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Location and scale testing in a mixed design

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Abstract
There are several situations in scientific research where the researcher is interested in either changes in means or variances if new treatments are applied and compared to a control. Recently, Khalawi and Magel proposed and compared nonparametric tests for the simple tree alternative for testing differences in means or variances in a mixed design (completely randomized (CRD) and a randomized complete block (RCBD)) under symmetric distributions. However, there are times in which the data come from a right-tailed skewed distribution. In this paper, we estimate and compare these nonparametric tests under an asymmetric distribution via Monte Carlo simulation.

Keywords: Simple tree alternative, completely randomized design, randomized complete block design, location-scale problem, power

1. Introduction
The simple tree alternative tests the null hypothesis of no differences among the control or any of the treatments against a one-sided alternative that at least one of the treatments means or parameters is greater than the mean or parameter for the control. The Fligner-Wolfe test is a nonparametric test that tests for differences in means in a completely randomized design (CRD) between any of the treatments and the control with the treatment mean being larger [2]. The sample observations from the combined set of control and treatment populations are ordered from smallest to largest and each observation is assigned a rank, r_{ij}, with i=1,2,…,k and j=1,2,,,,n_i with n_i being the sample size for the ith treatment, and k be the number of treatments. The control is denoted as treatment 1. The Fligner-Wolfe test statistic is: $T_{L1} = \sum_{i=2}^{k}\sum_{j=1}^{n_i} r_{ij}$. It is the sum of ranks of the combined treatment sample, not including the control, or treatment 1. When the null hypothesis is true, the mean and variance of the Fligner-Wolfe test are, respectively, $E(T_{L1}) = \frac{n_t(N+1)}{2}$ and $Var(T_{L1}) = \frac{n_c n_t(N+1)}{12}$. In this case, n_c and n_t denote the number of observations in the control sample and combined treatment sample, respectively, with $N = n_c + n_t$. The combined treatment sample includes treatments 2, 3, …, k. Under H_0, the standardized version of the Fligner-Wolf test has an asymptotic standard normal distribution. The null hypothesis is rejected for large values.
Olet and Magel [3], proposed a modified Page's test for a simple tree alternative in a randomized complete block design (RCBD) when testing for means. To compute the modified Page's test (T_{L2}), within each block, observations are arranged from smallest to largest and ranked within each block. Treatment 1 is designated as the control. The modified Page's test is: $T_{L2} = R_1 + 2\sum_{j=2}^{k} R_j = R_1 + 2[R_2 + R_3 + \cdots R_k]$ where R_j be the sum of the ranks for j^{th} treatment. In the modified Page's test, the sum of ranks of all the treatments (not including the control) are given the same weight instead of the weights increasing as in Page's original test. The expected value and variance of the modified Page's test are, respectively, $E(T_{L2}) = n_b E[L_1] = n_b\left(k^2 + \frac{k-1}{2}\right)$ and $Var(T_{L2}) = n_b Var(L_1) = n_b\left(\frac{k^2-1}{12}\right)$ In this case, (L_1) is the modified Page's test for one block with k treatment and n_b is the number of blocks. Under H_0, the standardized version of the modified Page's test (Z_{L2}) has an asymptotic standard normal distribution and will be rejected for large values..
A modified version of the Ansari-Bradley test for CRD for the simple tree alternative was introduced by Alsubie and Magel [4, 5].

Let n_c denote the number of observations in the sample from the control population and n_t denote the number of observations in the combined samples from the combined treatment population where $N = n_c + n_t$. The Ansari-Bradley test statistic is then calculated with one of the populations being the control population and the other population being the combined treatment population. The observations from both the control sample and the combined treatment sample are ranked together with the largest and smallest observations receiving a rank of 1, the second largest and second smallest observations receiving a rank of 2, etc. The test statistic is the sum of the ranks received by the control sample. When the null hypothesis is true, the expected value and variance of (T_{S1}) are, respectively, If $N = n_c + n_t$ is an even number: $E(T_{S1}) = \frac{n_c\,(N+2)}{4}$ and $Var(T_{S1}) = \frac{n_c\,n_t(N+2)(N-2)}{48(N-1)}$ If $N = n_c + n_t$ is an odd number: $E(T_{S1}) = \frac{n_c\,(N+1)^2}{4N}$ and $Var(T_{S1}) = \frac{n_c n_t(N+1)(3+N^2)}{48N^2}$. The standardized version of modified Ansari-Bradley for CRD has an asymptotic standard normal distribution and the null hypothesis is rejected for large values.

