}$$

$$\times\frac{\left[\left(\alpha_{u_k}\right)\right]_{m_k}[(a_g)]_{m_1}(\sigma)_m(-\lambda)^m(-\lambda_1)_{m_1}\mu^{m_1}\left(\mu_2x_2^{e_2}\right)^{m_2}}{\left[\left(\beta_{v_k}\right)\right]_{m_k}[(b_h)]_{m_1}\xi^m\, m!\, m_1!\, y^{em+e_1m_1+e_2m_2}\, m_2!}$$

$$\times\frac{\left(\mu_mx_m^{e_k}\right)^{m_k}\left(\lambda x_1^{e}\right)^{n,em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k}}{m_k!\,(n,em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k)!} \qquad \ldots (1.2)$$

2. Notations

I. (i) $(n) = 1, 2, \ldots\ldots n-1, n$.

(ii) $(a_p) = a_1, a_2, a_3, \ldots\ldots a_p$.

(iii) $(a_p; i) = a_1, a_2, a_3, \ldots\ldots a_{i-1}, a_{i+1} \ldots\ldots a_p$.

II. (i) $[(a_p)] = a_1, a_2, a_3 \ldots\ldots a_p$.

(ii) $\left[(a_p)\right]_n = \prod_{i=1}^{p}(a_i)_n = (a_1)_n(a_2)_n(a_3)_n \ldots \left(a_p\right)_n$

III. (i) $\Delta(a;b) = \frac{b}{a}, \frac{b+1}{a} + \cdots \ldots \frac{b+a-1}{a}$.

(ii) $\Delta(a(1);b) = \frac{b}{a}, \frac{b+1}{a}, \frac{b+2}{a}, \ldots\ldots \frac{b+a-2}{a}$

(iv) $\Delta(a; b \pm c \pm d) = \Delta(a; b+c+d), \Delta(a; b+c-d)$,
$\Delta(a; b-c+d), \Delta(a; b-c-d)$,

IV. (i) $\Delta_k[a;b] = \prod_{r=1}^{a}\left(\frac{b+r-1}{a}\right)_k = \left(\frac{b}{a}\right)_k\left(\frac{b+1}{a}\right)_k \ldots \left(\frac{b+a-1}{a}\right)_k$

(ii) $\Delta_k[a(1);b] = \left(\frac{b}{a}\right)_k\left(\frac{b+1}{a}\right)_k \ldots \left(\frac{b+a-2}{a}\right)_k$

(iii) $\Delta_k\left[m;\left(a_p\right)\right] = \prod_{i=1}^{b}\prod_{r=1}^{m}\left(\frac{a_i+r-1}{m}\right)_k$

V. (i) $\Gamma\left[\left(a_p\right)\right] = \prod_{i=1}^{p}(a_i)$

(ii) $\Gamma\left[\left(a_p\right);(s)\right] = \prod_{i=s+1}^{p}(a_i)$

(iii) $\Gamma[(a;b)] = \prod_{r=1}^{a}\Gamma\left(\frac{b+r-1}{a}\right)$

(iv) $\Gamma\left[\Delta(m);\left(a_p\right)\right] = \prod_{i=1}^{p}\prod_{r=1}^{m}\Gamma\left(\frac{(a_i)+r-1}{m}\right)$

VI. (i) $\Gamma(a \pm b) = \Gamma(a+b)\Gamma(a-b)$

(ii) $\Gamma_{**}(a+b) = \Gamma(a+b)\Gamma(a+b)$

$$M = \frac{[(A_r)]_n\left[(C_p)\right]_n\left(\lambda x_1^{d}\right)^n}{[(B_s)]_n\left[(D_q)\right]_n n!}$$

