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Shrinkage estimation of stress strength reliability P (Y<X<Z) for Lomax distribution based on records

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Abstract
The present paper deals with the Shrinkage Estimation of stress strength reliability model R=P[Y<X<Z] Where X is the random strength and Y and Z are independent random stress variables follows Lomax Distribution based on record values. In this paper we obtain the shrinkage estimation of R based on record values using Classical and Bayesian Estimation method when the scale parameter λ which is common for all the distribution is known. We illustrated the performance of the estimators using simulation study.

Keywords: Lomax distribution, stress strength reliability, maximum likelihood estimation, record values, shrinkage estimator, modified thompson type shrinkage estimator, bayesian estimation, linex loss function, squared error loss function

1. Introduction

In the literature the problem of estimating the stress strength reliability R= P[X>Y] has been considered as both distribution free and parametric frame works. In stress strength reliability analysis the strength X and the stress Y are considered as random variables. In reliability studies the stress- strength model describes the life of a component which has a random strength X and is subjected to a random stress Y. In stress strength model the system fails if at any time the applied stress is exceeds its strength, if the system functions only if its inherent random strength is greater than the random stress applied on it. The stress strength reliability have wide applications in Quality control, Engineering Statistics, Medical Statistics, Bio statistics etc. An important case is estimation of R =P(Y<X<Z) which represents the situation where the strength X should not only be greater than stress Y but also be smaller than the stress Z. The model of P(Y<X<Z) have wide application in various subareas of engineering, psychology, genetics, clinical trials etc. Minimum Variance Unbiased (MVU), Maximum Likelihood and Empirical Estimator of R= P[$Y < X < Z$] was discussed by Singh (1980) [15]. Dutta and Sriwastav (1986) [5] deals with the estimation of R when Y, Z and X are exponential random variables. Maximum Likelihood Estimate and Uniformly Minimum Variance Unbiased Estimate of R when Y, Z and X either uniform or exponential random variable with the unknown location parameter was considered by Ivshin (1998) [11]. Hassan *et al.* (2013) [10] focused on the estimate of R= P[Y $< X < Z$], where Y and Z be a random stress and X be a random strength have Weibull Distribution in presence of k outliers. Hameed *et al.* (2020) [9] focused on the estimate of R= P[Y $< X < Z$], when Y, Z and X are independent and that these stress and strength variable follows Kumaraswamy Distribution. Karam and Ali (2021) [13] discuss the estimation of Stress – Strength Reliability for P[Y<X<Z] using Dagum Distribution.

A shrinkage estimator is a new estimate produced by shrinking a raw estimate. For example two extreme mean values can be combined to make one more centralized mean value; repeating this for all means in a sample will result in a revised sample mean that has "shrunk" towards the true population mean. Shrinkage is where extreme values in a sample are "shrunk" towards a central value and the shrinkage estimator performs better than the usual estimator when the initial value is close to the true value of the parameter Thompson (1968) [17] suggested the single stage shrinkage estimators for the mean of a normal distribution when an initial value μ_0 of the mean is available by moving the usual estimator towards initial value.

Glifin Francis et.al. (2022) [8] obtained the shrinkage estimator of stress strength reliability R= P[X<Y] when X and Y are geometric distribution using record values.

Record values can be viewed as order statistics from a sample whose size is determined by the values and the order of occurrence of the observations. In any field, whenever a high or low value is observed, in connection with the phenomena under study, it becomes a part of history and will be designated as a record. Record values and associated statistics have wide application in meteorology, hydrology, sports and life tests. Chandler (1952) [3] introduced and studied some properties of record values. Feller (1966) [7] presented the some examples of record values and record value times. Balakrishnan and Ahsanullah (1994) [2] discussed the Relations for single and product moments of record values from Lomax Distribution. Ahsanullah (1991) [1] obtained the Record values of the Lomax Distribution.

The layout of this paper is organized as follows. In section 2 we obtain the shrinkage estimation of R based on record values. In section 3 we obtain the Bayesian shrinkage estimation of R based on record values. In Section 4 we illustrate the performance of the estimator by simulation study. Finally in section 5 conclusions are presented.

2. Shrinkage Estimation of R based on Record Values

Let X be the strength of the random variable following Lomax distribution with parameters $L(\alpha_1,\lambda)$, where α_1 is the shape parameter and λ is scale parameter and Y and Z be the stress of the random variable following Lomax distribution with parameter $L(\alpha_2,\lambda)$ and $L(\alpha_3,\lambda)$ corresponding probability density functions are given below.

$$f(x,\alpha_1,\lambda) = \frac{\alpha_1\lambda^{\alpha_1}}{(x+\lambda)^{\alpha_1+1}};\ x > 0,\ \alpha_1 > 0,\ \lambda > 0 \tag{2.1}$$

$$f(y,\alpha_2,\lambda) = \frac{\alpha_2\lambda^{\alpha_2}}{(y+\lambda)^{\alpha_2+1}};\ y > 0,\ \alpha_2 > 0,\ \lambda > 0 \tag{2.2}$$

$$f(z,\alpha_3,\lambda) = \frac{\alpha_3\lambda^{\alpha_3}}{(z+\lambda)^{\alpha_3+1}};\ z > 0,\ \alpha_3 > 0,\ \lambda > 0. \tag{2.3}$$

The stress strength reliability $R = P[Y < X < Z] = \int_0^\infty P(Y < x, Z > x / X = x)\, dF_x(x)$

$= \int_0^\infty F_y(x)\, \overline{F_z}(x)\, f(x)\, dx$

$$= \frac{\alpha_1\alpha_2}{(\alpha_1+\alpha_3)((\alpha_1+\alpha_2+\alpha_3)}:\ 0<R<1. \tag{2.4}$$

Let $\underline{r} = (r_0, r_1, \ldots, r_{n_1})$ be the first (n_1+1) upper record values with $r_0 = x_1$, from (2.1) then its likelihood function is given by

$$L(\underline{r}\,|\alpha_1,\lambda) = f(r_n)\left[\prod_{i=0}^{n_1-1}\frac{f(r_i)}{\bar{F}(r_i)}\right] = {\alpha_1}^{n_1+1}\,\lambda^{\alpha_1}\,(r_{n_1}+\lambda)^{-\alpha_1}\,\prod_{i=1}^{n_1}\frac{1}{(r_i+\lambda)} \tag{2.5}$$

Let $\underline{y} = (y_1, y_2, \ldots, y_{n_2})$ be the random sample of n_2 observation from (2.2) then its likelihood function is given by

$$L(\underline{y}\,|\alpha_2,\lambda) = \prod_{j=1}^{n_2}\frac{\alpha_2\lambda^{\alpha_2}}{(y+\lambda)^{\alpha_2+1}} = {\alpha_2}^{n_2}\,\lambda^{n_2\alpha_2}\,\prod_{j=1}^{n_2}(y_j+\lambda)^{-(\alpha_2+1)} \tag{2.6}$$

Let $\underline{z} = (z_1, z_2, \ldots, z_{n_3})$ be the random sample of n_3 observation from (2.3) then its likelihood function is given by

$$L(\underline{z}\,|\alpha_3,\lambda) = \prod_{k=1}^{n_3}\frac{\alpha_3\lambda^{\alpha_3}}{(z+\lambda)^{\alpha_3+1}} = {\alpha_3}^{n_3}\,\lambda^{n_3\alpha_3}\,\prod_{k=1}^{n_3}(z_k+\lambda)^{-(\alpha_3+1)} \tag{2.7}$$

The joint likelihood function is given by

$$L(\underline{r},\underline{y},\underline{z}|\alpha_1,\alpha_2,\alpha_3,\lambda) = {\alpha_1}^{n_1+1}\,\lambda^{\alpha_1}\,(r_{n_1}+\lambda)^{-\alpha_1}\,\prod_{i=1}^{n_1}(r_i+\lambda)^{-1}\,{\alpha_2}^{n_2}\,\lambda^{n_2\alpha_2}$$

$$\prod_{j=1}^{n_2}(y_j+\lambda)^{-(\alpha_2+1)}\,{\alpha_3}^{n_3}\,\lambda^{n_3\alpha_3}\,\prod_{k=1}^{n_3}(z_k+\lambda)^{-(\alpha_3+1)} \tag{2.8}$$

From (2.8) we can obtain the MLE of $\alpha_1, \alpha_2, \alpha_3$, as

$$\widehat{\alpha_1} = \frac{n_1+1}{\log(1+\frac{r_{n_1}}{\lambda})} \tag{2.9}$$

$$\widehat{\alpha_2} = \frac{n_2}{\sum_{j=1}^{n_2}\log(1+\frac{y_j}{\lambda})} \tag{2.10}$$

$$\widehat{\alpha_3} = \frac{n_3}{\sum_{k=1}^{n_3} \log(1+\frac{z_k}{\lambda})} \tag{2.11}$$

Now using (2.9), (2.10) and (2.11) the MLE of R is given by

$$\hat{R}_{mle} = \frac{\widehat{\alpha_1}\widehat{\alpha_2}}{(\widehat{\alpha_1}+\widehat{\alpha_3})(\widehat{\alpha_1}+\widehat{\alpha_2}+\widehat{\alpha_3})};\ 0<R<1, \tag{2.12}$$

2.1 Shrinkage Estimation with Constant shrinkage factor
In this case we obtain the shrinkage estimate given by Abdul Hameed (2020)

$$\hat{\beta}_{sh} = \psi(\hat{\beta})\hat{\beta}_{ub} + (1-\psi(\hat{\beta}))\hat{\beta}_0$$

with $\psi(\hat{\beta}) = 0.01$ the constant shrinkage weight factor suggested by Hameed et.al. (2020) this leads the Shrinkage estimates of $\alpha_1, \alpha_2,$ and α_3 as

$$\hat{\alpha}_{1sh} = 0.01\hat{\alpha}_{1ub} + 0.99\hat{\alpha}_{10} \tag{2.13}$$

$$\hat{\alpha}_{2sh} = 0.01\hat{\alpha}_{2ub} + 0.99\hat{\alpha}_{20} \tag{2.14}$$

And

$$\hat{\alpha}_{3sh} = 0.01\hat{\alpha}_{3ub} + 0.99\hat{\alpha}_{30} \tag{2.15}$$

where $\hat{\alpha}_{1ub} = = \frac{n_1-1}{\sum_{i=1}^{n_1}\log\left(1+\frac{x_i}{\lambda}\right)}, \hat{\alpha}_{2ub} = \frac{n_2-1}{\sum_{j=1}^{n_2}\log(1+\frac{y_j}{\lambda})}$ and $\hat{\alpha}_{3ub} = \frac{n_3-1}{\sum_{k=1}^{n_3}\log(1+\frac{z_k}{\lambda})}$.