Khalawi and Magel [1] proposed a version of the modified Ansari-Bradley test for the simple tree alternative in a randomized complete block design RCBD when it assumed that there is only one observation for each treatment in each block. The modified Ansari-Bradley for a randomized complete block design (RCBD) is calculated by computing the Ansari-Bradley test for each block based on the control observation and the combined treatment observations. Observations within each block are ranked as described in the previous paragraph and the test statistic is given by $T_{S2} = \sum_{j=1}^{n_b} C_j$ where n_b is the number of blocks and C_j is the rank the control observation received in block i. Under H_0, the expected value and variance of T_{S2} are, respectively, if k is an even number: $E(T_{S2}) = n_b\left(\frac{(k+2)}{4}\right)$ and $Var(T_{S2}) = n_b\left(\frac{(k-1)(k+2)(k-2)}{48(k-1)}\right)$. If k is an odd number: $E(T_{S2}) = n_b\left(\frac{(k+1)^2}{4k}\right)$ and $Var(T_{S2}) = n_b\left(\frac{(k-1)(k+1)(3+k^2)}{48k^2}\right)$. The standardized version of modified Ansari-Bradley has an asymptotic standard normal distribution and the null hypothesis is rejected for larger values.

Researchers may want to know if a treatment that has been applied has had an effect on either the mean or variance or both. Lepage [6] proposed a two-sample test for testing for differences between two means or two variances or possibly differences in both means and variances. His test combined the Wilcoxon rank-sum and Ansari-Bradley's test statistics. Duran *et al.* [7] developed a slightly different test based on Lepage's test. In Duran's test, the Ansari-Bradley test was replaced by Mood's test.

Khalawi and Magel [1] proposed six nonparametric tests for comparing treatment means with a control mean to see if at least one treatment mean is larger while also comparing the treatment variances with the control variance to see if one of the treatment variances is larger. This was done for a RCBD and CRD mixed design. The proposed tests are a combination of the Fligner-Wolfe test, modified Page's test, and modified Ansari-Bradley test for CRD and RCBD. The null and alternative hypotheses for these proposed tests are:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k, and\ H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k \quad (1)$$

$$H_a: \mu_1 \leq [\mu_2, \mu_3, \ldots \mu_k]\ \ and/or\ H_a: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{ with at least one strict inequality}$$

where μ_i is the location parameter of population i and σ_i is the scale parameter of population i the control population, while populations 2 through k are the treatment populations.

The first test proposed by [1], Z_1, was developed using the standardized Fligner-Wolfe test for CRD Z_{L1}, the standardized modified Page's test for RCBD Z_{L2}, the standardized modified Ansari-Bradley test for CRD Z_{S1}, and the standardized modified Ansari-Bradley test for RCBD Z_{S2}. Their test one is given below:

$$Z_1 = \frac{(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}{\sqrt{(4)}} \quad (2)$$

The second test proposed by [1], Z_2, uses the Fligner-Wolfe test for CRD T_{L1}, the modified Page's test for RCBD T_{L2}, the modified Ansari-Bradley test for CRD T_{S1}, and the modified Ansari-Bradley test for RCBD T_{S2}. Their test two is given below:

$$Z_2 = \frac{(T_{L1} + T_{L2} + T_{S1} + T_{S2}) - (E(T_{L1}) + E(T_{L2}) + E(T_{S1}) + E(T_{S2}))}{\sqrt{Var(T_{L1}) + Var(T_{L2}) + Var(T_{S1}) + Var(T_{S2})}} \quad (3)$$

where $E(T_{L1})$ and $Var(T_{L1})$ are the expected value and variance of the Fligner Wolfe test T_{L1}, $E(T_{L2})$ and $Var(T_{L2})$ are the expected value and variance of the modified Page's test T_{L2}, $E(T_{S1})$ and $Var(T_{S1})$ are the expected value and variance of the modified Ansari-Bradley test T_{S1} for CRD, and $E(T_{S2})$ and $Var(T_{S2})$ are the expected value and variance of the modified Ansari-Bradley test T_{S2} for RCBD.

The third test proposed by [1], Z_3, adds various weights to the components of their first test. The CRD portions each had a weight of $\frac{n_a}{n}$ and the RCBD portions each had a weight of $\frac{n_b}{n}$. Their test three is given below:

$$Z_3 = \frac{\left(\frac{n_a}{n}Z_{L1} + \frac{n_b}{n}Z_{L2} + \frac{n_a}{n}Z_{S1} + \frac{n_b}{n}Z_{S2}\right)}{\sqrt{\left(\frac{n_a^2}{n^2} + \frac{n_b^2}{n^2} + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2}\right)}} \quad (4)$$

Where, n is the sum of the sample size for each treatment $n_a$under the CRD portion and the number of blocks n_b under the RCBD portion $n = n_a + n_b$.

The fourth test proposed by [1], Z_4, used the same components as their test 2, but added relative weights associated with the sample sizes as in test three. Their test four is given below:

$$Z_4 = \frac{\left(\frac{n_a}{n}T_{L1}+\frac{n_b}{n}T_{L2}+\frac{n_a}{n}T_{S1}+\frac{n_b}{n}T_{S2}\right)-\left(\frac{n_a}{n}E(T_{L1})+\frac{n_b}{n}E(T_{L2})+\frac{n_a}{n}E(T_{S1})+\frac{n_b}{n}E(T_{S2})\right)}{\sqrt{\frac{n_a^2}{n^2}Var(T_{L1})+\frac{n_b^2}{n^2}Var(T_{L2})+\frac{n_a^2}{n^2}Var(T_{S1})+\frac{n_b^2}{n^2}Var(T_{S2})}} \quad (5)$$

Where,
$n = n_a + n_b$.