3. Theorem

For $e_2 > 1, \ldots\ldots, e_k > 1$, we have

$$J_n\{(x_m),y\} = M\frac{\Gamma(\mu+\nu)}{\Gamma(\mu)\Gamma(\nu)}\int_0^{'}\int_0^{'}\frac{(1-\xi)^{\mu-1}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}}$$

$$\times F_{s+q+h+u:1:\beta_{v_1},\beta_{v_2}\ldots\ldots\beta_{v_k}}^{r+p+g+v+1:1:1:\alpha_{u_1};\alpha_{u_2}\ldots\ldots\alpha_{u_k}}\left[\begin{matrix}[(-n):e,e_1,e_2;\ldots\ldots e_k]\\ \rule{3cm}{0.4pt},\end{matrix}\right.$$

$$[(1-(B_s)-n): e, e_1, e_2-1 \ldots e_k-1], \left[(1-(D_q)-n): e, e_1, e_2; \ldots\ldots e_k\right]$$
$$[(1-(A_r)-n): e, h_1, e_2-1 \ldots e_k-1], \left[(1-(C_p)-n): e, e_1, e_2; \ldots\ldots e_k\right]$$

$$\left[\begin{matrix}\left[(\alpha_{u_1}):1\right],\left[(\alpha_{u_2}):1\right]\ldots\ldots\left[(\alpha_{u_k}):1\right]\left[(a_g):1\right][\sigma:1][\lambda_1:1]\left[\left(\frac{\mu+\nu}{2}\right):1\right]\left[\left(\frac{\mu+\nu+1}{2}\right):1\right]\\ \left[(\beta_{v_1}):1\right],\left[(\beta_{v_2}):1\right]\ldots\ldots\left[(\beta_{v_k}):1\right],[(b_h):1][(\mu):1][(\mu):1]\end{matrix}\right.$$

$$\frac{-\lambda(-1)^{e(r+s+p+q+g+h+u+v+1)}}{\xi^e(\mu x_1^d y)^e}, \frac{\mu_1(-1)^{e_1(r+s+p+q+g+h+u+v+1)}}{(\mu x_1^d y)^{e_1}},$$

$$\frac{\mu_2 x_2^{e_2}(-1)^{e_2(r+s+p+q+g+h+u+v+1)+r+s}}{(\mu x_1^d y)^{e_2}}, \ldots\ldots \frac{\mu_k x_k^{e_k}(-1)^{e_k(r+s+p+q+g+h+u+v+1)+r+s}}{(\mu x_1^d)^{e_k}}\Bigg] d\xi d\eta \qquad \ldots (3.1)$$

Proof: we have

$$I_2 = \int_0' \int_0' \frac{(1-\xi)^{\mu-1}\eta^{\mu}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}} \sum_{m=0}^{\left[\frac{n}{e}\right]} \sum_{m_1=0}^{\left[\frac{n-em}{e_1}\right]} \sum_{m_2=0}^{\left[\frac{n-em-e_1m_1}{e_2}\right]}$$

$$\times \sum_{m_k=0}^{\left[\frac{n-em-e_1m_1\ldots\ldots-e_{k-1}m_{k-1}}{e_k}\right]} \frac{[(A_r)]_{n-em-e_1m_1-(e_2-1)m_2-\cdots\ldots-(e_k-1)m_k}}{[(B_s)]_{n-em-e_1m_1-(e_2-1)m_2-\cdots\ldots-(e_k-1)m_k}}$$

$$\times \frac{[(C_p)]_{n-em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k}\left[\left(\alpha_{u_k}\right)\right]_{m_k}[(a_g)]_{m_1}}{[(D_q)]_{n-em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k}\left[\left(\beta_{v_k}\right)\right]_{m_k}[(b_h)]_{m_1}}$$

$$\times \frac{(\sigma)_m(-\lambda)^m(\lambda_1)_{m_1}\mu_1^{m_1}\left(\mu_2 x_2^{e_2}\right)^{m_2}\ldots\left(\mu_k x_k^{e_k}\right)^{m_k}}{\xi^m\, m!\, m_1!\, y^{em+e_1m_1+e_2m_2}\, m_2!\ldots(m_k)!}$$

