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On minimal and maximal iβ-continuous function

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Abstract
The present paper discusses new class of sets depended on the concept of iβ-open set. The importance concepts which introduced in this paper are minima iβ-open and maximal iβ-open sets. Besides, new types of topological spaces introduced which called $Ti\beta max$ and $Ti\beta min$ spaces. Also, we present new two functions of continuity which called minimal iβ-continuous and maximal iβ-continuous.

Keywords: M_{in}iβ-open set, M_{ax}iβ-open set, M_{in}iβ-continuous and M_{ax}iβ-continuous

1. Introduction

Minimal and maximal sets play an important role in the researches of generalized topological spaces, Nakaoka and Oda introduced these concepts in [2, 6, 3] and they used them to investigate many topological properties. In this paper we introduced the notion of minimal iβ-open and maximal iβ-open and their complements. In [2] A proper nonempty open(closed) subset L of a topological space X is said to be minimal open (closed) set if any open set which is contained in L is φ or L. A proper nonempty open subset L of a topological space X is said to be maximal open(closed) set if any open(closed) set which is contains L is L or X. [4, 3]. In [5] Let A a subset of a topological space X, then the union of all open subset of X which contained in A is called the interior of A and denoted by int(A) and the intersection of all closed subset of X which contain A is called the closure of A and denoted by $cl(A)$, A subset A of a space X is called a β-open set if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$. The complement of a β-open set is defined to be a β-closed set [1].

2. iβ-continuous iβ-open set

Definition (2.1): A subset A of a topological space X is called iβ-open set if there exists β-open set L ($L \neq$X,φ) such that $A \subseteq cl(A \cap L)$.

Definition (2.2): Let X and Y be topological spaces and $f: X \to Y$ is a function then f is called a iβ-continuous function if $f^{-1}(A)$ is a iβ-open set in X for every open set A in Y Minimal and Maximal iβ -open sets.

Definition (2.3): A proper iβ-open subset B of a topological space X is said to be a minimal iβ-open set if any iβ-open set which is contained in B is φ or B.

Definition (2.4): A proper nonempty iβ-open subset Bof a topological space X is said to be a maximal iβ-open set if any iβ-open set which contains B is X or B.

Definition (2.5): A proper nonempty iβ-closed subset Fof a topological space X is said to be aminimal iβ-closed set if any iβ-closed set which is contained in F is φ or F.

Definition (2.6): A proper nonempty iβ-closed subset F of a topological space X is said to be a maximal iβ-closed set if any iβ-closed set which contains F is X or F.

Remarks (2.7)
1. The family of all minimal iβ-open (resp. minimal iβ-closed) sets of a topological space X

is denoted by MiniβO(X)(resp. MiniβC(X)).

2. The family of all maximal iβ-open (resp. maximal iβ-closed) sets of a topological spacX is denoted by MaxiβO(X)(resp. MaxiβC(X)).

Remark (2.8): The concept of M_{in}iβ-open, M_{ax}iβ-open, $M_{in}i\beta$-closed and Maxiβ-closed are independent of each other as in the following example.

Example (2.9): Let X = {p, q, r} and $J = \{\varphi, \{p\}, \{p,q\}, X\}$ so $i\beta O(X) = \{\varphi, \{p\}, \{p,q\}, \{p,r\}, X\}, M_{in}i\beta O(X) = \{\{p\}, \{r\}\}, M_{ax}i\beta(X) = \{\{p,r\}, \{p,q\}\}, M_{in}i\beta C(X) = \{\{q\}, \{r\}\}, M_{ax}i\beta C(X) = \{\{p\}, \{q,r\}\}$ ،So the new sets are independent each other.

Theorem (2.10): Let F be a subset of a topological space X, then F is a M_{in} $i\beta$-closed if and only if X-F is $M_{ax}i\beta$-open set.
Proof: ⟹ F be a $M_{in}i\beta$-closed, so X-F is $i\beta$-open. We have to show that X-F is $M_{ax}i\beta$-open suppose not, so there is a $i\beta$-open subset D of X such that $X - F \subset D$ hence $X - D \subset F$ and this contradicted being F is M_{in} $i\beta$-closed.

⟸ let F be a $i\beta$-closed subset of X, suppose that there is a $i\beta$-closed $K \neq \varphi$ such that $K \subset F$ thus $X - F \subset X - K$ but $X - K$ is prope $i\beta$-open set. Contradictedion to the assumption of being X-F is M_{ax} $i\beta$-open.

