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Bayesian inference in control charts using normal prior

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Abstract
The Control Chart is a method of quality control which utilizes statistical methods to monitor and manage processes and based on the sampling inspections and chart performance. This issue is currently being researched into the economic planning of control chart. The traditional approach to design control chart utilizes the traditional structure of control plot to determine the value of parameters of the chart, namely: The sample size, sample interval and control chart limits for achieving economic needs. Under Bayes estimate framework, focus on defining the optimal control policy based on posterior probabilities, thereby reducing total expected costs in the given finite time horizon or the expected average long-term costs.

Keywords: Statistical process control, bayesian inference, prior and posterior distribution, normal distribution

1. Introduction

SPC (Statistical Process Control) is a statistical technique used to display a system manages and is a proactive method. Unlike recognition sampling plans, it is also used because of the system in which the nonconformities merchandise have already occurred, SPC is used to sign the system while it's miles out of control, after which one has taken essential preventive steps to alter the manufacturing system. Specifically, it's miles and utility of statistical strategies for the gathering of the records observations, charting the records, after which tracking the variety of a selected system of hobby over a time variety relative to the Lower and Upper control limits typically set at three Standard deviations beneath and above from the system goal line. One of the thrilling new methods in commercial SQC (Statistical Quality Control), is the Bayesian method, primarily based totally on subjective possibility of earlier facts approximately the system wherein many varieties of uncertainty are blanketed with inside the version and expressed. Bayesian context consists of a top level view of the notion with inside the system earlier and the product after staring at the records, additionally the cap potential to dynamically replace the control chart additives and anticipated parameters as they arise for the gathering of latest records. Bayesian fashions are designed to interesting system parameters as variables that behave in a distribution of unknown possibility. The Bayesian version affords the shape of observable and unobservable variables, parameters, and their dependencies. This shape continues in view greater flexibility and attempts to clear up the parameters of the manufacturing machine. Ulrich (2002, 2007) [7, 9] used the Bayes framework to affirm the goal and the variety of manage charts, respectively. Constructed control chart, proposed through Ulrich (2010) [10], while the goal of the system and its variance are unknown of ordinary distribution. In 2015, Aamir proposed the chart the use of posterior control limits in which he used the non-informative and informative priors` context to replace the system suggest and show the control limits. Many researchers have additionally prolonged the MLE method to a state of affairs wherein the prior isn't always known. The most effective speculation they formulated changed into that the form of the modifications belongs to a fixed of monotonous effects.

The Bayesian method to modeling and statistical evaluation bureaucracy the idea for estimating a precies of earlier distributions and cutting-edge pattern records. From the perspective of statistical theory, the statistical system is received through seeking out the out the common performance of all viable records. This is the contradiction to the Bayesian system due to the fact greater attention is paid to the conduct of the system in a given state of affairs. In addition, in contrast to frequentist procedures, Bayesian methods formally use the facts to be had from confirms aside from statistical surveys. This fact, beyond experiences, describes all capacity values' possibility distribution of the unknown parameter in a statistical version. The Bayesian Approach is provided in section 2. Section 3 contains The Bayesian Control limits using Normal Prior. Section 4 illustrates a numerical example and section 5 describe conclusion.

2. Bayesian approach

Bayesian approaches are a reliable model for questionable statistical decisions and statistical conclusions. They provide an often-used solution to the problems faced by standard statistical methods, as well as increasing the usefulness of statistics. Statistical inference using Bayes' theorem should be implemented in a way that accounts for uncertainty caused by evident values.

Posterior ∝ (Prior x Likelihood)

Where, "Posterior" is later information related to the size of the sample data. "Prior" means knowing the interest's quantity of the probability distribution; and "Likelihood" is a prototype observation. Therefore, Bayes theorem is attributed to three random quantities of y and parameter σ^2, s^2 and prior μ as.

$$g(\mu|y) \propto g(\mu)\, x\, f\,(y|\mu)$$

The prior distribution is combined with the likelihood function to give another distribution called as posterior distribution.