$$\times \frac{(\mu x_1^d)^{n-em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k}\left(\frac{\mu+\nu}{2}\right)_{m_1}\left(\frac{\mu+\nu+}{2}\right)_{m_1}}{(n-em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k)!\,(\mu)_{m_1}(\nu)_{m_1}}$$

$$= \sum_{m=0}^{\left[\frac{n}{e}\right]} \sum_{m_1=0}^{\left[\frac{n-em}{e_1}\right]} \sum_{m_2=0}^{\left[\frac{n-em-e_1m_1}{e_2}\right]} \ldots\ldots \sum_{m_k=0}^{\left[\frac{n-em-e_1m_1\ldots\ldots-e_{k-1}m_{k-1}}{e_k}\right]}$$

$$\times \frac{[(A_r)]_{n-em-e_1m_1-(e_2-1)m_2-\cdots\ldots-(e_k-1)m_k}[(C_p)]_{n-em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k}}{[(B_s)]_{n-em-e_1m_1-(e_2-1)m_2-\cdots\ldots-(e_k-1)m_k}[(D_q)]_{n-em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k}}$$

$$\times \frac{\left[\left(\alpha_{u_k}\right)\right]_{m_k}[(a_g)]_{m_1}(\sigma)_m(-\lambda)^m(\lambda_1)_{m_1}\mu_1^{m_1}\left(\mu_2 x_2^{e_2}\right)^{m_2}}{\left[\left(\beta_{v_k}\right)\right]_{m_k}[(b_h)]_{m_1}\,\xi^m\, m!\, m_1!\, y^{em+e_1m_1+e_2m_2}\, m_2!}$$

$$\times \frac{\left(\mu_k x_k^{e_k}\right)^{m_k}(\mu x_1^d)^{n-em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k}\left(\frac{\mu+\nu}{2}\right)_{m_1}\left(\frac{\mu+\nu+}{2}\right)_{m_1}}{m_k!\,(n-em-e_1m_1-e_2m_2-\cdots\ldots-e_km_k)!\,(\mu)_{m_1}(\nu)_{m_1}}$$

$$\times \int_0' \int_0' \frac{(1-\xi)^{\mu+m_1-1}\eta^{\mu+m_1}(1-\eta)^{\nu+m_1-1}}{(1-\xi\eta)^{\mu+\nu-1+2m_1}} d\xi dn.$$

$$= M\frac{\Gamma(\mu)\Gamma(\nu)}{\Gamma(\mu+\nu)} \sum_{m,m_1,m_2\ldots m_k}^{\infty} \frac{[1-(B_s)-n]_{em+e_1m_1+(e_2-1)m_2+\cdots\ldots+(e_k-1)m_k}}{[1-(A_r)-n]_{em+e_1m_1+(e_2-1)m_2+\cdots\ldots+(e_k-1)m_k}}$$

$$\times \frac{[1-(D_q)-n]_{em+e_1m_1+e_2m_2+\cdots\ldots+e_km_k}\left[\left(\alpha_{u_k}\right)\right]_{m_k}[(a_g)]_{m_1}}{[1-(C_p)-n]_{em+e_1m_1+e_2m_2+\cdots\ldots+e_km_k}\left[\left(\beta_{v_k}\right)\right]_{m_k}[(b_h)]_{m_1}}$$

$$\times \frac{(\sigma)_m(\lambda_1)_{m_1}(-\lambda)^m(-1)^{e(r+s+p+q+g+h+u+v+1)m}}{m!\,\xi^m(\mu x_1^d y)^{em}}$$

$$\times \frac{\mu_1(-1)^{e_1(r+s+p+q+g+h+u+v+1)m_1}\left(\mu_2 x_2^{e_2}\right)^{m_2}(-1)^{\{e_2(r+s+p+q+g+h+u+v+1)r+s\}m_2}}{m_1!\,(\mu x_1^d y)^{e_1m_1}\, m_2!\,(\mu x_1^d y)^{e_2m_2}}$$