The fifth test proposed by [1], Z_5, was similar to their tests 1 and 3 with different weight values. In this case, the weights were switched with the weights used in 3 between statistics used in the RCBD portion and statistics used in the CRD portion. Their test five is given below:

$$Z_5 = \frac{\left(\frac{n_b}{n}Z_{L1}+\frac{n_a}{n}Z_{L2}+\frac{n_b}{n}Z_{S1}+\frac{n_a}{n}Z_{S2}\right)}{\sqrt{\left(\frac{n_b^2}{n^2}+\frac{n_a^2}{n^2}+\frac{n_b^2}{n^2}+\frac{n_a^2}{n^2}\right)}} \quad (6)$$

where $n = n_a + n_b$.
The sixth test proposed by [1], Z_6, was similar to tests one and four with different weight values. The weights used between statistics used in the RCBD portion and the CRD portion were switched with those used in test four. Their test six is given below:

$$Z_6 = \frac{\left(\frac{n_b}{n}T_{L1}+\frac{n_a}{n}T_{L2}+\frac{n_b}{n}T_{S1}+\frac{n_a}{n}T_{S2}\right)-\left(\frac{n_a}{n}E(T_{L1})+\frac{n_a}{n}E(T_{L2})+\frac{n_a}{n}E(T_{S1})+\frac{n_a}{n}E(T_{S2})\right)}{\sqrt{\frac{n_b^2}{n^2}Var(T_{L1})+\frac{n_a^2}{n^2}Var(T_{L2})+\frac{n_b^2}{n^2}Var(T_{S1})+\frac{n_a^2}{n^2}Var(T_{S2})}} \quad (7)$$

Under H_0, all of the tests have an asymptotic standard normal distribution and the null hypothesis is rejected for large values.
Fouad and Magel [1] considered only symmetric populations in their simulation study. The recommendation overall was to use Z_1 *or* Z_3. If the number of blocks for the RCBD portion is less than the sample size for each treatment in the CRD portion, they recommended test three, Z_3. Otherwise Z_1 is recommended.
This paper will estimate and compare these six nonparametric tests under an asymmetric distribution via Monte Carlo simulation.

2. Materials and Methods

A simulation study was performed using SAS version 9.4 [8] as described in [1]. Significance levels were estimated first for each proposed test and each sample size arrangement. All of the significance levels were set at alpha equal to 0.05. The estimated significance levels were found by simulating 10,000 sets of samples when the means and variances were equal to each other for all populations, conducting each test for each set of samples, keeping count of the number of times each test rejected the null hypothesis, and then dividing these counts by 10,000 to obtain the estimates of the alpha values. If the estimated alpha values were approximately 0.05, the estimated powers of tests were compared to each other. Estimated powers were found by keeping track of the number of times each test rejected under a certain condition when 10,000 sets of samples were simulated and dividing this number by 10,000.
The exponential distribution was considered as an asymmetric distribution because it often occurs and is right-tailed skewed. The call function (X= Rand ("Exponential", 1)) was used to generate the random sample from a standard exponential distribution with a mean equal to one and variance equal to one.
In this study, only equal sample sizes for the CRD portion were considered. Powers were estimated when the number of blocks in the RCBD portion equaled the sample size for each sample in the CRD portion, n_b and n_a both equal 10, when the number of blocks was greater, $n_b = 10, n_a = 5$, and when the number of blocks was less, $n_b = 5, n_a = 10$.
The data used in this study was generated from a mixed design consisting of a CRD and RCBD portion. Cases were considered in which the variances of both portions were equal and then when the variance of the CRD portion was twice as large as the variance of the RCBD portion.
Powers for the proposed tests were estimated under a variety of location and scale parameter arrangements for 3, 4, and 5 populations as in [1].

3. Results and Discussion

Tables 1-6 show the estimated powers obtained for three, four, and five treatments (k=3, 4, 5) with various parameters under the exponential distribution. In the tables, (μ, σ^2) represent the mean and variance of the random variable generated from an exponential distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. The first 3 tables (Tables 1-3) estimate powers when the variance of the CRD portion is equal to the variance of the RCBD portion. The last 3 tables (Tables 4-6) estimate powers when the variance of the CRD portion is twice the variance of the RCBD portion. The tests all maintained their estimated alpha values as given when all the parameters are equal to 1. Powers were then estimated for each of the various tests when at least some of the parameters (mean or variance) changed in the treatment groups.
Tests one, three and five are the same as well as tests two, four, and six are the same when the sample size for each treatment in the CRD portion is equal to the number of blocks in the RCBD ($n_b = 10, n_a = 10$). In this case, test two, Z_2, has the highest powers (Table 1) overall when the variances of the CRD and RCBD portions are equal. However, when the variance in the CRD portion is larger, test one, Z_1, has the highest powers (Table 4).