$\hat{\alpha}_{10}, \hat{\alpha}_{20}$ and $\hat{\alpha}_{30}$ is taken as the boot strap estimate of $\alpha_1, \alpha_{2,}\alpha_{3,}$.

$$\hat{R}_{sh} = \frac{\hat{\alpha}_{1sh}\hat{\alpha}_{2sh}}{(\hat{\alpha}_{1sh}+\hat{\alpha}_{2sh}+\hat{\alpha}_{3sh})(\hat{\alpha}_{1sh}+\hat{\alpha}_{3sh})};\ 0<R<1, \tag{2.16}$$

From (2.16) expression, it is very difficult to find the exact variance and distribution of $\hat{R}_{sh}$. So we use the multivariate delta method (See Wasserman, (2003) [18], Soliman *et al.* (2013) [16], Dhanya, M. and Jeevavand, E. S. (2018) [4], Khan, M.J.S. and Khatoon, B. (2019) [12] to find the approximate estimate of the asymptotic variance of $\hat{R}_{sh}$ which is obtained as follows

Let the Fisher Information matrix $\emptyset$

$$\emptyset\ (\propto_1, \propto_2, \propto_3) = \begin{bmatrix} E\left(\frac{-\partial^2 lnL}{\partial\propto_1^2}\right) & E\left(\frac{-\partial^2 lnL}{\partial\propto_1\partial\propto_2}\right) & E\left(\frac{-\partial^2 lnL}{\partial\propto_1\partial\propto_3}\right) \\ E\left(\frac{-\partial^2 lnL}{\partial\propto_2\partial\propto_1}\right) & E\left(\frac{-\partial^2 lnL}{\partial^2\propto_2}\right) & E\left(\frac{-\partial^2 lnL}{\partial\propto_2\partial\propto_3}\right) \\ E\left(\frac{-\partial^2 lnL}{\partial\propto_3\partial\propto_1}\right) & E\left(\frac{-\partial^2 lnL}{\partial\propto_3\partial\propto_2}\right) & E\left(\frac{-\partial^2 lnL}{\partial\propto_3^2}\right) \end{bmatrix} \tag{2.17}$$

and

$$B' = \begin{bmatrix} \frac{\partial R}{\partial\propto_1} & \frac{\partial R}{\partial\propto_2} & \frac{\partial R}{\partial\propto_3} \end{bmatrix} = [b_1\ b_2\ b_3] \tag{2.18}$$

Then σ_R^2 = V(R) = B' $\emptyset^{-1}$B.
In this case

$$\emptyset\ (\propto_1, \propto_2, \propto_3) = \begin{bmatrix} \frac{n_1+1}{\propto_1^2} & 0 & 0 \\ 0 & \frac{n_2}{\propto_2^2} & 0 \\ 0 & 0 & \frac{n_3}{\propto_3^2} \end{bmatrix} \text{ so } \emptyset^{-1} = \begin{bmatrix} \frac{\propto_1^2}{n_1+1} & 0 & 0 \\ 0 & \frac{\propto_2^2}{n_2} & 0 \\ 0 & 0 & \frac{\propto_3^2}{n_3} \end{bmatrix}$$

Also

$$b_1 = \frac{\partial R}{\partial \alpha_1} = \frac{-\alpha_2(\alpha_1^2 - \alpha_2\alpha_3 - \alpha_2^2)}{(\alpha_1+\alpha_2)^2\,(\alpha_1+\alpha_2+\alpha_3)^2} \tag{2.19}$$

$$b_2 = \frac{\partial R}{\partial \alpha_2} = \frac{-\alpha_1(\alpha_2^2 - \alpha_1\alpha_3 - \alpha_1^2)}{(\alpha_1+\alpha_2)^2\,(\alpha_1+\alpha_2+\alpha_3)^2} \tag{2.20}$$

And

$$b_3 = \frac{\partial R}{\partial \alpha_3} = - \frac{-\alpha_1\alpha_2}{(\alpha_1+\alpha_3)(\alpha_1+\alpha_2+\alpha_3)^2}. \tag{2.21}$$

Then

$$\sigma^2_{R_{sh}} = V(R) = B^1\,\emptyset^{-1}B = \frac{b_1^2\alpha_1^2}{n_1+1} + \frac{b_2^2\alpha_2^2}{n_2} + \frac{b_3^2\alpha_3^2}{n_3}. \tag{2.22}$$

By replacing the parameters with their shrinkage estimate we get the estimate $\hat{\sigma}^2_{R_{sh}}$ of $\sigma^2_{R_{sh}}$. In this case he asymptotic distribution of $\hat{R}_{sh}$ is N(R, $\hat{\sigma}^2_{Rmle}$).

2.2 The Modified Thompson Type shrinkage estimator

Here we use two the modified Thompson type shrinkage weight factor to find out the shrinkage estimator.
(a) suggested by Hameed et.al. (2020) here we take the weight factor as

$$\phi(\hat{R}) = \frac{\hat{R}_{ub}-\hat{R}_0}{(\hat{R}_{ub}-\hat{R}_0)^2+var(\hat{R}_{ub})}(0.001) \tag{2.23}$$

where $\hat{R}_{ub} = \frac{\widehat{\alpha}_{1ub}\,\widehat{\alpha}_{2ub}}{(\widehat{\alpha}_{1ub}+\widehat{\alpha}_{2ub}+\widehat{\alpha}_{3ub})(\widehat{\alpha}_{1ub}+\widehat{\alpha}_{3ub})}$ and $var(\hat{R}_{ub})$ is as defined in (2.22). So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{Th} = \phi(\hat{R})\hat{R}_{ub} + (1-\phi(\hat{R}))\hat{R}_0 \tag{2.24}$$

(b) Shrinkage weight factor suggested by Mehta & Srinivasan (1971) here we take the weight factor as

$$\varphi(\hat{R}) = a.exp\left\{-\frac{b(\hat{R}_{ub}-\hat{R}_0)^2}{var(\hat{R}_{ub})}\right\} \tag{2.25}$$

where $0<a<1$and $b>0$ So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{MS} = \varphi(\hat{R})\hat{R}_{ub} + (1-\varphi(\hat{R}))\hat{R}_0. \tag{2.26}$$

3. Bayesian Shrinkage Estimation of R based on Record Values

In this section we estimate the Bayesian shrinkage estimate of R under Squares Error and Linex Loss functions when λ is known.
Let $\underline{r} = (r_0, r_1, \ldots\ldots r_{n_1})$ be the first (n_1+1) upper record values with $r_0 = x_1$, from (2.1)
The gamma prior for α_1 is g (α_1) α $\alpha_1{}^{p-1}e^{-\alpha_1\tau}$, $\alpha_1, \tau, p > 0$ (3.1)

Combining the likelihood function (2.5) and the prior distribution (3.1) and the posterior density of α_1 is derived as follows.

The posterior density of α_1 is given by

$$f(\alpha_1 \,|\underline{x})\ \alpha\ \alpha_1{}^{n_1+p}e^{-\alpha_1(\tau+\,log\left(\frac{r_{n_1}+\lambda}{\lambda}\right))}\ \alpha\ \alpha_1{}^{n_1+p}e^{-\alpha_1 U}$$

$$f(\alpha_1 \,|\underline{x})\ \alpha\ \alpha_1{}^{V-1}e^{-\alpha_1 U}. \tag{3.2}$$

where U= $\tau + log\left(\frac{r_{n_1}+\lambda}{\lambda}\right)$, V= n_1+ p+1

Similarly let $\underline{y} = (y_1, y_2 \ldots . y_{n_2})$ be the random sample of n_2 observation from (2.2)
The gamma prior for α_2 is g (α_2) α $\alpha_2{}^{q-1}e^{-\alpha_2\Psi}$, $\alpha_2, \Psi, q > 0$ (3.3)
Combining the likelihood function (2.6) and the prior distribution (3.3) and the posterior density of α_2 is derived as follows.
The posterior density of α_2 is

$$f(\alpha_2 | \underline{y}) \propto {\alpha_2}^{n_2+q-1} e^{-\alpha_2(\Psi+\sum_{j=1}^{n_2} log(y_j+\lambda)-n_2 log\lambda)} \propto {\alpha_2}^{Q-1} e^{-\alpha_2 M} \quad (3.4)$$

Where Q= n_2+ q, M= $\Psi + \sum_{j=1}^{n_2} log\left(y_j + \lambda\right) - n_2 log\lambda$
Similarly let $\underline{z}$ = $(z_{1,} z_2 \dots . z_{n_3})$ be the random sample of n_3 observation from (2.3)

The gamma prior for α_3 is g $(\alpha_3) \propto {\alpha_3}^{r-1} e^{-\alpha_3 \eta}$, $\alpha_3, \eta, r > 0$ (3.5)

Combining the likelihood function (2.7) and the prior distribution (3.5) and the posterior density of α_3 is derived as follows. The posterior density α_3 of is

$$f(\alpha_3 | \underline{z}) \propto {\alpha_3}^{n_3+r-1} e^{-\alpha_3(\eta + \sum_{k=1}^{n_3} log(z_k+\lambda)-n_3 log\lambda)} \propto {\alpha_3}^{W-1} e^{-\alpha_3 N} \quad (3.6)$$

