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The use of matrices in hotel management

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Abstract
Hotel rooms are the primary inventory kept by hotel managers. The demand for hotel rooms, the number of rooms in demand and the number of days the rooms are in demand forms the hotel's matrix of demand. We use this matrix of demand to determine the level of patronage of the hotel and compare the profitability of two or more hotels.

Keywords: Hotel, rooms, demand, patronage, matrix

1. Introduction
Matrix is a rectangular array of numbers arranged in rows and columns. For example the matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \dots a_{1n} \\ a_{21} & a_{22} \dots a_{2n} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{m1} & a_{m2} \dots a_{mn} \end{pmatrix}$$

Has m rows and n columns and is described as an $m \times n$ matrix. In this work, we use the matrix notation to represent the number of rooms in demand and how long they are in demand. Recall that the number of rooms in a hotel is fixed and subdivided into subunits [1, 2, 3, 4]. Some common subunits in Nigeria include; single rooms, double rooms, standard rooms, kings rooms, queens rooms, suites, presidential etc. also, the daily demand for rooms in the hotel cut across all the subunits of the hotel, such that, the total number of rooms in demand in a day is the sum of demand from all sub units. The zero entries indicate no patronage, while a nonzero entry indicate room (s) in demand. The hotel manager is concerned about the entries of the matrix, as zero entries indicate no or low patronage.

This work is focused on the rooms in a hotel. The total number of rooms in the hotel is fixed and are classified into subunits based on the type of bed, number of occupants, number of beds, décor, specific furnishing or features, views and service provided. Some common subunits in hotels according to setupmyhotel.com are;

$k_1 =$ **Single Room:** A room with one or more beds that is allocated to one person. The size of the room or area measures $37m^2$ to $45m^2$.

$k_2 =$ **Double Room:** A room with one or more beds that is allocated to two people. The size of the room or area measures $40m^2$ to $45m^2$.

$k_3 =$ **Triple Room:** A room with three twin beds or one double bed and one twin bed that can accommodate three people. The measurement is mostly $45m^2$ to $65m^2$.

$k_4 =$ **Quad Room:** A room that can accommodate four people and measures $70m^2$ to $85m^2$.

k_5 = **Queen Room:** A room that can accommodate one or more people with a queen-sized bed and measures $32m^2$ to $50m^2$.

k_6 = **King Room**: A room that can accommodate one or more people with a king-sized bed and measures $32m^2$ to $50m^2$.

k_7 = **Double-double Room:** A room that can accommodate two to four people with two double beds and measures $50m^2$ to $70m^2$.

k_8 = **Suite/ Executive Suite:** One or more bedrooms are linked by a parlour or living room and measures $70m^2$ to $100m^2$.

k_9 = **Mini Suite/ Junior Suite:** This is a single room that has a bed and sitting area. The sleeping area is sometimes in a bedroom separated from the parlour or living room and measures $60m^2$ to $80m^2$.

k_{10} = **Presidential Suite:** This is the most expensive room a hotel has to offer. Usually in a hotel, only one presidential suite is available. A presidential suite, similar to the normal suites, always has one or more bedrooms and a living area, with an emphasis on magnificent in-room decorating, high-quality facilities and supplies, and custom-tailored services. This measure $80m^2$ to $350m^2$.

k_{11} = **Apartments:** This room type is common in-service apartments and hotels that cater to long-term visitors. Things that are usually available in the room are open kitchens, cooking equipment, dryers, washers, and other amenities. This measure $96m^2$ to $250m^2$.

k_{12} = **Accessible Room/ Disabled Room:** This room type is mainly designed for disabled guest. This measure $30m^2$ to $42m^2$.

k_{13} = **Cabana Room:** This type of room is either located next to a swimming pool or has a private pool attached to it. This measure $30m^2$ to $45m^2$

k_{14} = **Villa Room:** This is a specific type of accommodation that can be found in some resort hotels. It's a kind of stand-alone residence that allows hotel guests more privacy and space. This measure $100m^2$ to $150m^2$.

k_{15} = **Smoking/ Non-smoking Room:** To reduce the effects of second hand smoke exposure on non-smoking guests, several hotels offer both smoking and non-smoking rooms.

Note that the number of rooms in each subunit is fixed and from time to time. Some rooms maybe converted from one subunit to another depending on demand.