Test four, Z_4, has the highest powers when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion ($n_b = 10, n_a = 5$). This may be seen in Tables 2 and 5. This result is consistent between when the variances of both portions are equal and when the variance of the CRD portion is twice as large.

Figures 1 and 2 plot the powers of all the tests when the number of blocks in the RCBD portion increases and the sample size for each treatment in the CRD portion is held constant ($n_a = 5$). The variance in the CRD portion is equal to the variance in the RCBD portion in Figure 1. In Figure 2 the variance in the CRD portion is twice as large as the variance in the RCBD.

Test six, Z_6, has the highest powers (Table 3) when the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion ($n_b = 5, n_a = 10$), and when the variances from both portions are equal. However, when the variance in the CRD portion is twice as large as the variance in the RCBD portion, test three, Z_3, has the highest powers (Table 6).

Figures 3 and 4 plot the powers of all the tests when the sample size for each treatment in the CRD portion increases and the number of blocks in the RCBD portion is held constant ($n_b = 5$). The results in Figure 3 are when the variances of both portions are equal. It is noted in Figure 3 that the top 3 tests have estimated powers that are close as the sample size increases. The results given in Figure 4 are when the variance in the CRD portion is twice as large as the variance in the RCBD portion.

Table 1: Estimated power of tests for CRD and RCBD design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

$(\mu_1, \sigma_1^2)\ (\mu_2, \sigma_2^2)\ (\mu_3, \sigma_3^2)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0489	0.0505	0.0489	0.0505	0.0489	0.0505
(1,1) (2,4) (2,4)	0.4517	0.4737	0.4517	0.4737	0.4517	0.4737
(1,1) (1,1) (3,9)	0.4349	0.3633	0.4349	0.3633	0.4349	0.3633
(1,1) (2,4) (3,9)	0.5946	0.6413	0.5946	0.6413	0.5946	0.6413
(1,1) (2.5,6.25) (5,25)	0.7599	0.8453	0.7599	0.8453	0.7599	0.8453
(1,1) (2,4) (5,25)	0.7478	0.7931	0.7478	0.7931	0.7478	0.7931
(1,1) (3,9) (3.5,12.25)	0.7324	0.8101	0.7324	0.8101	0.7324	0.8101
(1,1) (6,36) (8,64)	0.8644	0.9766	0.8644	0.9766	0.8644	0.9766

Table 2: Estimated power of tests for CRD and RCBD design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

$(\mu_1, \sigma_1^2)\ (\mu_2, \sigma_2^2)\ (\mu_3, \sigma_3^2)\ (\mu_4, \sigma_4^2)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0525	0.0584	0.0516	0.0524	0.0542	0.0554
(1,1) (2,4) (2,4) (2,4)	0.3726	0.4102	0.3606	0.4728	0.3333	0.3327
(1,1) (1,1) (3,9) (3,9)	0.4643	0.4481	0.4518	0.5257	0.4063	0.3597
(1,1) (2,4) (3,9) (4,16)	0.5354	0.6453	0.5093	0.7332	0.4858	0.5369
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.6321	0.8276	0.5985	0.9061	0.5702	0.7131
(1,1) (2,4) (5,25) (1,1)	0.4916	0.4590	0.4757	0.5405	0.4328	0.3693
(1,1) (3,9) (3.5,12.25) (2,4)	0.5224	0.6172	0.4967	0.7064	0.4749	0.5113
(1,1) (4,16) (6,36) (8,64)	0.6104	0.8792	0.5709	0.9415	0.5512	0.7816

Table 3: Estimated power of tests for CRD and RCBD design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

$(\mu_1, \sigma_1^2)\ (\mu_2, \sigma_2^2)\ (\mu_3, \sigma_3^2)\ (\mu_4, \sigma_4^2)(\mu_5, \sigma_5^2)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0515	0.0522	0.0516	0.0538	0.0517	0.0546
(1,1) (2,4) (2,4) (2,4) (2,4)	0.4233	0.4999	0.4480	0.4904	0.3389	0.5256
(1,1) (1,1) (3,9) (3,9) (3,9)	0.5408	0.6080	0.5677	0.5971	0.4407	0.6432
(1,1) (2,4) (3,9) (4,16) (5,25)	0.6282	0.8096	0.6589	0.7980	0.5035	0.8412
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.6554	0.9371	0.6879	0.9303	0.5184	0.9549
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	0.6246	0.6947	0.6482	0.6825	0.5083	0.7298
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.6353	0.8157	0.6619	0.8038	0.5078	0.8481
(1,1) (4,16) (6,36) (8,64) (10,100)	0.6131	0.9605	0.6481	0.9544	0.4751	0.9737

Table 4: Estimated power of tests for CRD and RCBD design under the exponential distribution with different means and variances; the variance CRD=2RCBD; K=3; $n_b = 10, n_a = 10$.