Where W= n_3+ r, N= $\eta + \sum_{k=1}^{n_3} log\left(z_k + \lambda\right) - n_3 log\lambda$

Assume that $\alpha_1 \alpha_2 and\ \alpha_3$ are independently distributed the joint posterior density can be written as

$$f(\alpha_1, \alpha_{2,} \alpha_3) = {\alpha_1}^{V-1} e^{-\alpha_1 U} {\alpha_2}^{Q-1} e^{-\alpha_2 M} {\alpha_3}^{W-1} e^{-\alpha_3 N} = {\alpha_1}^{V-1} {\alpha_2}^{Q-1} {\alpha_3}^{W-1} e^{-(\alpha_1 U+\alpha_2 M+\alpha_3 N)} \quad (3.7)$$

Applying the transformation R = $\frac{\alpha_1 \alpha_2}{(\alpha_1+\alpha_3)(\alpha_1+\alpha_2+\alpha_3)}$, S = $\alpha_1 + \alpha_2 + \alpha_3$ and T= $\alpha_{1+} \alpha_3$ S>0, T>0,0<R<1 which gives $\alpha_1 = \frac{RST}{S-T}$,$\alpha_2 = S - T, and\ \alpha_3 = \frac{[(1-R)TS-T^2]}{S-T}$

$$f(R,S,T | \underline{x}, \underline{y}, \underline{z}) \propto \left(\frac{RST}{S-T}\right)^{V-1} (S-T)^{Q-1} \left(\frac{[(1-R)TS-T^2]}{S-T}\right)^{W-1} e^{-\left(\left(\frac{RST}{S-T}\right)U+(S-T)M+\left(\frac{[(1-R)TS-T^2]}{S-T}\right)N\right)} \frac{ST}{S-T}$$

$$\propto R^{V-1} S^V T^V (S-T)^{Q-V-W} \left((1-R)TS-T^2\right)^{W-1} e^{-\left(\left(\frac{RST}{S-T}\right)U+(S-T)M+\left(\frac{[(1-R)TS-T^2]}{S-T}\right)N\right)} \quad (3.8)$$

$$f(R,S,T | \underline{x}, \underline{y}, \underline{z}) = \left(C_1(0)\right)^{-1} R^{V-1} S^V T^V (S-T)^{Q-V-W} \left((1-R)TS-T^2\right)^{W-1} e^{-\left(\left(\frac{RST}{S-T}\right)U+(S-T)M+\left(\frac{[(1-R)TS-T^2]}{S-T}\right)N\right)},$$
S>0,T>0,0<R<1 (3.9)

where C is the normalizing constant
The Bayes estimate of R under the squared error loss function is given by

$$\hat{R}_{SELF} = E(R | \underline{x}, \underline{y}, \underline{z}) = \int_0^1 R f(R | \underline{x}, \underline{y}, \underline{z})\, dR = \frac{C_1(1)}{C_1(0)} \quad (3.10)$$

where $C_1(d) = \int_0^1 \int_0^\infty \int_0^\infty R^d R^{V-1} S^V T^V (S-T)^{Q-V-W} \left((1-R)TS-T^2\right)^{W-1} e^{-\left(\left(\frac{RST}{S-T}\right)U+(S-T)M+\left(\frac{[(1-R)TS-T^2]}{S-T}\right)N\right)} dTdSdR$

$$= \int_0^1 \int_0^\infty \int_0^\infty R^{V+d-1} S^V T^V (S-T)^{Q-V-W} \left((1-R)TS-T^2\right)^{W-1} e^{-\left(\left(\frac{RST}{S-T}\right)U+(S-T)M+\left(\frac{[(1-R)TS-T^2]}{S-T}\right)N\right)} dTdSdR \quad (3.11)$$

Under Linex Loss Function the Bayes estimate of R is given by

$$\hat{R}_{LLF} = \frac{1}{k} \log G_1 \quad (3.12)$$

Where

$$G_1 = (C_1(0))^{-1} \int_0^1 \int_0^\infty \int_0^\infty R^{V-1} S^V T^V (S-T)^{Q-V-W} \left((1-R)TS-T^2\right)^{W-1} e^{-\left(\left(\frac{RST}{S-T}\right)U+(S-T)M+\left(\frac{[(1-R)TS-T^2]}{S-T}\right)N+kR\right)} dTdSdR \quad (3.13)$$

3.1 Shrinkage Estimation
In this case we obtain the shrinkage Bayes estimate $\hat{R}_{SELFsh}$ and $\hat{R}_{ELFsh}$ as

$$\hat{R}_{SELFsh} = \psi(\hat{\beta})\hat{R}_{SELF} + (1-\psi(\hat{\beta}))\hat{R}_0 \quad (3.14)$$

And

$$\hat{R}_{LLF} = \psi(\hat{\beta})\hat{R}_{LLF} + (1-\psi(\hat{\beta}))\hat{R}_0 \quad (3.15)$$

with $0 \le \psi(\hat{\beta}) \le 1$ and $\hat{R}_0$ is the boot strap estimate of R obtained by taking prior parameters $p = q = r = \tau = \Psi = \eta = 0$

4. Simulation Study

To obtain the boot strap estimate of R we use the following boot strap algorithm (Efron 1982).

Step-1. Simulate a random sample from Uniform (0,1). Using this simulated value compute random sample for $X \sim \mathrm{L}(\lambda,\ \alpha_1)$, $Y \sim \mathrm{L}(\lambda,\ \alpha_2)$ and $Z \sim \mathrm{L}(\lambda,\ \alpha_3)$ respectively.

Compute the MLE of α_1, α_2, α_3 say $\hat{\alpha}_{1sh}$, $\hat{\alpha}_{2sh}$, $\hat{\alpha}_{3sh}$ given in setion-2.

Step-2. Generate an independent parametric bootstrap sample using $\hat{\alpha}_{1sh}$, $\hat{\alpha}_{2sh}$, $\hat{\alpha}_{3sh}$ instead of α_1, α_2, α_3.

Step-3. Calculate the maximum likelihood estimate of $\hat{\alpha}_{1sh}$, $\hat{\alpha}_{2sh}$, $\hat{\alpha}_{3sh}$ obtained in step-2 say $\hat{\alpha}'_{1sh}$, $\hat{\alpha}'_{2sh}$, $\hat{\alpha}'_{3sh}$

Step-4. Repeat the step-2 and step-3 N times to obtained the parametric bootstrap estimates $\hat{\alpha}_{10}$, $\hat{\alpha}_{20}$ and $\hat{\alpha}_{30}$. Then using these values, calculate $\hat{R}_{sh}$. In the absence of real data finally, we study the performance of the estimates obtained in the above section using Monte Carlo Simulated data sets. All the computations are done by using R Program. Generate the sample of sizes $(n_1 = n_2, n_3)(10,10), (10,25), (10,50), (25,10), (25,25), (25,50), (50,10), (50,25), (50,50)$ from Lomax Distribution with parameter values 0.5, 2, 3.5 for α_1, α_2 and α_3. The bias, mean square error of estimates of R given in the following table.

Table 1: Bias, MSEs (in parentheses), of the estimates of, $\hat{R}_{sh}$, $\hat{R}_{Th}$, $\hat{R}_{MS}$

$n_1 = n_2$	n_3	$\alpha_1 = \alpha_2$	α_3	$\hat{R}_{sh}$ Bias (MSE)	$\hat{R}_{Th}$ Bias (MSE)	$\hat{R}_{MS}$ Bias (MSE)
10	10	0.5	0.5	-0.00701	-0.20758	-0.07514
				(0.00116)	(0.06457)	(0.01668)
	25			-0.02565	-0.17593	-0.04839
				(0.00312)	(0.03719)	(0.00861)
	50			-0.0058	-0.21804	-0.01492
				(0.00092)	(0.07003)	(0.00627)
25	10			0.08572	-0.22092	-0.16167
				(0.00771)	(0.09948)	(0.010782)
	25			0.07823	-0.19545	-0.06298
				(0.00645)	(0.06981)	(0.01592)
	50			0.08142	-0.23407	-0.11294
				(0.00712)	(0.06299)	(0.02260)
50	10			0.09930	-0.20892	-0.01861
				(0.01017)	(0.06356)	(0.00953)
	25			0.09656	-0.06563	-0.0113
				(0.00945)	(0.00513)	(0.00050)
	50			0.08088	0.20116	-0.03554
				(0.00678)	(0.07384)	(0.00585)
10	10	0.5	2	-0.03095	-0.017106	-0.02739
				(0.00146)	(0.05878)	(0.012126)
	25			-0.02023	-0.08261	-0.00382
				(0.00096)	(0.02445)	(0.00024)
	50			-0.01662	-0.04911	-0.01169
				(0.00055)	(0.00761)	(0.00047)
25	10			-0.0129	-0.06309	-0.01212
				(0.00078)	(0.00906)	(0.00026)
	25			-0.01053	-0.04558	-0.00254
				(0.00028)	(0.00565)	(0.00008)
	50			-0.00947	-0.02666	-0.00532
				(0.00029)	(0.00213)	(0.00010)
50	10			-0.01101	-0.0406	-0.01202
				(0.00057)	(0.00360)	(0.00039)
	25			-0.00945	-0.0139	-0.00166
				(0.00015)	(0.00045)	(0.00003)
	50			-0.00828	-0.03595	-0.00505
				(0.00018)	(0.00285)	(0.00009)
10	10	0.5	3.5	-0.02125	-0.12268	0.00732
				(0.00192)	(0.08335)	(0.00064)
	25			-0.03046	-0.1301	-0.00482
				(0.00237)	(0.06001)	(0.00011)
	50			-0.01720	-0.0617	-0.00786
				(0.00052)	(0.01401)	(0.00012)
25	10			-0.01614	-0.06476	-0.00342
				(0.00060)	(0.01402)	(0.00010)
	25			-0.02148	-0.0975	0.00116
				(0.00076)	(0.02243)	(0.00016)
	50			-0.02540	-0.06253	-0.00935