Theorem (2.11): Let U and V be M_{ax} $i\beta$-open subsets of a topological space X, then $U \cup V$ =X or U=V.
Proof: If $U \cup V = X$ then the proof is complete. If not, i.e. $U \cup V \neq X$ so we have to show that U=V. Since $U \cup V \neq X$ so $U \subset U \cup V$ and $V \subset U \cup V$. But U is $M_{ax}i\beta$-open set, so $U \cup V$ = X or $U \cup V$ =U. Thus $U \cup V$ =U and so $V \subset U$. Now since $V \subset U \cup V$ and V is maximal f-open set, so $U \cup V$ =X or $U \cup V$ =V, but $U \cup V \neq X$ so $U \cup V$ =V and hence $U \subset V$ therefore U=V.

Theorem (2.12): Let U be a M_{ax} $i\beta$-open and V be a $i\beta$-open subsets of a topological space X then $U \cup V$ =X or V=U.
Proof: If $U \cup V$ =X then the proof is complete. If $U \cup V \neq X$, U⊂ $U \cup V$ and V⊂ $U \cup V$. Since U isM_{ax} $i\beta$-open an U⊂ $U \cup V$ so by definition of $M_{ax}i\beta$-open we have that $U \cup V$ =X or $U \cup V$ =U, but $U \cup V \neq$ X so $U \cup V$=U and hence V⊂U.U.

Theorem (2.13): Let U be a $M_{ax}i\beta$-open subset of a Topological space X with $x \in X\backslash U$ then $X\backslash U \subset V$ for any $i\beta$-open subset of X with $x \in V$.
Proof: Let $x \in X\backslash U$ and $x \in V$, so $V \not\subset U$, thus by (2.12) we have that $U \cup V = X \Longrightarrow (X\backslash U) \cap (X\backslash U)= \varphi \Longrightarrow X\backslash U \subset V$.

Theorem (2.14) Let F be a M_{in} $i\beta$-closed and K be a $i\beta$-closed subsets of a topological space X then F∩K = φ or F⊂K.

Proof: Let F be a M_{in}iβ-closed and V is a β- closed. If F∩V= ∅. The proof is complete. If F∩V≠∅, then we have to prove F⊆ V. Since F∩V⊆ F and F is a M_{in}iβ- closed and F∩V is a β- closed. Then F∩V=F or F∩V=∅. since F∩V≠∅, then F∩V=F, since $F \subseteq V$.

Theorem (2.15): Let F and V be a $M_{in}i\beta$-closed subsets of a topological space X. Then F∩V=∅ or F=V.

Proof: Let F and V be a M_{in}-iβ-closed. If F∩V= ∅. The proof is complete. If F∩V≠∅, then we have to prove F=V. Then F∩V⊆F and F∩V =V. Since F∩V⊆F and F is a M_{in}iβ-closed, Then F∩V = F or F∩V = ∅. since F∩V≠∅, Then F∩V=F, hence F⊆V. If F∩V⊆V and V is a M_{in}iβ- closed, Then F∩V=V or F∩V=∅. since F∩V≠∅, Then F∩V=V, hence V ⊆F. Therefore F=V.

Theorem (2.16): Let U, V and W be $M_{ax}i\beta$-open subsets of a topological space X such that $U \neq V$, if U∩V⊆W, then either U=W or V=W.

Proof: Suppose that U∩V⊆ W, if U =W then the proof is complete. If U≠W we have to show that V=W
$V \cap W = V \cap (X \cap W) = V \cap [W \cap (U \cup V)]$ by(2.9), $V \cap [(W \cap U) \cup (W \cap V)] = (V \cap W \cap U) \cup (V \cap W \cap V) = (U \cap V) \cup (V \cap W)$, Since $U \cap V \subseteq W = (U \cup W) \cap V = X \cap V$, since $U \cup W = X$ thus. $(V \cap W = V) = V$. implies V⊆W but V is $M_{ax}i\beta$-open, therefore V=W or $U \cup W = X$ but $U \cup W \neq X$ then V=W.

Theorem (2.17): Let U, V and W be M_{ax} $i\beta$-open subsets of a topological space X which are different from each other, then $U \cap V \nsubseteq U \cap W$.

Proof: Let $U \cap V \subseteq U \cap W \Longrightarrow (U \cap V) \cup (W \cap V) \subseteq (U \cap W) \cup (W \cap V) \Longrightarrow (U \cap W) \cup V \subseteq (U \cap V) \cup W \Longrightarrow X \cup V \subseteq X \cup W \Longrightarrow V \subseteq W$, But V is $M_{ax}i\beta$-open and W is proper subset of X so V=U, this result contradicteds the fact that U,V and W are different from each other. Hence $U \cap V \nsubseteq U \cap W$.