$$\times \frac{\left(\mu_k x_k^{e_k}\right)^{m_k}(-1)^{\{e_k(r+s+p+q+g+h+u+v+1)r+s\}m_2}}{m_k!\,(\lambda x_1^d)^{e_km_k}}$$

$$\times (-n)_{em+e_1m_1+e_2m_2+\cdots\ldots+e_km_k} \qquad \ldots (3.2)$$

The single terminating factor $(-n)_{em+e_1m_1+e_2m_2+\cdots\ldots+e_km_k}$ makes all summation in (3.2) runs upto ∞. Then we finally achieve.

$$I_2 = \frac{\Gamma(\mu)\Gamma(\nu)}{\Gamma(\mu+\nu)} J_n\{(x_m), y\}.$$

Hence, the theorem
We have from [2].

$$\int_0^{'}\int_0^{'} \frac{(1-\xi)^{\mu-1}\eta^{\mu}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}}d\xi dn = \beta(\mu,\nu)$$

Particular Cases of (3.1)

(i) On making the substitution $p = 0 = q = r = s = g = u_k = v_k$; $m = m_1 = h = 0 = \lambda_1 = -\lambda = \mu_1 = y = \mu$; $b_1 = 1 + \alpha$, and $x_1 = \frac{1}{y}$, in (3.1) we get

$$L_n^{(\alpha)}(y) = \frac{\Gamma(\mu+\nu)(1+\alpha)_n}{\Gamma(\mu)\,\Gamma(\nu)\,y^n\,n!}\int_0^{'}\int_0^{'}\frac{(1-\xi)^{\mu-1}\eta^{\mu}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}}$$

$$\times F\begin{bmatrix} -n, \frac{\mu+\nu}{2}, \frac{\mu+\nu-1}{2}; \\ y \\ 1+\alpha, \mu, \nu; \end{bmatrix} d\xi d\eta$$

Where $L_n^{(\alpha)}(y)$ are the laguerre polynomials

(ii) On taking $r = 0 = s = p = g = u_k = v_k$; $q = h = y = d = e = e_1 = v_1 = \mu_1 = \sigma = \lambda_1 = -\lambda$; $\mu = \frac{1}{2}$; $= D_1 = \frac{1}{2} = b_1$; and putting $x_1 = \frac{x-1}{x+1}$, we arrive at

$$T_n(x) = \frac{\Gamma(\mu+\nu)}{n!\,\Gamma(\mu)\,\Gamma(\nu)}\left(\frac{x-1}{2}\right)^n \int_0^{'}\int_0^{'}\frac{(1-\xi)^{\mu-1}\eta^{\mu}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}}$$

$$\times F\begin{bmatrix} -n, \frac{1}{2}-n, \frac{\mu+\nu}{2}, \frac{\mu+\nu-1}{2}; \\ \frac{x+1}{x-1} \\ \frac{1}{2}, \mu, \nu; \end{bmatrix} d\xi d\eta$$

Where, $T_n(x)$are the Tchebicheffe Polynomials of First Kind

(iii) If we take $r = 0 = s = p = g = u_k = v_k$; $q = 1 = h = d = e = e_1 = \xi = \sigma = \lambda_1 = -\lambda = y$; $\mu_1 = \frac{1}{2} = \mu$; $= D_1 = \frac{3}{2} = b_1$; and writing $\frac{x-1}{x+1}$, for x, we achieve

$$U_n(x) = \frac{\Gamma(\mu+\nu)(n+1)!}{\Gamma(\mu)\,\Gamma(\nu)n!}\left(\frac{x-1}{2}\right)^n \int_0^{'}\int_0^{'}\frac{(1-\xi)^{\mu-1}\eta^{\mu}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}}$$

$$\times F\begin{bmatrix} -n, \frac{-1}{2}-n, \frac{\mu+\nu}{2}, \frac{\mu+\nu-1}{2}; \\ \frac{x+1}{x-1} \\ \frac{3}{2}, \mu, \nu; \end{bmatrix} d\xi d\eta$$

Where $U_n(x)$ are the Tchebicheffe Polynomials of Second Kind.