				(0.00088)	(0.00591)	(0.00010)
50	10			-0.01615	-0.04646	-0.00815
				(0.00033)	(0.00408)	(0.00010)
	25			-0.0182	-0.06934	-0.00083
				(0.00039)	(0.00751)	(0.00012)
	50			-0.01981	0.06253	-0.00944
				(0.00045)	(0.00951)	(0.00010)
10	10	2	0.5	0.07580	-0.03661	0.04311
				(0.01815)	(0.01856)	(0.00324)
	25			0.05269	-0.21105	-0.12856
				(0.02050)	(0.09588)	(0.07451)
	50			0.06592	-0.21946	-0.03181
				(0.02050)	(0.09899)	(0.00548)
25	10			-0.03612	-0.20845	0.01071
				(0.00354)	(0.073041)	(0.00221)
	25			-0.06057	-0.26658	-0.01921
				(0.01217)	(0.09346)	(0.00270)
	50			-0.03808	-0.2774	0.00283
				(0.00464)	(0.08461)	(0.00164)
50	10			0.02926	-0.20393	0.05149
				(0.00492)	(0.09729)	(0.00395)
	25			0.01559	-0.11548	0.02186
				(0.00478)	(0.03601)	(0.00103)
	50			0.03052	0.18074	0.04785
				(0.00354)	(0.06296)	(0.00373)
10	10	2	2	-0.0264	-0.18963	-0.00191
				(0.00603)	(0.05543)	(0.000179)
	25			-0.00321	-0.10259	0.02894
				(0.00231)	(0.02821)	(0.00181)
	50			0.00478	-0.16248	0.01924
				(0.00290)	(0.06628)	(0.00227)
25	10			-0.03938	-0.20862	-0.06496
				(0.00285)	(0.08678)	(0.00728)
	25			-0.01123	-0.0626	-0.00457
				(0.00050)	(0.00794)	(0.00010)
	50			-0.00521	-0.17162	-0.03161
				(0.00049)	(0.06574)	(0.00183)
50	10			-0.01246	-0.19903	-0.01983
				(0.00213)	(0.06816)	(0.00138)
	25			0.01045	-0.06173	0.01934
				(0.00124)	(0.01823)	(0.00113)
	50			0.01882	-0.05177	-0.00342
				(0.00064)	(0.00754)	(0.00010)
10	10	2	3.5	-0.02149	-0.12207	0.02982
				(0.00332)	(0.08970)	(0.00294)
	25			-0.02169	-0.0853	0.02855
				(0.00260)	(0.04194)	(0.00088)
	50			-0.02889	-0.15902	0.02889
				(0.00356)	(0.09628)	(0.00132)
25	10			-0.00543	-0.13705	-0.00903
				(0.00123)	(0.03538)	(0.00110)
	25			-0.01145	-0.13158	0.00335
				(0.00082)	(0.04930)	(0.00017)
	50			-0.01903	-0.12326	-0.01346
				(0.00105)	(0.02365)	(0.00025)
50	10			-0.01226	-0.09124	0.00770
				(0.00545)	(0.03720)	(0.00090)
	25			-0.01852	-0.09225	0.02056
				(0.00553)	(0.01971)	(0.00054)
	50			-0.00645	-0.10034	0.00144
				(0.00101)	(0.02147)	(0.00023)
10	10	3.5	0.5	-0.04111	0.05690	0.3267
				(0.01207)	(0.07114)	(0.07409)
	25			-0.05424	0.13450	0.28117
				(0.00831)	(0.08507)	(0.096151)
	50			-0.03176	0.20938	0.27899
				(0.00577)	(0.05707)	(0.09212)
25	10			0.01210	0.06035	0.29035
				(0.00423)	(0.05079)	(0.08737)

	25			0.01084	-0.12604	0.28852
				(0.00194)	(0.09642)	(0.08482)
	50			0.01789)	-0.1666	0.26743
				(0.00294)	(0.09883)	(0.07214)
50	10			0.02293	-0.14975	0.14911
				(0.00232)	(0.03821)	(0.02371)
	25			-0.01046	-0.21434	0.14916
				(0.00338)	(0.08174)	(0.02374)
	50			0.00494	-0.16677	0.13720
				(0.00168)	(0.04966)	(0.02152)
10	10	3.5	2	0.03993	0.13473	0.17621
				(0.00902)	(0.09751)	(0.03339)
	25			0.039241	-0.12197	-0.02427
				(0.00489)	(0.07512)	(0.00120)
	50			0.058056	0.06207	0.14659
				(0.00396)	(0.02035)	(0.02243)
25	10			-0.03511	0.10015	0.10224
				(0.00778)	(0.04100)	(0.01126)
	25			-0.00374	0.08097	0.13958
				(0.00272)	(0.05094)	(0.02136)
	50			0.01293	0.09000	0.13845
				(0.00213)	(0.00962)	(0.02060)
50	10			-0.01243	-0.05436	-0.05248
				(0.006789)	(0.03444)	(0.00515)
	25			-0.016987	-0.25981	0.01887
				(0.00230)	(0.08767)	(0.00245)
	50			-0.01243	-0.04837	0.00271
				(0.00078)	(0.00788)	(0.00039)
10	10	3.5	3.5	0.04112	-0.14898	0.03428
				(0.00519)	(0.09120)	(0.00225)
	25			0.03312	-0.11114	0.04547
				(0.00496)	(0.02559)	(0.00349)
	50			0.02391	-0.26474	-0.03116
				(0.00409)	(0.08696)	(0.00254)
25	10			-0.0214	-0.19241	0.02849
				(0.00810)	(0.07846)	(0.00222)
	25			-0.05039	-0.16102	0.07814
				(0.01143)	(0.08775)	(0.00638)
	50			-0.07402	-0.23652	0.02429
				(0.01517	(0.06999)	(0.00089)
50	10			-0.02711	-0.1667	0.02235
				(0.01131)	(0.06656)	(0.00130)
	25			-0.05905	-0.21635	0.05292
				(0.01206)	(0.00626)	(0.00305)
	50			-0.04303	-0.14337	0.02231
				(0.00094)	(0.04500)	(0.00079)