Theorem (2.18): Let F be a $M_{in}i\beta$-closed subset of a topological space X, if $x \in F$ then $F \subseteq K$ for any $i\beta$-closed subset K of X containing x.

Proof: Suppose $x \in K$and $F \nsubseteq K$ so $F \cap K \subseteq F$ and $F \cap K \neq \varphi$since $x \in F \cap K$, But F is $M_{in}i\beta - closed$ so $F \cap K = F. or$ $F \cap K = \varphi$. hence $F \cap K = F$ which contract the relation $F \cap K \subseteq F$. Therefore $F \subseteq K$.

Theorem (2.19): Let F and $F_\alpha(\alpha \in A)$ be $M_{in}i\beta$-closed sets if $F \subseteq \cup_{\alpha \in A} F_\alpha$ then there exists $\alpha_0 \in A$ such that $F = F_{\alpha_0}$.

Proof: First we have to show that $F \cap F_{\alpha_0} \neq \varphi$ suppose that $F \cap F_{\alpha_0} = \varphi$ then $F_{\alpha_0} \subseteq X\backslash F$ and $F \subseteq \cup_{\alpha \in A} F_\alpha \subseteq X\backslash F$ which is a contradicted ion. So $F \cap F_{\alpha_0} \neq \varphi$ and hence $F \cap F_{\alpha_0} \subseteq F$ and $F \cap F_{\alpha_0} \subseteq F_{\alpha_0}$ since $F \cap F_{\alpha_0} \subseteq F$ and F is M_{in}-$i\beta$-closed then $F \cap F_{\alpha_0} = F$ or $F \cap F_{\alpha_0} = \varphi$ thus $F \cap F_{\alpha_0} = F$ and hence $F_{\alpha_0} \subseteq F$. Now since $F \cap F_{\alpha_0} \subseteq F_{\alpha_0}$ and $F_{\alpha_0}$is M_{in}-$i\beta$-closed then $F \cap F_{\alpha_0} \subseteq F_{\alpha_0}$or $F \cap F_{\alpha_0} = \varphi$, Thus $F \cap F_{\alpha_0} \subseteq F_{\alpha_0}$and hence $F \subseteq F_{\alpha_0}$. Therefore $F = F_{\alpha_0}$.

3. Minima iβ-open ($T_{i\beta min}$) and Maximal iβ-open ($T_{i\beta max}$) space

Definition (3.1): A topological space X is said to be $T_{i\beta min}$ space if every nonempty proper $i\beta$-open subset of X is $M_{in}i\beta$-open set.

Definition (3.2): A topological space X is said to be $T_{i\beta max}$ space if every nonempty proper $i\beta$-open subset of X is $M_{ax}i\beta$-open set.

Example (3.3): Let X={a, b} and J = {φ,{a},X} thus

βO(X)= $i\beta$O(X) = J, it is clear that {a} is maximal and minimal $i\beta$-open sets thus the space X is both $T_{i\beta min}$ and $T_{i\beta max}$.

Remark (3.4): $T_{i\beta min}$ and $T_{i\beta max}$ spaces are identical.

Theorem (3.5): A space X is $T_{i\beta min}$ if and only if it is $T_{i\beta max}$.

Proof: ⟹Let X is $T_{i\beta min}$ space. Suppose that X is not $T_{i\beta max}$, so there is a proper $i\beta$ -open subset K of X which is not maximal. This mean there exist a $i\beta$-open subset of X with $K \subseteq H \neq \varphi$, Thus we get that H is not minimal which is contradicted of being X is $T_{i\beta max}$.
Let⟹ X is $T_{i\beta max}$ space. Suppose that X is not $T_{i\beta min}$, so there is a proper $i\beta$ -open subset K of X which is not minimal. This mean there exist an $i\beta$-open subset of X with $\varphi \neq H \subseteq K$, Thus we get that H is not maximal which is contradicted of being X is $T_{i\beta max}$.

Theorem (3.6): A topological space X is $T_{i\beta min}$ space if and only if every nonempty proper $i\beta$-closed subset of X is M_{ax} $i\beta$-closed set in X.

Proof: ⟹ Let F be proper $i\beta$-closed subset of X and suppose F is not maximal. So there an $i\beta$-closed subset Kof X with $K \neq X$ such that $F \subseteq K$. Thus $X - K \subseteq X - F$. Hence $X - F$ is a proper $i\beta$-open which is not minimal and this contradicted of being X is $T_{i\beta min}$ space.
Suppose ⟹ U is proper X-U is a proper $i\beta$-open subset of X, thus X-U is proper $i\beta$-closed subset of X. so X-U is $M_{ax}i\beta$-closed subset of X. and by (2.10) U is $M_{in}i\beta$-open. thus X is $T_{i\beta min}$ space.