$(\mu_1, \sigma_1^2)\ (\mu_2, \sigma_2^2)\ (\mu_3, \sigma_3^2)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0532	0.0513	0.0532	0.0513	0.0532	0.0513
(1,1) (2,4) (2,4)	0.4726	0.3408	0.4726	0.3408	0.4726	0.3408
(1,1) (1,1) (3,9)	0.4128	0.2424	0.4128	0.2424	0.4128	0.2424
(1,1) (2,4) (3,9)	0.6195	0.4596	0.6195	0.4596	0.6195	0.4596
(1,1) (2.5,6.25) (5,25)	0.7972	0.6546	0.7972	0.6546	0.7972	0.6546
(1,1) (2,4) (5,25)	0.7658	0.5917	0.7658	0.5917	0.7658	0.5917
(1,1) (3,9) (3.5,12.25)	0.7687	0.6195	0.7687	0.6195	0.7687	0.6195
(1,1) (6,36) (8,64)	0.9222	0.8516	0.9222	0.8516	0.9222	0.8516

Table 5: Estimated power of tests for CRD and RCBD design under the exponential distribution with different means and variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0505	0.0538	0.0499	0.0501	0.0476	0.0507
(1,1) (2,4) (2,4) (2,4)	0.3809	0.3112	0.3587	0.3812	0.3461	0.2437
(1,1) (1,1) (3,9) (3,9)	0.4548	0.3336	0.4393	0.4194	0.3999	0.2530
(1,1) (2,4) (3,9) (4,16)	0.5853	0.5116	0.5358	0.6211	0.5501	0.3965
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.7205	0.6972	0.6577	0.8161	0.6962	0.5628
(1,1) (2,4) (5,25) (1,1)	0.4836	0.3360	0.4756	0.4347	0.4171	0.2539
(1,1) (3,9) (3.5,12.25) (2,4)	0.5721	0.4886	0.5234	0.6029	0.5290	0.3765
(1,1) (4,16) (6,36) (8,64)	0.7334	0.7564	0.6541	0.8709	0.7208	0.6297

Table 6: Estimated power of tests for CRD and RCBD design under the exponential distribution with different means and variances; the variance in CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) $(\mu_4, \sigma_4^2)(\mu_5, \sigma_5^2)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0516	0.0501	0.0530	0.0518	0.0477	0.0505
(1,1) (2,4) (2,4) (2,4) (2,4)	0.4532	0.3407	0.4892	0.333	0.3638	0.3723
(1,1) (1,1) (3,9) (3,9) (3,9)	0.5630	0.4129	0.5972	0.4006	0.4557	0.4545
(1,1) (2,4) (3,9) (4,16) (5,25)	0.7367	0.6493	0.7911	0.6335	0.5742	0.6956
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.8456	0.8254	0.8884	0.8115	0.6743	0.8593
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	0.6498	0.4942	0.6851	0.4791	0.5223	0.5383
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.7397	0.6581	0.7965	0.6428	0.5812	0.7067
(1,1) (4,16) (6,36) (8,64) (10,100)	0.8572	0.8633	0.9020	0.8489	0.6738	0.8907