The observation y is a random variable taken from a Normal distribution with mean μ and variance σ^2 which is assumed known.

$$g(\mu) \propto e^{-\frac{1}{2\sigma^2}(y-\mu)^2}$$

We have a prior distribution that is Normal with mean m and variance s^2. The shape of the prior density is given by

$$g(\mu) \propto e^{-\frac{1}{2s^2}(\mu-m)^2}, -\infty < \mu < \infty \text{ and } \sigma^2 > 0.$$

Usually we have a random sample y_1, y_2. y_n of observations instead of a single observation. The observations in a random sample are all independent of each other, so the joint likelihood of the sample is the product of the individual observation likelihoods. This gives

$$f\,(y_1,.,y_n|\mu) = f\,(y_1|\mu) \times f\,(y_2|\mu) \times . \times f\,(y_n|\mu).$$

Prior times likelihood will be a $N(\mu_n, \sigma^2)$.

As we know s^2 and σ^2, we can ignore the constant term, then we have

$$g(\mu/y_1, y_2 \,.\, y_n) \propto g(\mu)\; x\; f(y_1 y_2 . y_n/\mu)$$

$$\propto exp\left\{-\frac{1}{2s^2}(\mu-m)^2\right\} x\; exp\left\{-\frac{n}{2\sigma^2}(\mu-\bar{y})^2\right\}$$

After simplification we get the mean and variance of Posterior distribution and Normal Distribution

Mean = $\frac{\sigma^2 m+s^2 y}{\sigma^2+s^2}$

Variance = $\frac{\sigma^2 s^2}{(\sigma^2+s^2)}$

3. Control limits using normal prior

The Bayesian control chart is a graphical tool used to determine the probability of two out-of-control conditions in a process. This information can be useful when making decisions about process state. The prior distribution determines how much information about the structure of the process should be known before using this chart. However, if you do not have enough data for the posterior distribution, then its variance will be smaller than that of the prior distribution

UCL = $\frac{\sigma^2 m+s^2 y}{\sigma^2+s^2} + 3\,\frac{\sigma s}{(\sigma+s)}$

CL = $\frac{\sigma^2 m+s^2 y}{\sigma^2+s^2}$

LCL = $\frac{\sigma^2 m+s^2 y}{\sigma^2+s^2}$ - 3 $\frac{\sigma s}{(\sigma+s)}$

4. Numerical example

An example of how Bayesian control limits can be used is illustrated by considering a situation where data from an experiment falls within the range of values that have been observed in past experiments. Control limits for this type of scenario are determined using simulated data sets with different values for μ and σ, as shown in Tables 4.2-4.5 below. The UCL, CL and LCL values are reported below.

Table 1: Bayesian Control Limits in Normal Prior when $\sigma = 4$

Y	M	S=2			S=3		
		CL	UCL	LCL	CL	UCL	LCL
2	0.3	0.64	4.64	0	0.912	6.054857	0
3	0.5	1	5	0	1.4	6.542857	0
5	0.8	1.64	5.64	0	2.312	7.454857	0
10	1	2.8	6.8	0	4.24	9.382857	0
15	2	4.6	8.6	0.6	6.68	11.82286	1.537143
20	5	8	12	4	10.4	15.54286	5.257143
25	10	13	17	9	15.4	20.54286	10.25714

Table 2: Bayesian Control Limits in Normal Prior when $\sigma = 7$

Y	M	S=2			S=3		
		CL	UCL	LCL	CL	UCL	LCL
2	0.3	0.428302	5.094969	0	0.563793	6.863793	0
3	0.5	0.688679	5.355346	0	0.887931	7.187931	0
5	0.8	1.116981	5.783648	0	1.451724	7.751724	0
10	1	1.679245	6.345912	0	2.396552	8.696552	0
15	2	2.981132	7.647799	0	4.017241	10.31724	0
20	5	6.132075	10.79874	1.465409	7.327586	13.62759	1.027586
25	10	11.13208	15.79874	6.465409	12.32759	18.62759	6.027586