Theorem (3.7): A topological space X is $T_{i\beta max}$ space if and only if every nonempty proper $i\beta$-closed subset of X is M_{in} $i\beta$-closed set in X.

Proof: ⟹ Let F be proper $i\beta$-closed subset of X, and suppose F is not $M_{in}i\beta$-closed in X, So there is a proper $i\beta$-closed subset of X *such that* $K \subseteq F$. Thus $X - F \subseteq X - K$. But $X - F$ is a proper $i\beta$-open in X so $X - F$ is not maximal in X. Contradictedion to the fact $X - F$ is $M_{ax}i\beta$-open.
⟹ let U be a proper $i\beta$-open subset of X, thus X-U is proper $i\beta$-closed subset of X. so X-U. is $M_{ax}i\beta$-closed set. by (2.10) we get that U is $M_{ax}i\beta$-open.

Theorem (3.8): Every pair of different M_{in} $i\beta$-open sets of $T_{i\beta min}$ are disjoint.
Proof: Let U and V be minimal f-open subsets of $T_{i\beta min}$space X such that $U \neq V$ to show $U \cap V = \varphi$, suppose not i.e. $U \cap V \neq \varphi$. $U \cap V \subseteq U$ and $U \cap V \subseteq V$ Since $U \cap V \subseteq U$ and Uis M_{in} $i\beta$ -open then $U \cap V = U$ or $U \cap V = \varphi$ thus $U \cap V = U$. Since $U \cap V \subseteq V$ and V is M_{in} $i\beta$-open then $U \cap V = V$or $U \cap V = \varphi$ thus $U \cap V = V$. Hence we get that U=V this result contradicteds, the fact that U and Vare defferent. Therefore $U \cap V = \varphi$.

Theorem (3.9): Union of every pair of different $M_{ax}i\beta$-open sets in $T_{i\beta max}$space X is X.

Proof: Let U and V be $M_{ax}i\beta$ -open subsets of $T_{i\beta max}$space X such that $U \neq V$ to show that $U \cup V, X$ suppose not i.e. $U \cup V \neq X$. So $U \subseteq U \cup V$ and $V \subseteq U \cup V$. Since $U \subseteq U \cup V$ and U is $M_{ax}i\beta$-open then $\cup V = U$ *or* $U \cup V = X$. Thus $U \cup V = U$ … … (1).
Now since $V \subseteq U \cup V$ and V is $M_{ax}i\beta$ -open then $U \cup V = V$ or $U \cup V = X$ Thus $U \cup V = V$ … … (2)
Hence from (1) and (2) we get that U=V this result contradicted the fact that U and V are different. Therefore $U \cup V = X$. Continuity with Minimal and Maximal $i\beta$ -open.

Definition (3.10): Let X and Y be topological spaces, a function $f: X \to Y$ is called $M_{in}i\beta$-continuous if $f^{-1}(U)$ is $M_{in}i\beta$-continuous $M_{in}i\beta$-open subset U of Y.

Example (3.11): Let X = Y = {a, b, c} and $f: (X, J) \to (Y, L)$ is the identity function,where $J = \{\varphi, \{a\}, \{a, b\}, X\}$ and $L = \{\varphi, \{a, c\}, Y\}$, then f is $M_{in}i\beta$ -continuous since the only proper open subset of Y is {a,c} and $f^{-1}(\{a, c\}) = \{a, c\}$ is $M_{in}i\beta$ -open in X.

Theorem (3.12): Every $M_{in}i\beta$-continuous function is $i\beta$-continuous.

Proof: Let $f: X \to Y$ be a $M_{in}i\beta$-continuous function and U be open subset of Y. then $f^{-1}(U)$ is $M_{in}i\beta$-open in X and so $f^{-1}(U)$ is X.

Remark (3.13): The converse is not true in general as in the following example.

Example (3.14): Let X = Y = {a, b, c} and $f: (X, J) \to (Y, L)$ is the identity function, where $J = \{\varphi, \{a\}, \{c\}, \{a, c\}, X\}$ and $L = \{\varphi, \{a, c\}, Y\}$, then f is $i\beta$-continuous but f is not $M_{in}i\beta$-continuous. Sinc $f^{-1}(\{a, c\}) = \{a, c\}$ is not $M_{in}i\beta$-open sinc $\{a\} \in i\beta O(X)$ and $\varphi \neq \{a\} \subset \{a, c\}$.