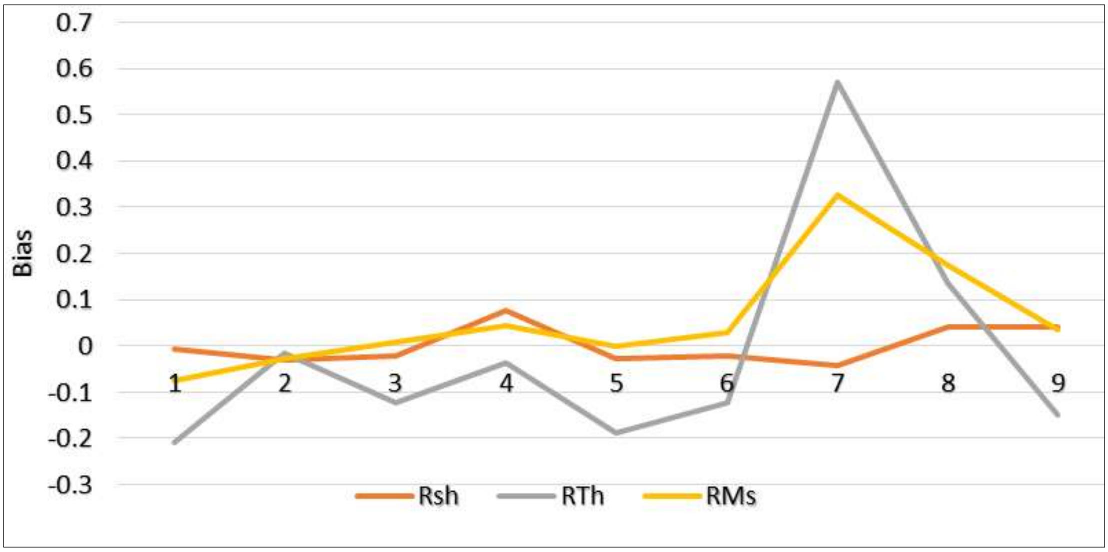

Fig 1: Bias of the estimate when n=10

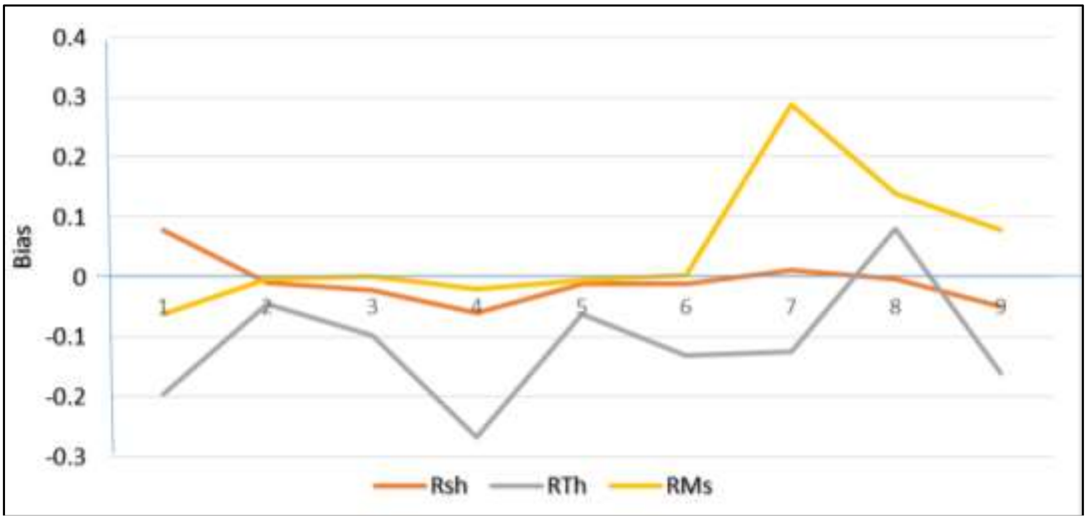

Fig 2: Bias of the estimate when n=25

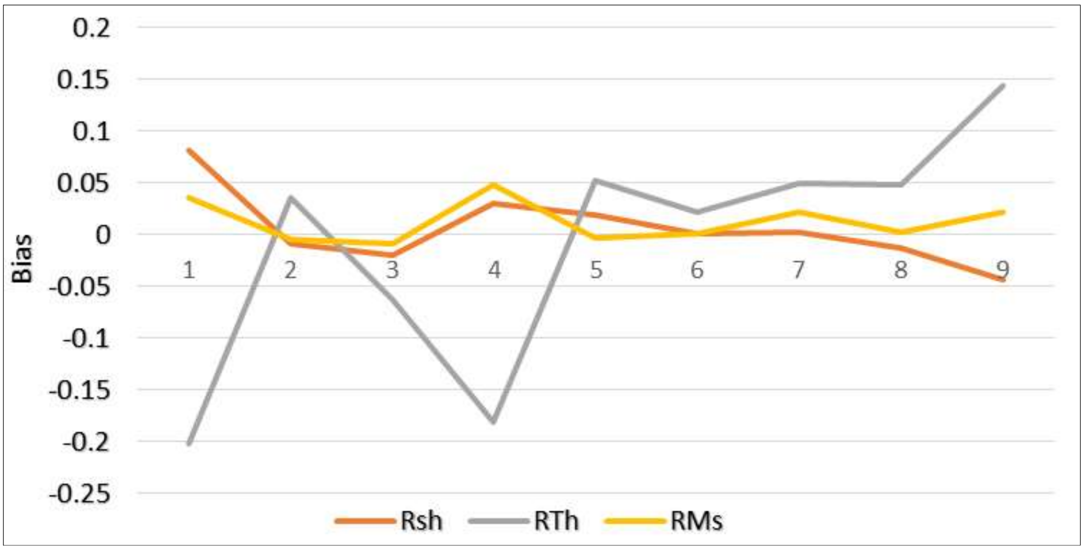

Fig 3: Bias of the estimate when n=50

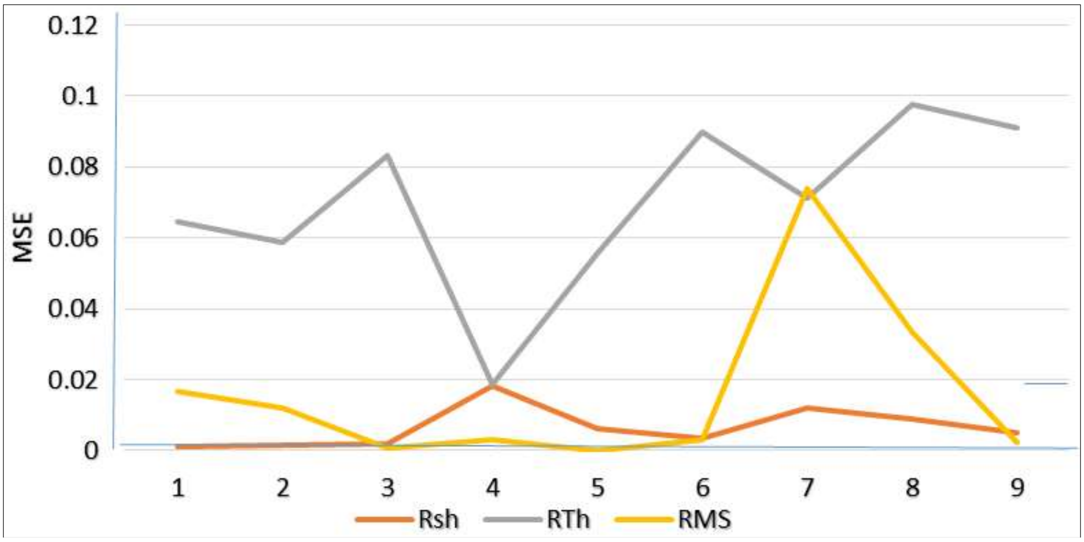

Fig 4: MSE of the estimate n=10

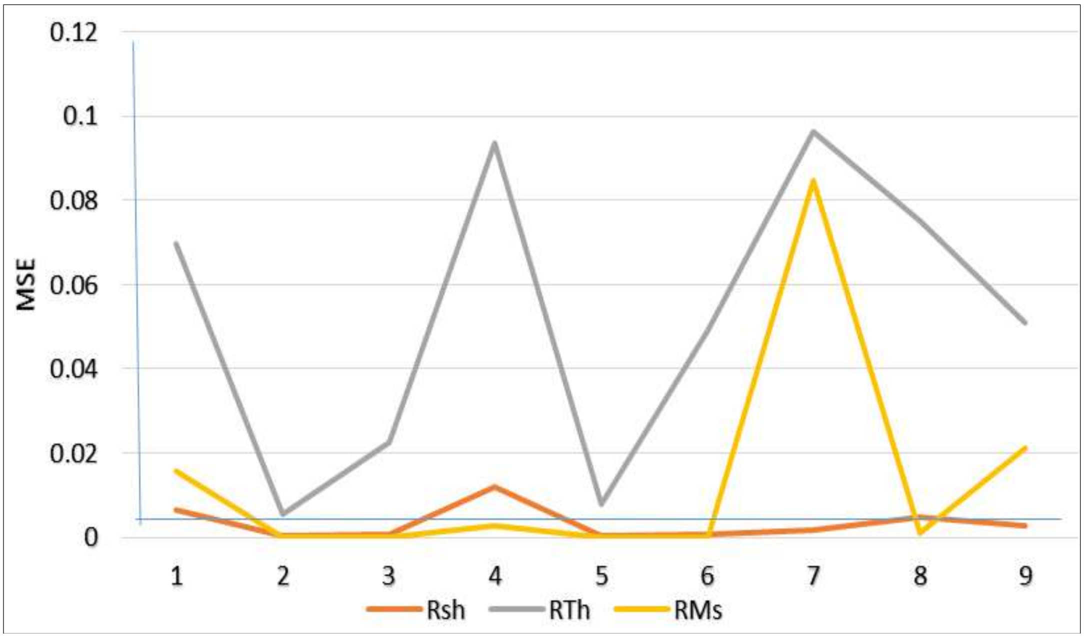

Fig 5: MSE of the estimate n=25

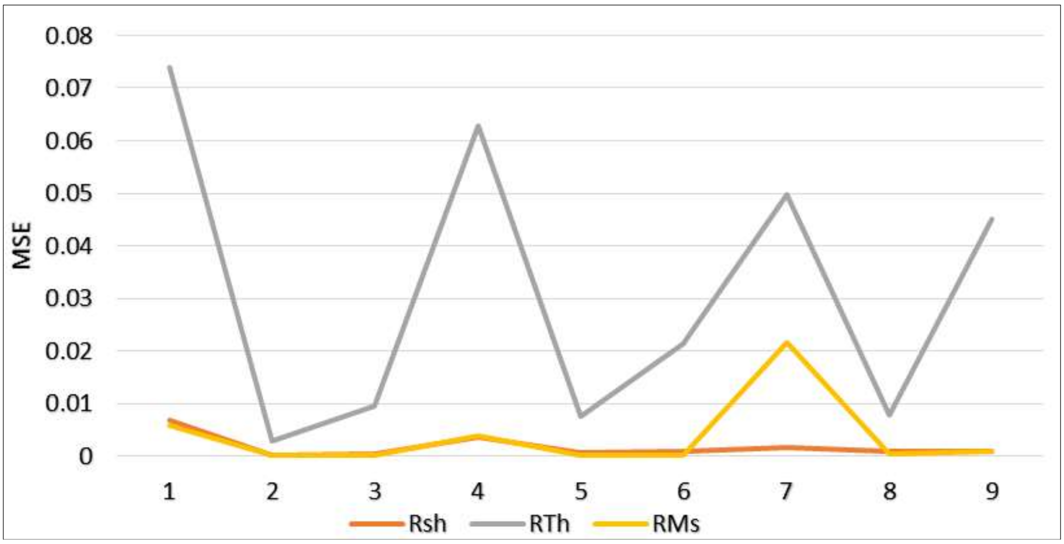

Fig 6: MSE of the estimate n=50

Table 2: Bias and MSEs (in parentheses) of the shrinkage Bayes estimates of Reliability under squared error loss function and Linex Loss function

$n_1=n_2$	n_3	$\alpha_1=\alpha_2$	α_3	$\Psi(\widehat{\beta})$=0.99 $\hat{R}_{SELFsh}$ Bias (MSE)	$\Psi(\widehat{\beta})$=0.01 $\hat{R}_{SELFsh}$ Bias (MSE)	$\Psi(\widehat{\beta})$=0.99 $\hat{R}_{LLFsh}$ Bias (MSE)	$\Psi(\widehat{\beta})$=0.01 $\hat{R}_{LLFsh}$ Bias (MSE)
10	10	0.5	0.5	-0.01220	0.15700	0.24745	0.07959
				(0.00125)	(0.024906)	(0.07963)	(0.00637)
	25			-0.03051	0.16536	0.12661	0.07884
				(0.00307)	(0.02734)	(0.04180)	(0.00623)
	50			-0.01104	0.16267	0.02411	0.09355
				(0.00104)	(0.02647)	(0.03887)	(0.00948)
25	10			0.007724	0.16661	0.27096	0.07620
				(0.00629)	(0.02776)	(0.08354)	(0.00581)
	25			0.06982	0.16574	0.16815	0.07579
				(0.00531)	(0.02747)	(0.03257)	(0.00575)
	50			-0.01103	0.16634	0.25143	0.07636
				(0.00101)	(0.02766)	(0.07033)	(0.00583)
50	10			0.09062	0.16658	0.28835	0.07521
				(0.00853)	(0.02775)	(0.08339)	(0.00565)
	25			0.08796	0.16487	0.28805	0.07544
				(0.00804)	(0.02718)	(0.08322)	(0.00569)
	50			0.07254	0.16657	0.28361	0.07673