(iv) If we set $r = 0 = s = p = g = \alpha_{u_k} = \beta_{v_k}$; $q = 1 = h = y = \xi = d = e = e_1 = -\lambda = \sigma = \lambda_1$; $\mu = \frac{1}{2} = \mu_1$; $D_1 = 1 + \alpha = b$; and writing $\frac{x+1}{x-1}$ for x, we obtain

$$P_n^{(\alpha,\alpha)}(x) = \frac{\Gamma(\mu+\nu)(1+\alpha)}{\Gamma\mu\,\Gamma\nu\,n!}\left(\frac{x+1}{2}\right)^n \int_0^{'}\int_0^{'}\frac{(1-\xi)^{\mu-1}\eta^{\mu}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}}$$

$$\times F\begin{bmatrix} -n, -\alpha-n, \frac{\mu+\nu}{2}, \frac{\mu+\nu-1}{2}; \\ \frac{x-1}{x+1} \\ 1+\alpha, \mu, \nu; \end{bmatrix} d\xi d\eta$$

Where $P_n^{(\alpha,\alpha)}(x)$ are the Ultrashpherical Polynomials

(v) If we setting $r = 0 = s = p = g = \alpha_{u_k} = \beta_{v_k}$; $q = 1 = h = e = e_1 = d = \xi = y = -\lambda = \sigma = \lambda_1$; $\mu = \frac{1}{2} = \mu_1$; $D_1 = 1 + \beta$; and $b_1 = 1 + \alpha$; $x_1 = \frac{X+1}{X-1}$, we get

$$P_n^{(\alpha,\beta)}(x) = \frac{\Gamma(\mu+\nu)(1+\alpha)_n}{\Gamma(\mu)\,\Gamma(\nu)\,n!}\left(\frac{x+1}{2}\right)^n \int_0^{'}\int_0^{'}\frac{(1-\xi)^{\mu-1}\eta^{\mu}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}}$$

$$\times F\begin{bmatrix} -n, -\beta - n, \frac{\mu+\nu}{2}, \frac{\mu+\nu-1}{2}; \\ \left(\frac{x+1}{x-1}\right) \\ 1+\alpha, \mu, \nu; \end{bmatrix} d\xi d\eta$$

Where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials

(vi) For $r = 0 = s = p = g =$; $q = 1 = h = e = e_1 = d = y = \sigma = -\lambda = \lambda_1 = y$; $\mu = \frac{1}{2} = \mu_1$; $D_1 = \frac{1}{2} + \lambda = b_1$ and writting $\frac{x+1}{x-1}$ for x_1, we obtain

$$C_n^{\lambda}(x) = \frac{\Gamma(\mu+\nu)(2\lambda)_n}{\Gamma(\mu)\,\Gamma(\nu)\,n!}\left(\frac{x+1}{2}\right)^n \int_0^{\prime}\int_0^{\prime} \frac{(1-\xi)^{\mu-1}\eta^{\mu}(1-\eta)^{\nu-1}}{(1-\xi\eta)^{\mu+\nu-1}}$$

$$\times F\begin{bmatrix} -n, \frac{1}{2} - \lambda - n, \frac{\mu+\nu}{2}, \frac{\mu+\nu-1}{2}; \\ \frac{x-1}{x+1} \\ \lambda + \frac{1}{2}, \mu, \nu; \end{bmatrix} d\xi d\eta$$

Where $C_n^{\lambda}(x)$ are the Gegenbauer Polynomials

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