2. Description of the model

The total number of rooms K in a hotel is fixed and is subdivided into subunits k_1 , k_2, . . .k_n. The number of rooms in each subunit is also fixed. The price for a room in each subunit is $x_1, x_2, .. x_n$. The rooms in each subunit is further divided into rooms in demand and rooms available to meet demand. The total revenue for the day is the sum of revenue from each subunit. Table 1 shows the subunits, price of rooms in each subunit, rooms in demand and rooms available to meet demand.

3. Description and Derivation of the Demand Matrix

The demand matrix tells us the number of rooms from each subunits that is in demand and the number of period(s) the room(s) will be in demand. If the total number of rooms in a hotel is K and there are n subunits in the hotel, then $K = k_1 + k_2 + k_3 + \ . \ . \ . + k_n$.

Where k_1 can be single rooms, k_2 double rooms, k_3 kings room, k_4 queens room, k_5 suites, and so on. A row in the matrix represent the rooms in demand from a subunit and the number of days they are in demand. For example, If 10 rooms are in demand from subunit k_1, such that

2 are in demand for 4 days
1 is in demand for 2 day
3 are in demand for 3 days
4 are in demand for 1 day
Then we have

$$t_1a_{11}x_1 + t_2a_{12}x_1 + t_3a_{13}x_1 + t_4a_{14}x_1$$

Where t_i is the number of day(s) the room(s) are in demand.

a_{11} is the number of rooms from subunit k_1 in demand for one period.

a_{12} is the number of rooms from subunit k_1 in demand for two period.

a_{13} is the number of rooms from subunit k_1 in demand for three period.

a_{14} is the number of rooms from subunit k_1 in demand for four period.

then the example yields $a_{11} = 4, \ a_{12} = 1, a_{13} = 3, a_{14} = 2$.

If we extend the example to other subunits $k_2, k_3, k_4, \ k_5$ we obtain a system of linear equations with $x_i, \ i = 1,2,3,4,5$ as unit price of rooms in subunits

$$k_1, k_2, k_3, k_4, \ k_5$$

$$t_1a_{11}x_1 + t_2a_{12}x_1 + t_3a_{13}x_1 + t_4a_{14}x_4$$
$$t_1a_{21}x_2 + t_2a_{22}x_2 + t_3a_{23}x_2 + t_4a_{24}x_2$$
$$t_1a_{31}x_3 + t_2a_{32}x_3 + t_3a_{33}x_3 + t_4a_{34}x_3$$
$$t_1a_{41}x_4 + t_2a_{42}x_4 + t_3a_{43}x_4 + t_4a_{44}x_4$$
$$t_1a_{51}x_5 + t_2a_{52}x_5 + t_3a_{53}x_5 + t_4a_{54}x_5$$

In matrix form, the rooms in demand will be given

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} \end{pmatrix}$$

If the hotel has n subunits and we extend to all subunits in the hotel, then

$t_1a_{11}x_1 + t_2a_{12}x_1 + t_3a_{13}x_1 + t_4a_{14}x_4 + \ldots$
$t_1a_{21}x_2 + t_2a_{22}x_2 + t_3a_{23}x_2 + t_4a_{24}x_2 + \ldots$
$t_1a_{31}x_3 + t_2a_{32}x_3 + t_3a_{33}x_3 + t_4a_{34}x_3 + \ldots$
$t_1a_{41}x_4 + t_2a_{42}x_4 + t_3a_{43}x_4 + t_4a_{44}x_3 + \ldots$
. . . .
. . . .
. . . .
$t_1a_{n1}x_n + t_2a_{n2}x_n + t_3a_{n3}x_n + t_4a_{n4}x_n + \ldots$

Which will yield the matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \ ..; \\ a_{21} & a_{22} & a_{23} & a_{24} \ldots \\ a_{31} & a_{32} & a_{33} & a_{34} \ldots \\ a_{41} & a_{42} & a_{43} & a_{44} \ldots \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} \ldots \end{pmatrix}$$

The dimension of the matrix depends on the room(s) with the highest period of demand. In the example above, the highest period of demand is 4 days, so the demand matrix is $n\times 4$. If it is 5days then the dimension of the demand matrix will be $n\times 5$.

The total revenue function for a non regular fixed lifetime inventory like the hotel was obtained in [5] and is given by Revenue function $= \sum_{i,j=1}^{n} t_i a_{i,j} x_i - v\int_k^{\infty}(d-k)f(d)dd - \theta\int_0^k (k-d)f(d)dd + P$ (1)

If in addition to the example above for subunit k_1, we have

20 rooms in demand from subunit k_2, with distribution as follows;
3 rooms in demand for 5 days
3 rooms in demand for 4 days
4 rooms in demand for 3days
6 rooms in demand for 2days
4 rooms in demand for 1day.