Table 3: Bayesian Control Limits in Normal Prior when $\sigma = 10$

Y	M	S=2			S=3		
		CL	UCL	LCL	CL	UCL	LCL
2	0.3	0.365385	5.365385	0	0.440367	7.363444	0
3	0.5	0.596154	5.596154	0	0.706422	7.629499	0
5	0.8	0.961538	5.961538	0	1.146789	8.069866	0
10	1	1.346154	6.346154	0	1.743119	8.666196	0
15	2	2.5	7.5	0	3.073394	9.996471	0
20	5	5.576923	10.57692	0.576923	6.238532	13.16161	0
25	10	10.57692	15.57692	5.576923	11.23853	18.16161	4.315455

From Tables 1 to Table 3, for a fixed value of Parameter y, the control limits rise as parameter m rises. The control limits rise whenever parameter y and m rise for a fixed value of and s. Around the CL, the UCL and LCL ought to be equally spaced. Whatever the LCL's negative turn, it can be rounded to zero. A control chart for the proposed Bayesian model is shown in Figure 1. A special case of these values are y = 10, m = 1, s = 3, and = 7.

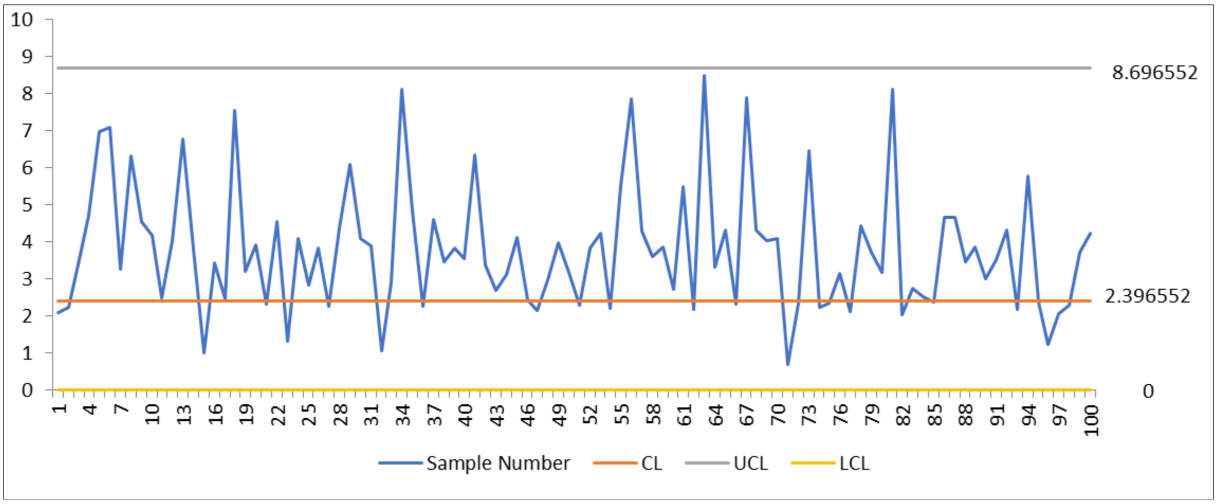

Fig 1: Bayesian control chart for normal prior

5. Conclusion

By giving the parameter a prior distribution, the Bayesian inference method incorporates prior information about the unknown parameter into the inference process. When creating control charts with Bayesian inference, the prior knowledge is combined with a likelihood function to obtain the posterior distribution. Using the posterior distribution, the standardized mean's future distribution is determined. It is a beautiful task to identify the uninformative prior in a multi-parameter problem. Before the control limits are established, a numerical problem utilizing a binary diffusion coupling is presented, along with a control chart showing that all process observations are under control.

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