				(0.00570)	(0.02774)	(0.08345)	(0.00588)
10	10	0.5	2	-0.04921	-0.01046	0.10282	-0.01423
				(0.00244)	(0.00039)	(0.01106)	(0.00186)
	25			-0.23995	0.00221	0.10825	-0.03883
				(0.08427)	(0.00153)	(0.013487)	(0.00181)
	50			-0.05891	-0.00612	0.05577	-0.01583
				(0.003772)	(0.00059)	(0.00313)	(0.00055)
25	10			-0.55163	-0.11814	0.04357	-0.05314
				(0.00361)	(0.05660)	(0.00201)	(0.00315)
	25			-0.05289	-0.01084	0.03894	-0.06500
				(0.00296)	(0.00020)	(0.00155)	(0.00433)
	50			-0.05181	-0.06305	0.04207	-0.06144
				(0.00287)	(0.01425)	(0.00180)	(0.00377)
50	10			-0.05369	0.00550	0.16071	-0.06009
				(0.00332)	(0.00022)	(0.09183)	(0.00362)
	25			-0.05214	0.03492	0.18363	-0.05997
				(0.00280)	(0.00709)	(0.09894)	(0.00360)
	50			-0.01724	0.03057	0.13408	-0.06062
				(0.00039)	(0.00093)	(0.09114)	(0.00368)
10	10	0.5	3.5	-0.02177	0.00269	-0.26415	-0.08531
				(0.00079)	(0.00438)	(0.09614)	(0.00728)
	25			-0.02535	0.0052	-0.10334	-0.08335
				(0.00103)	(0.00022)	(0.05988)	(0.00696)
	50			-0.02689	0.00549	-0.01388	0.08329
				(0.00105)	(0.00021)	(0.04795)	(0.00695)
25	10			-0.02556	-0.04017	0.01509	0.08244
				(0.00098)	(0.00919)	(0.03810)	(0.00680)
	25			-0.03077	0.00472	0.11171	0.08292
				(0.00124)	(0.00019)	(0.06022)	(0.00688)
	50			-0.03466	-0.04127	0.02859	-0.01388
				(0.00142)	(0.0078)	(0.04345)	(0.00041)
50	10			-0.02556	0.00163	0.10246	0.08330
				(0.00072)	(0.00062)	(0.04175)	(0.00694)
	25			-0.02920	-0.00422	0.10574	0.08313
				(0.00092)	(0.00139)	(0.04436)	(0.00691)
	50			-0.02776	0.00329	0.10812	0.04379
				(0.00085)	(0.00076)	(0.08301)	(0.00689)
10	10	2	0.5	-0.28474	-0.00483	0.27846	0.27378
				(0.09320	(0.0001)	(0.07895)	(0.07500)
	25			-0.28814	-0.00468	0.28660	0.33769
				(0.09767)	(0.00010)	(0.08380)	(0.06382)
	50			0.07077	0.02949	0.28233	0.27386
				(0.01657)	(0.00538)	(0.08119)	(0.07504)
25	10			0.18196	-0.02091	0.26383	0.27381
				(0.04845)	(0.00242)	(0.0698)	(0.07502)
	25			0.15346	-0.00487	0.26701	0.27383
				(0.04754)	(0.00011)	(0.07149)	(0.07502)
	50			0.17546	-0.02098	0.26419	0.27382
				(0.05208)	(0.00245)	((0.06996)	(0.07502)
50	10			0.17176	-0.0595	0.25689	0.34150
				(0.05012)	(0.02042)	(0.06632)	(0.06179)
	25			0.15836	-0.06129	0.25748	0.26987
				(0.05007)	(0.02176)	(0.06661)	(0.07289)
	50			0.16281	0.29523	0.24928	0.27383
				(0.05058)	(0.01041)	(0.06219)	(0.07502)
10	10	2	2	-0.03308	-0.06129	0.16528	0.08351
				(0.00632)	(0.02176)	(0.02732)	(0.00700)
	25			-0.01033	-0.03872	0.16471	0.08196
				(0.00334)	(0.00866)	(0.027136)	(0.00673)
	50			-0.00218	-0.00499	0.16442	0.081374
				(0.00120)	(0.00011)	(0.02704)	(0.00663)
25	10			-0.04548	-0.00677	0.16486	0.17431
				(0.00335)	(0.00114)	(0.02718)	(0.03038)
	25			-0.01801	-0.00604	0.16486	0.15187
				(0.0007)	(0.00010)	(0.02718)	(0.02496)
	50			-0.01173	-0.00713	0.16546	0.08458
				(0.00059)	(0.00012)	(0.02738)	(0.00716)
50	10			-0.0191	-0.00695	0.16350	0.084456
				(0.00231)	(0.00011)	(0.02673)	(0.00715)

	25			-0.21311	-0.01974	0.17016	0.09464
				(0.05154)	(0.02805)	(0.02896)	(0.00897)
	50			0.01182	0.15954	0.16443	0.08457
				(0.00042)	(0.02552)	(0.027038)	(0.00715)
10	10	2	3.5	-0.02954	-0.10357	0.09141	0.02241
				(0.00368)	(0.06256)	(0.00836)	(0.00134)
	25			-0.02988	-0.01139	0.09143	0.0208
				(0.00302)	(0.00052)	(0.00836)	(0.00110)
	50			-0.03686	-0.0103	0.09169	0.02478
				(0.00404)	(0.00051)	(0.00841)	(0.00171)
25	10			-0.01344	0.09511	0.09082	0.02347
				(0.00135)	(0.00904)	(0.00825)	(0.00176)
	25			-0.01961	-0.12165	0.09106	0.02279
				(0.00107)	(0.08649)	(0.00829)	(0.00162)
	50			-0.02693	-0.00354	0.09696	0.022136
				(0.00141)	(0.00003)	(0.00834)	(0.00136)
50	10			-0.02061	0.09467	0.090107	0.02386
				(0.00569)	(0.00896)	(0.00830)	(0.00174)
	25			-0.12407	-0.0058	0.091314	0.02048
				(0.02612)	(0.00013)	(0.00834)	(0.00112)
	50			-0.01461	0.09495	0.09087	0.01861
				(0.00117)	(0.00901)	(0.00825)	(0.000910
10	10	3.5	0.5	0.23451	0.18247	0.30507	0.312644
				(0.06765)	(0.08680)	(0.09365)	(0.09775)
	25			0.19818	0.18446	0.19785	0.31163
				(0.06068)	(0.09295)	(0.09252)	(0.09713)
	50			0.21992	0.18169	0.28555	0.31251
				(0.06976)	(0.09173)	(0.08302)	(0.09767)
25	10			0.22591	-0.27634	0.29665	0.31166
				(0.06902)	(0.07755)	(0.08830)	(0.09713)
	25			0.18647	-0.27852	0.29837	0.31167
				(0.06716)	(0.07589)	(0.08938)	(0.09714)
	50			0.21108	-0.274	0.29732	0.31166
				(0.07170)	(0.07637)	(0.08874)	(0.09714)
50	10			0.24427	-0.28392	0.19963	0.31019
				(0.08401)	(0.08150)	(0.08857)	(0.09623)
	25			0.21764	0.11677	0.18335	0.31003
				(0.07777)	(0.08886)	(0.07132)	(0.09613)
	50			0.23730	-0.28425	0.22433	0.31093
				(0.08705)	(0.08171)	(0.06493)	(0.09668)
10	10	3.5	2	0.03457	0.24192	0.18436	0.150809
				(0.00852)	(0.05853)	(0.03691)	(0.02274)
	25			0.03377	-0.01001	0.18421	0.15278
				(0.00448)	(0.00018)	(0.03713)	(0.02336)
	50			-0.19471	-0.0068	0.183662	0.15278
				(0.03854)	(0.00014)	(0.03684)	(0.02335)
25	10			0.10981	-0.00855	0.181536	0.149988
				(0.00135)	(0.00018)	(0.03587)	(0.02249)
	25			0.12095	-0.0289	0.18098	0.14998
				(0.01982)	(0.00272)	(0.03573)	(0.02249)
	50			0.12736	0.21857	0.17879	0.14998
				(0.02193)	(0.04966)	(0.03464)	(0.02248)
50	10			0.11614	0.23078	0.17927	0.15335
				(0.02076)	(0.05365)	(0.03474)	(0.02354)
	25			-0.02246	0.23500	0.18458	0.15315
				(0.00247)	(0.05542)	(0.03701)	(0.02348)
	50			0.12699	0.23561	0.17882	0.14938
				(0.02161)	(0.05542)	(0.03465)	(0.02231)
10	10	3.5	3.5	0.03402	0.16190	0.09607	0.07225
				(0.00463)	(0.02621)	(0.01191)	(0.00525)
	25			0.02582	0.12946	0.09641	0.07288
				(0.00452)	(0.02203)	(0.01194)	(0.00525)
	50			0.01695	0.12908	0.09681	0.07229
				(0.00379)	(0.02187)	(0.01196)	(0.005257)
25	10			0.02231	0.06180	0.09590	0.07228
				(0.03683)	(0.03789)	(0.01213)	(0.00525)
	25			-0.05763	0.10384	0.09974	0.07232
				(0.01223)	(0.01718)	(0.01286)	(0.00526)
	50			-0.08073	0.03783	0.10078	0.07233

				(0.01615)	(0.04957)	(0.01311)	(0.00526)
50	10			-0.03397	0.09425	0.09865	0.07231
				(0.01162)	(0.00916)	(0.01252)	(0.00526)
	25			-0.06628	0.15399	0.10004	0.072325
				(0.01299)	(0.02388)	(0.01288)	(0.00526)
	50			-0.04980	0.09425	0.098053	0.07034
				(0.01252)	(0.01781)	(0.012153)	(0.00496)