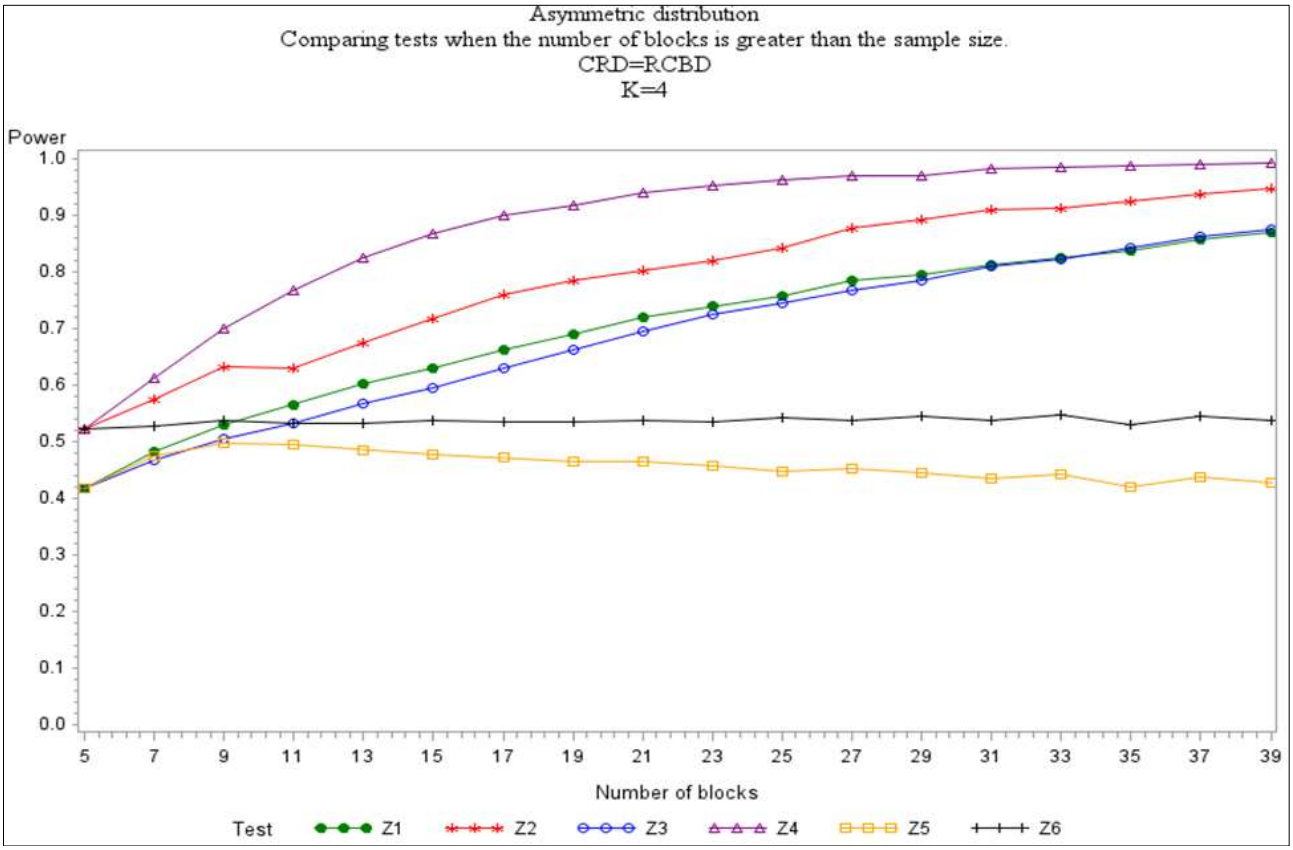

Fig 1: Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=RCBD; K=4; n_ a = 5, and n_ b= 5, 7, 9,…, 39.

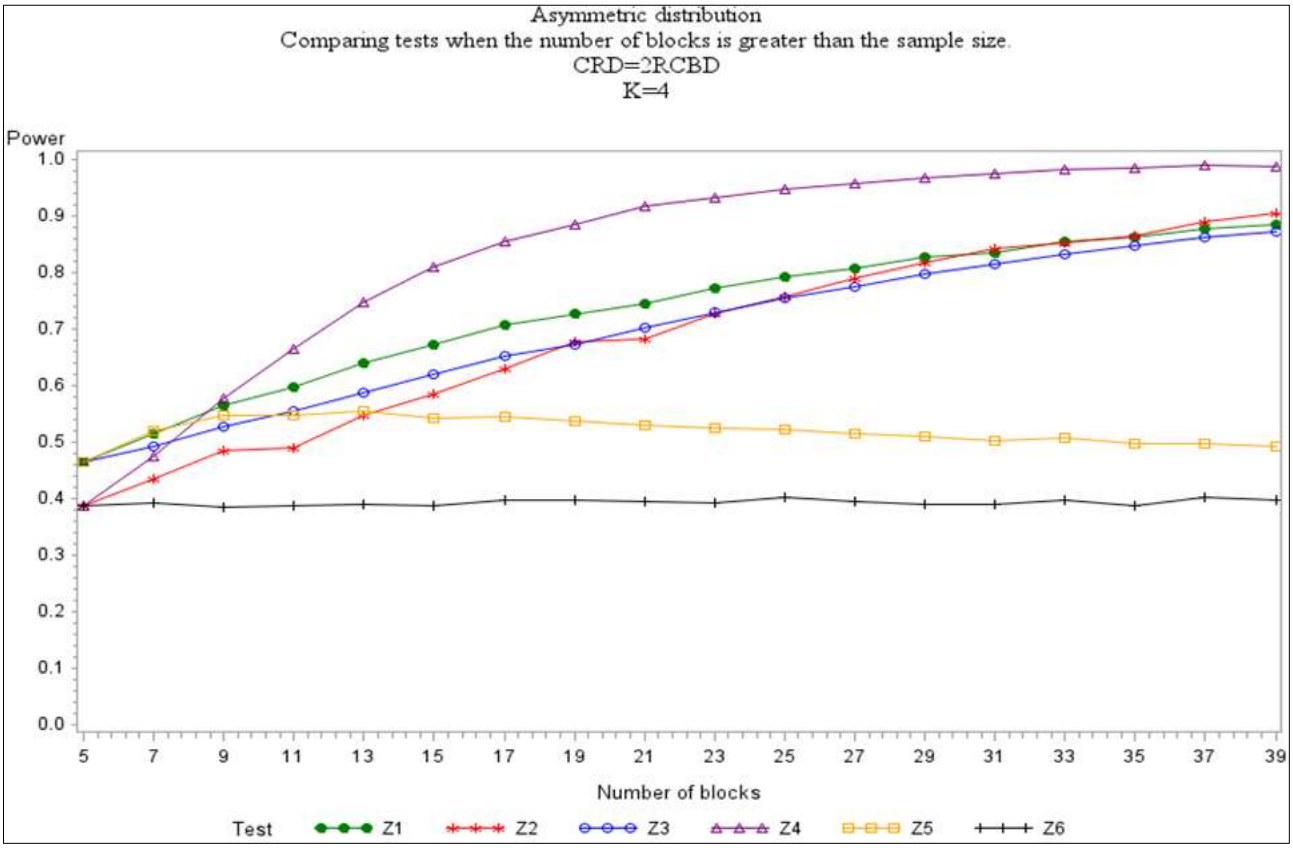

Fig 2: Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=2RCBD; K=4; n_ a=5, and n_ b=5, 7, 9,…, 39.