Theorem (3.15): Let X and Y be topological spaces, if $f: X \to Y$ is an $i\beta$-continuous onto function and X is $T_{i\beta min}$ space then if is $M_{in}i\beta$-continuous.

Proof: It is clear that the inverse image of φ and Yare $i\beta$-open subsets of X. So let U be a proper open subset of Y. Since f is $i\beta$-continuous so $M_{ax}i\beta$-continuous since so $f^{-1}(U)$ is proper $i\beta$-open subsets of X, but X is $T_{i\beta min}$ so $f^{-1}(U)$ $M_{in}i\beta$-open.

Remark (3.16): The converse is not true in general as in the following example.

Example (3.17): In (3.11) if $M_{in}i\beta$-continuous but X is not $T_{i\beta min}$.

Theorem (3.18): Let X and Y be topological spaces, if $f: X \to Y$ is a $i\beta$-continuous onto function and X is $T_{i\beta min}$space then f is M_{ax} $i\beta$-continuous.

Proof: It is clear that the inverse image of ϕ and Y are $i\beta$-open subset of X, so let U be a proper open subset of Y. sinc f is $i\beta$-continuous so $f^{-1}(U)$ is proper $i\beta$-open subsets of X, but X is $T_{i\beta min}$ so $f^{-1}(U)$ $M_{ax}i\beta$-open.

Remark (3.19): The converse is not true in general as in the following example.

Example (3.20): In (3.11) f is $M_{ax}i\beta$-continuous but X is not $T_{i\beta min}$ space.

Theorem (3.21): Every $M_{ax}i\beta$-continuous function is $i\beta$-continuous.

Proof: Let $f: X \to Y$ be a $M_{ax}i\beta$-continuous function and U be open subset of Y. then $f^{-1}(U)$ is $M_{ax}i\beta$-open in X and so $f^{-1}(U)$ is $i\beta$-open subset of X.

Remark (3.22): The Converse is not true in general as in the following example.

Example (3.23): Let X = Y = {a, b, c} and $f: (X, J) \to (Y, L)$ is the identity function, where $J = \{\varphi, \{a\}, \{a, c\}, X\}$ and $L = \{\varphi, \{a\}, Y\}$, then f is $i\beta$-continuous but f is not $M_{ax}i\beta$-continuous since $f^{-1}(\{a\}) = \{a\}$ is not $M_{ax}i\beta$-open since $\varphi \neq \{a, c\} \supset \{a\}$.

Remark (3.24): $M_{in}i\beta$-continuous and $M_{ax}i\beta$-continuous functions are independent of each other and the following examples show that.

Example (3.25): In (3.11) f is $M_{ax}i\beta$-continuous since $f^{-1}(\{a, c\}) = \{a, c\}$ is $i\beta$-open but f is not $M_{in}i\beta$-continuous.

Example (3.26): In (3.14) f is $M_{in}i\beta$-continuous but it is not $M_{ax}i\beta$-continuous. $f^{-1}(\{b\}) = \{b\}$ is not $M_{ax}i\beta$-open in X.

Theorem (3.27): Let $f: X \to Y$ a function and X and Y be topological spaces, then f is maximal (resp. minimal) $i\beta$-continuous if and only if $f^{-1}(F)$ is minimal (resp.maximal) $i\beta$-closed subset of X for each closed sudset F of Y.

Proof: Let F be closed set in Y. thus Y-F is open and so $f^{-1}(Y - F)$ is maximal (resp. minimal) $i\beta$-open. But $f^{-1}(Y - F) = X - f^{-1}(F)$ so $f^{-1}(F)$ is minimal (resp. maximal) $i\beta$-closed.

Theorem (3.28): Let X, Y and Z be topological spaces, if $f: X \to Y$ is a minimal (respect. maximal) $i\beta$-continuous function and $g: Y \to Z$ is a continuous function then $gof: X \to Z$ minimal (resp. maximal) $i\beta$-continuous function.

Proof: Let U be an open subset of Z, since g is cntinuous so $g^{-1}(U)$ is an open subset of Y. but f is minimal (resp. maximal) $i\beta$-continuous thus $f^{-1}(g^{-1}(U)) = (gof)^{-1}\, is.$ minimal (resp. maximal) $i\beta$-open subset of X. ents.

4. Conclusion

In this paper we get some theorems and properties of $M_{in}i\beta$-open and maximal $M_{ax}i\beta$ -open and theirof continuity here.

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