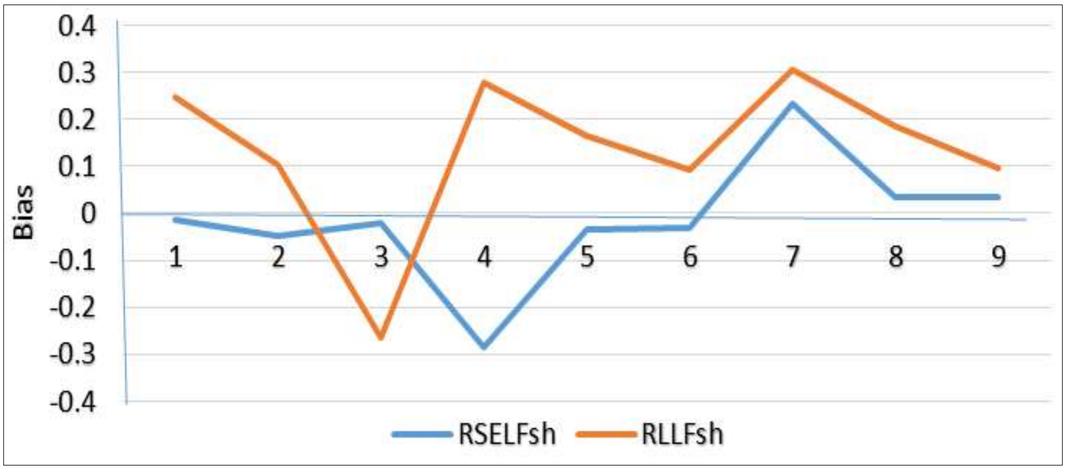

Fig 7: Bias of the estimate when n=10, $\Psi(\widehat{\beta})$= 0.99

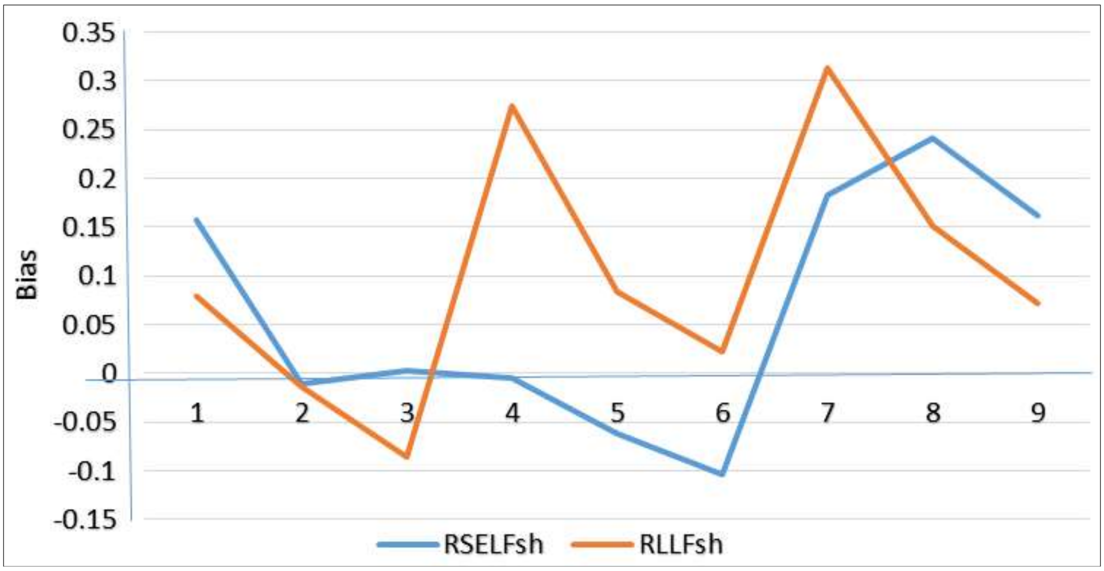

Fig 8: Bias of the estimate when n=10, $\Psi(\widehat{\beta})$= 0.01

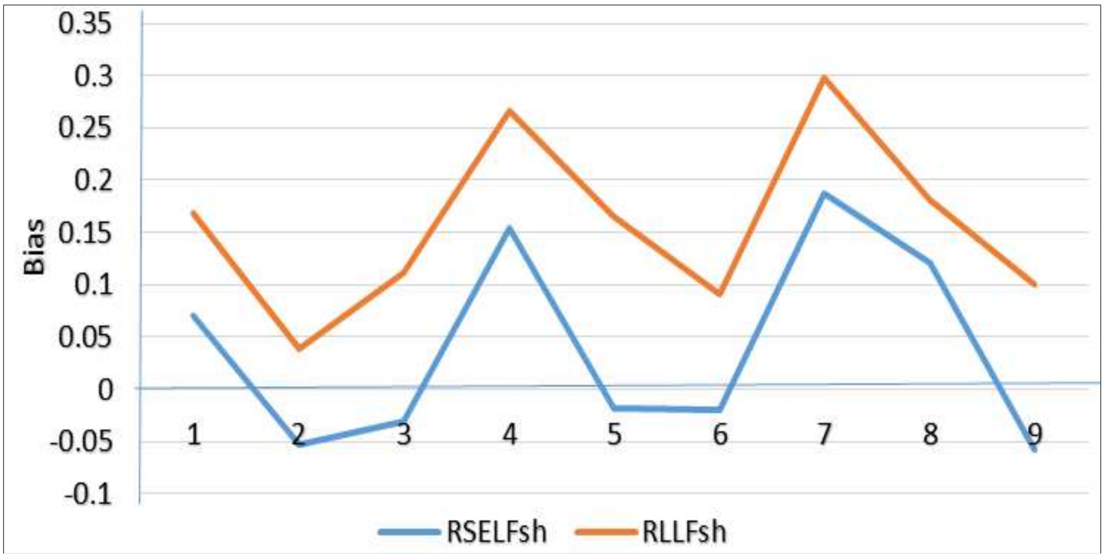

Fig 9: Bias of the estimate when n=25, $\Psi(\widehat{\beta})$= 0.99

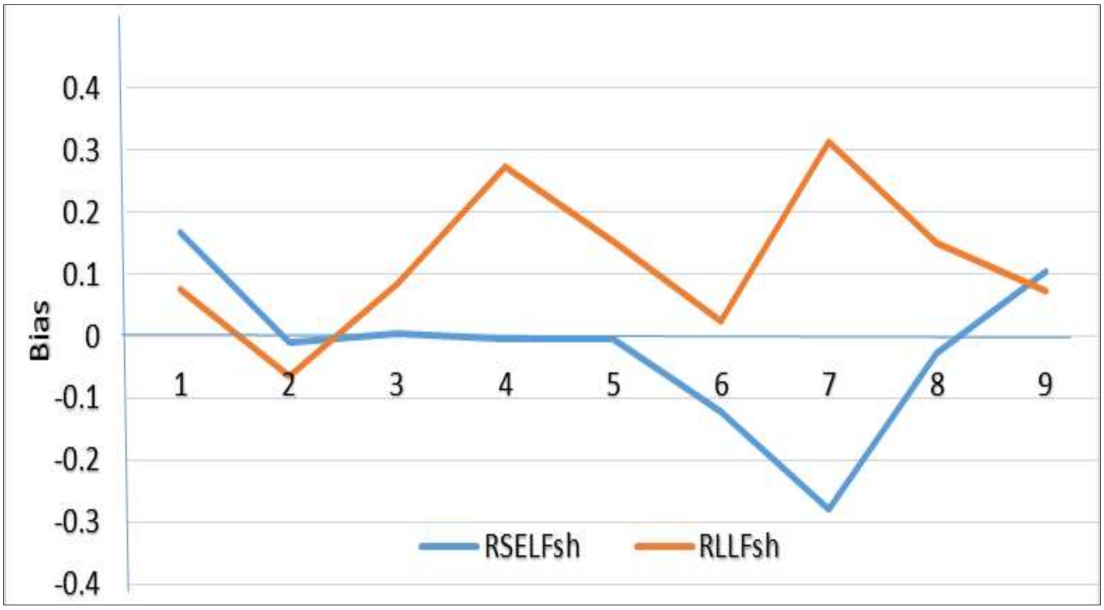

Fig 10: Bias of the estimate when n=25, $\Psi(\widehat{\beta})$= 0.01

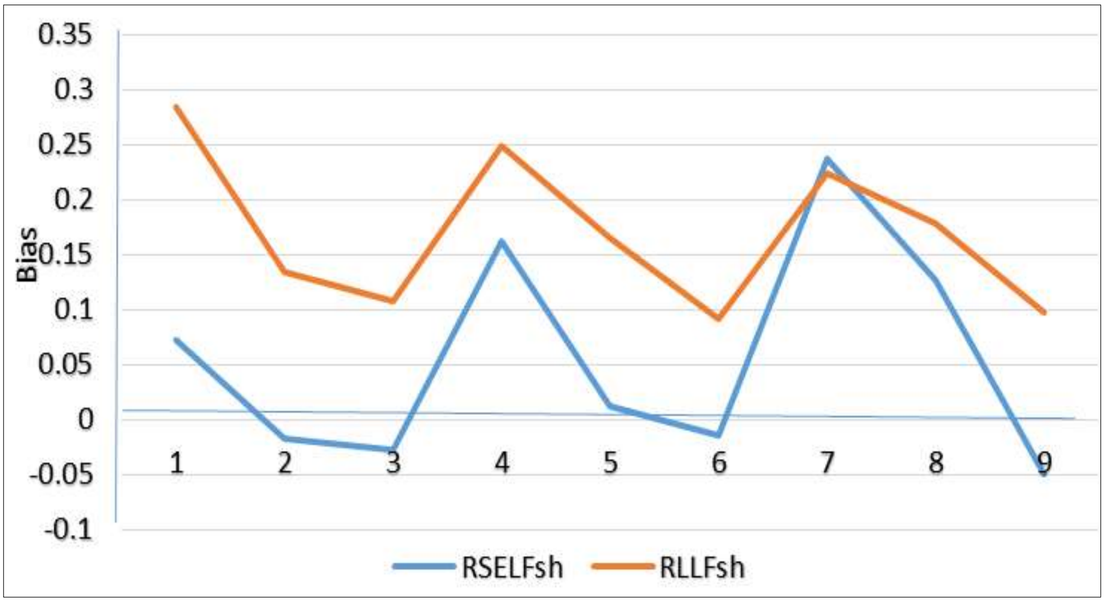