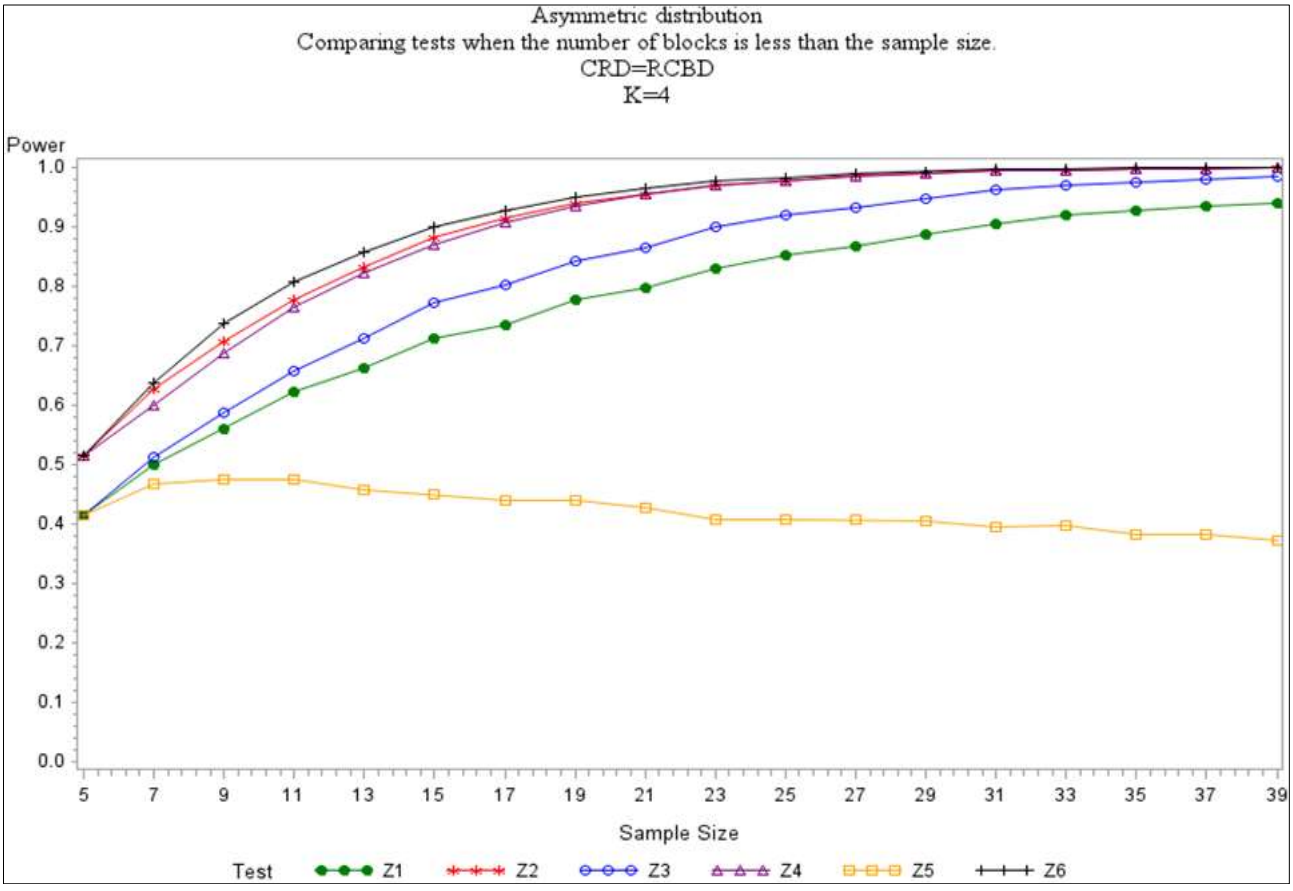

Fig 2: Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=RCBD; K=4; n_ a=5, 7, 9,…, 39, and n_ b=5.