Then $a_{21} = 4,\ a_{22} = 6, a_{23} = 4, a_{24} = 3, a_{25} = 3$.

30 rooms in demand from subunit k_3, with distribution as follows;
4 rooms in demand for 4 days.
6 rooms in demand for 3 days.
6 rooms in demand for 2 days.
14 rooms in demand for 1 day.

Then $a_{31} = 14,\ a_{32} = 6, a_{33} = 6, a_{34} = 4$.

25 rooms in demand from subunit k_4, with distribution as follows;
5 rooms in demand for 4 days.
4 rooms in demand for 2 day.
16 rooms in demand for 1 day.

Then $a_{41} = 16,\ a_{42} = 4, a_{43} = 0, a_{44} = 5$.

5 rooms in demand from k_5 with distribution as follows;
2 rooms in demand for 2 days
3 rooms in demand for 1 day.

Then $a_{51} = 3,\ a_{52} = 2, a_{53} = 0, a_{54} = 0$.

Our interest is to use the matrix notation to represent the rooms in demand.

$$\begin{pmatrix} a_{11}=4 & a_{12}=1 & a_{13}=3 & a_{14}=2 & a_{15}=0 \\ a_{21}=4 & a_{22}=6 & a_{23}=4 & a_{24}=3 & a_{25}=3 \\ a_{31}=18 & a_{32}=6 & a_{33}=6 & a_{34}=4 & a_{35}=0 \\ a_{41}=16 & a_{42}=4 & a_{43}=0 & a_{44}=5 & a_{45}=0 \\ a_{51}=3 & a_{52}=2 & a_{53}=0 & a_{54}=0 & a_{55}=0 \end{pmatrix}$$

The matrix of demand for the hotel is

$$\begin{pmatrix} 4 & 1 & 3 & 2 & 0 \\ 4 & 6 & 4 & 3 & 3 \\ 18 & 6 & 6 & 4 & 0 \\ 16 & 4 & 0 & 5 & 0 \\ 3 & 2 & 0 & 0 & 0 \end{pmatrix}$$

Hotel managers always want the entries in the matrix to be nonzero, as zero entry imply no patronage or low level of patronage.

Numerical Example
We shall now compare the level of patronage between two hotels A and B in Benin City having the set of subunits.

Table 1: Rooms/ prices in hotel A.

Room type	Number of rooms	Price per room (₦)
Single room	15	4000
Double room	10	9000
Queen room	10	15000
King room	10	15000
Suites	4	20000
Presidential	1	30000

Table 2: Number of rooms in demand, days in demand and revenue from hotel A.

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
1	Single room	15	4000	4 (1) 124 (2) 4 (3)	3	96000	216000
	Double room	10	9000	5 (1)	5	45000	
	Queen room	10	15000	2 (1)	8	30000	
	King room	10	15000	3 (1)	7	45000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
2	Single room	15	4000	4 (1) 4 (2) 5 (1)	2	20000	230000
	Double room	10	9000	62 (3) 4 (1)	4	90000	
	Queen room	10	15000	4 (1)	6	60000	
	King room	10	15000	4 (1)	6	60000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
3	Single room	15	4000	` 4 (1) 8 (1)	3	32000	221000
	Double room	10	9000	2 (2) 6 (1)	2	54000	
	Queen room	10	15000	4 (1)	6	60000	
	King room	10	15000	5 (1)	5	75000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
4	Single room	15	4000	10 (1)	5	40000	115000
	Double room	10	9000	2 (1) 5 (2)	3	45000	
	Queen room	10	15000	2 (1) 3 (3)	5	30000	
	King room	10	15000	1 (1) 2 (2) 1 (3)	6	0	
	Suites	4	20000	1 (2)	3	0	
	Presidential	1	30000	0	1	0	
5	Single room	15	4000	14(1)	1	48000	189000
	Double room	10	9000	4 (1) 1 (2) 1 (3)	4	36000	
	Queen room	10	15000	3 (1)	7	45000	
	King room	10	15000	4 (1)	6	60000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
6	Single room	15	4000	10 (1)	5	40000	250000
	Double room	10	9000	5 (1)	5	45000	
	Queen room	10	15000	6 (1)	4	90000	
	King room	10	15000	5 (1)	5	75000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
7	Single room	15	4000	11 (1)	4	44000	209000
	Double room	10	9000	0	10	0	
	Queen room	10	15000	4 (1)	6	60000	
	King room	10	15000	7 (1)	3	105000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	

Table 3: Rooms/prices in hotel B.