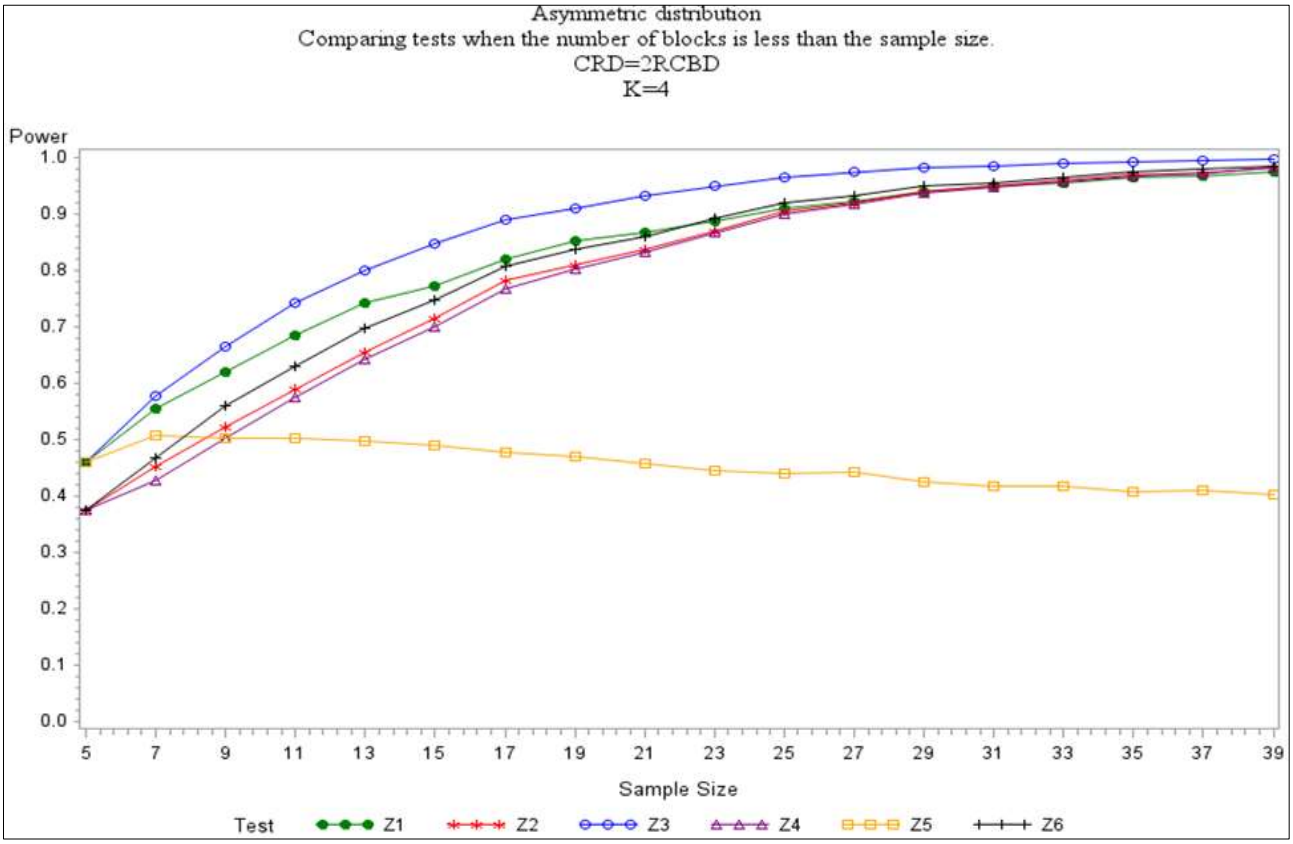

Fig 4: Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=2RCBD; K=4; n_ a=5, 7, 9… 39, and n_ b=5.

4. Conclusion

In this paper, the observations are assumed to come from an asymmetric distribution (exponential). We overall recommend the 4th test statistic, Z_4, when the number of blocks is greater than the sample size in the CRD portion. In the case where the number of blocks is less than the sample size and the variances are equal, the recommendation is to use the 6th test statistic, Z_6. However, in this last case, Z_4 may also be used since the powers of Z_4 are close to those of Z_6 and become closer as the sample size increases in relation to the number of blocks (see Figure 3). When the number of blocks is equal to or less than the common sample size in the CRD portion and the variance of the CRD is greater than the variance of the RCBD, we recommend the 3rd test statistic, Z_3. In summary, we can recommend that the researcher may use Z_4 when the underlying distribution is assumed to be right skewed, with the possible exception as to when the number of blocks is less than the sample size and the variance of the CRD portion is greater than the variance of the RCBD portion. In this case Z_3 is recommended.

5. References

1. Fouad K, Rhonda M. Proposed Nonparametric Tests for the Simple Tree Alternative for Location and Scale Testing in a Mixed Design. International Journal of Engineering Science Invention. 2022;11(3):60-68.
2. Fligner M, Wolfe D. Distribution-free tests for comparing several treatments with a control. Statistica Neerlandica. 1982;36(3):119-127.
3. Susan O, Rhonda M. A Comparison of Nonparametric Tests in a Mixed Design for the Simple Tree Alternative. International Journal for Research in Business Management and Accounting. 2017;3(3):1-23.
4. Ansari AR, Bradley RA. Rank-Sum Tests for Dispersion. Annals of Mathematical Statistics. 1960;31:1174-1189.
5. Lepage Y. A combination of Wilcoxon's and Ansari-Bradley's statistics. Biometrika. 1971;58(1):213-217.
6. Duran BS, Tsai WS, Lewis TO. A class of location-scale tests, Biometrika. 1976;63(1):173-176.
7. SAS Institute Inc. SAS® 9.3 Foundation and CALL Routines: Reference. Cary, NC: SAS Institute Inc.; c2011.