Room type	Number of rooms	Price per room (₦)
Single room	15	4000
Double room	10	9000
Queen room	10	15000
King room	10	15000
Suites	4	20000
Presidential	1	30000

Table 4: Number of rooms in demand, days in demand and revenue from hotel B.

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
1	Single room	15	4000	4 (1) 103 (2) 3 (3)	5	76000	223000
	Double room	10	9000	21 (1) 1 (2)	8	27000	
	Queen room	10	15000	0	10	0	
	King room	10	15000	42 (1) 2 (3)	6	120000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
2	Single room	15	4000	3 (1) 3 (2) 5 (1)	4	20000	233000
	Double room	10	9000	1 (1) 6 (2)	3	108000	
	Queen room	10	15000	1 (2)	9	30000	
	King room	10	15000	2 (2) 5 (1)	3	75000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
3	Single room	15	4000	` 3 (1) 7 (1)	5	28000	97000
	Double room	10	9000	6 (1) 1 (1)	3	9000	
	Queen room	10	15000	1 (1) 2 (1)	7	30000	
	King room	10	15000	2 (1) 2 (1)	6	30000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
4	Single room	15	4000	42 (2) 2 (3)	11	40000	145000
	Double room	10	9000	31 (1) 2 (2)	7	45000	
	Queen room	10	15000	2 (1)	8	30000	
	King room	10	15000	2 (1)	8	30000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
5	Single room	15	4000	2 (1) 2 (2) 5 (1)	6	20000	230000
	Double room	10	9000	2 (1) 0	8	0	
	Queen room	10	15000	6 (1)	4	90000	
	King room	10	15000	4 (2)	6	120000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
6	Single room	15	4000	2 (1) 7 (1)	6	28000	148000
	Double room	10	9000	0	10	0	
	Queen room	10	15000	3 (2)	7	90000	
	King room	10	15000	4 (1) 1 (2)	5	30000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	
7	Single room	15	4000	8 (1)	7	32000	131000
	Double room	10	9000	1 (1)	9	9000	
	Queen room	10	15000	3 (1) 2 (1)	5	30000	
	King room	10	15000	1 (1) 4 (1)	5	60000	
	Suites	4	20000	0	4	0	
	Presidential	1	30000	0	1	0	

The matrix of demand for the first five days is derived and used to compare their level of patronage.

Table 5: Matrix of demand for hotels A and B.

Day	Hotel A	Hotel B
1	$\begin{pmatrix} 4 & 4 & 4 \\ 5 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 12zero entries	$\begin{pmatrix} 4 & 3 & 3 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 11zero entries
2	$\begin{pmatrix} 9 & 4 & 0 \\ 4 & 0 & 2 \\ 4 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 12zero entries	$\begin{pmatrix} 8 & 3 & 0 \\ 1 & 6 & 0 \\ 0 & 1 & 0 \\ 5 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 11zero entries
3	$\begin{pmatrix} 12 & 0 & 0 \\ 6 & 2 & 0 \\ 4 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 13 zero entries	$\begin{pmatrix} 10 & 0 & 0 \\ 7 & 0 & 0 \\ 3 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 14 zero entries
4	$\begin{pmatrix} 15 & 0 & 0 \\ 2 & 5 & 0 \\ 2 & 0 & 3 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 9 zero entries	$\begin{pmatrix} 0 & 2 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 12 zero entries
5	$\begin{pmatrix} 14 & 0 & 0 \\ 4 & 1 & 1 \\ 3 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 12 zero entries	$\begin{pmatrix} 2 & 2 & 0 \\ 2 & 0 & 0 \\ 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 13 zero entries
Total	58 zero entries	61 zero entries

From Table 5, hotel A has 58 zero entries after five days and hotel B has 61 zero entries after five days, hence hotel A had more patronage in the period under review.

4. Conclusion

The demand matrix is used by hotel managers as a tool to determine their level of patronage or customers inflow. It is the desire of hotel managers that the entries of the demand matrix should be non-zero entries, as a zero matrix or almost zero matrix will indicate no patronage or low patronage.

Also, the demand matrix can be used to compare the level of patronage (demand) between two hotels. The hotel with the most zero entries in its matrix of demand has lower patronage (demand) when compared with the hotel with the least zero entries in its matrix of demand.

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