Fig 11: Bias of the estimate when n=50, $\Psi(\widehat{\beta})$= 0.99

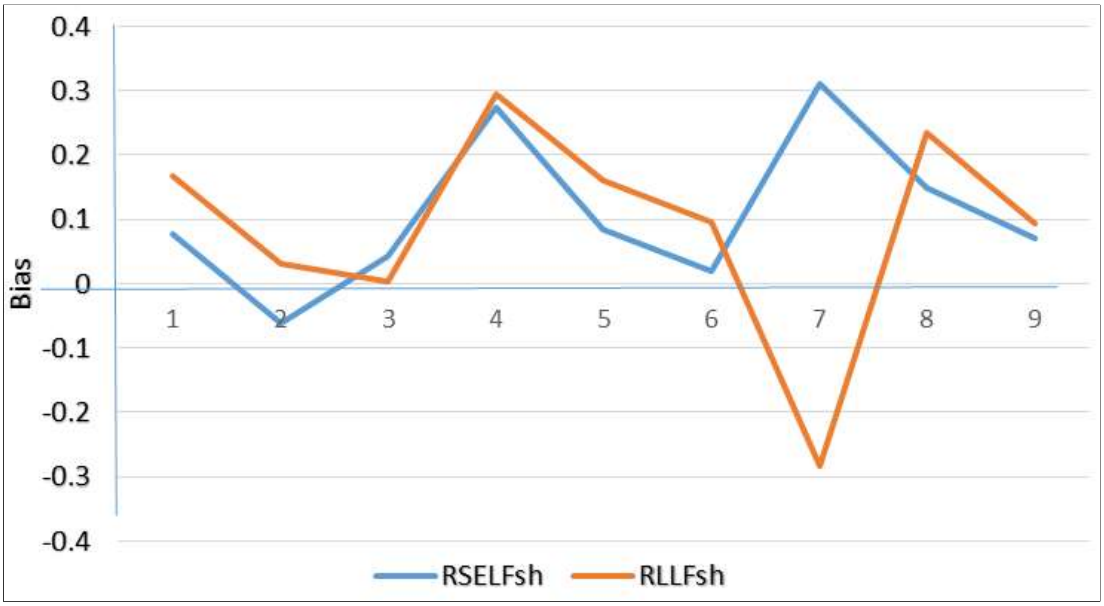

Fig 12: Bias of the estimate when n=50, $\Psi(\widehat{\beta})$= 0.01

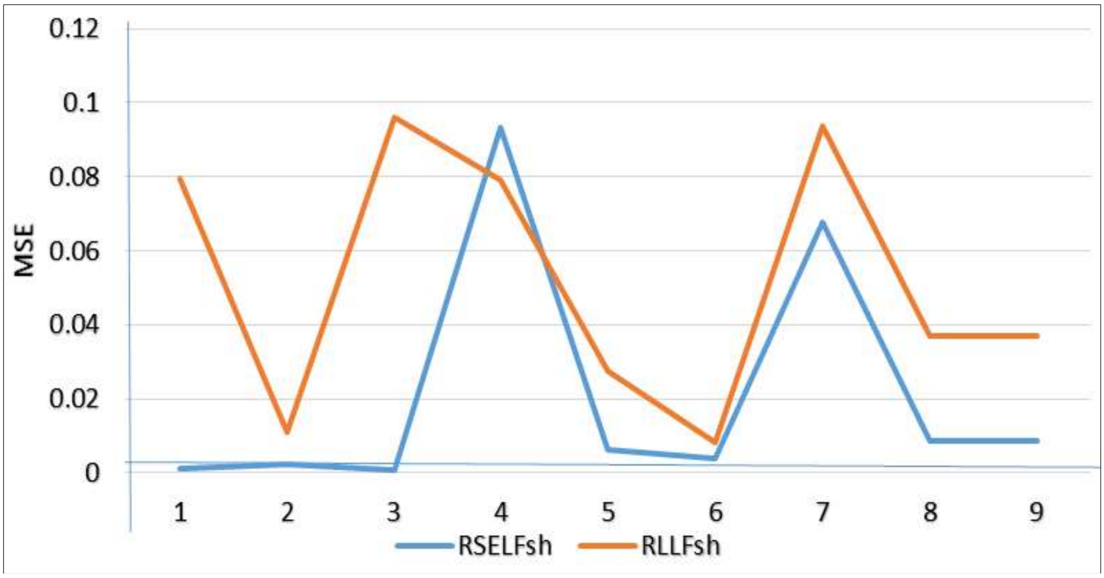

Fig 13: MSE of the estimate when n=10 $\Psi(\widehat{\beta})$= 0.99

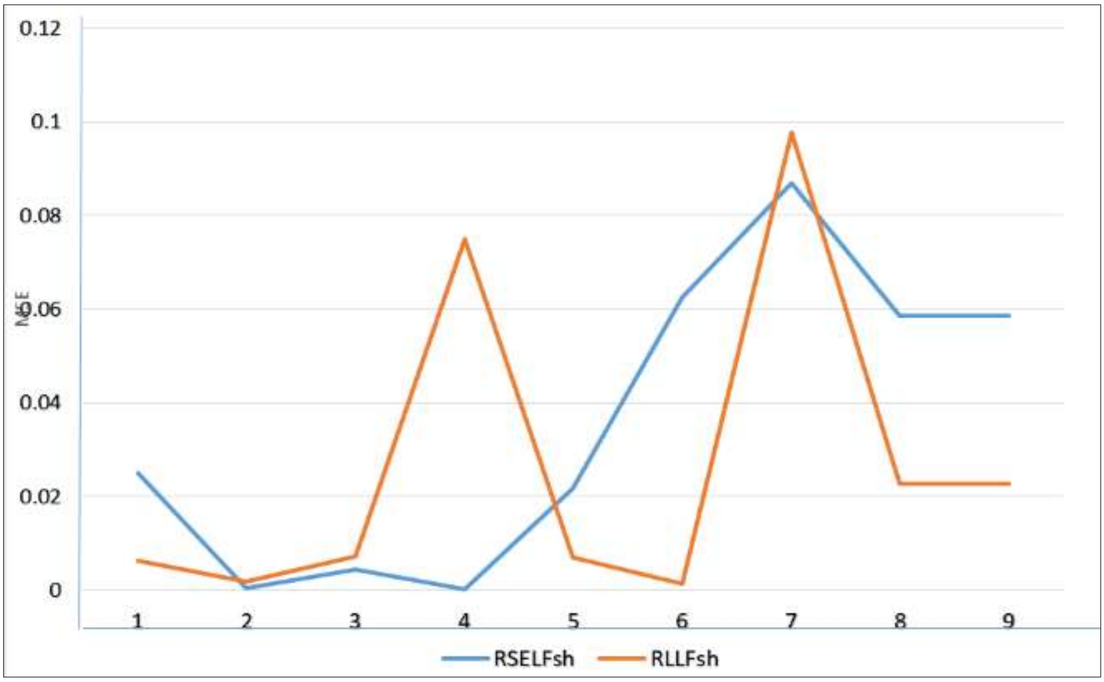

Fig 14: MSE of the estimate when n=10 $\Psi(\widehat{\beta})$= 0.01

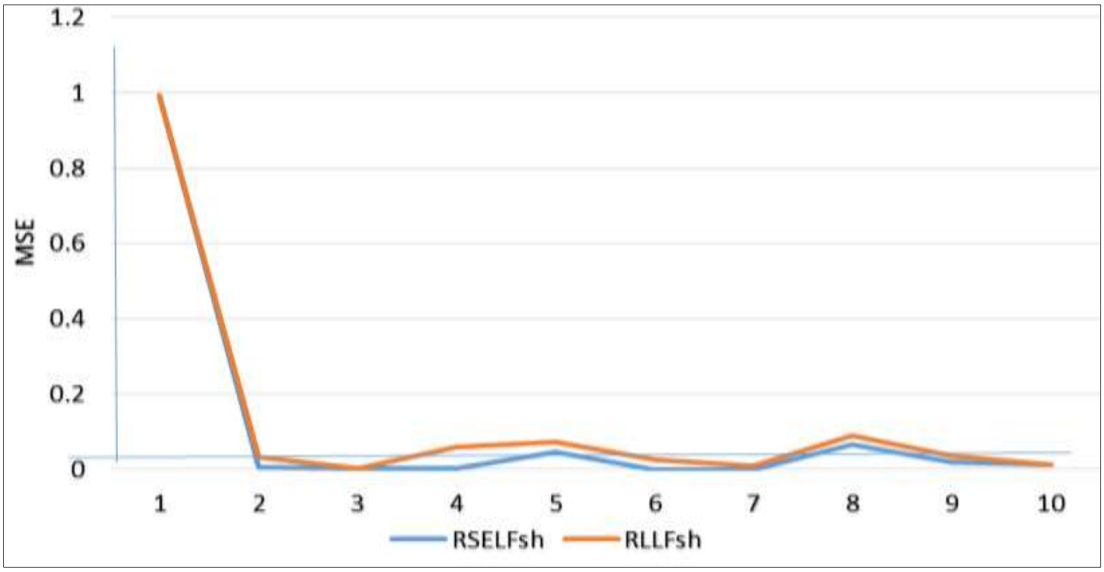

Fig 15: MSE of the estimate when n=25 $\Psi(\widehat{\beta})$= 0.99

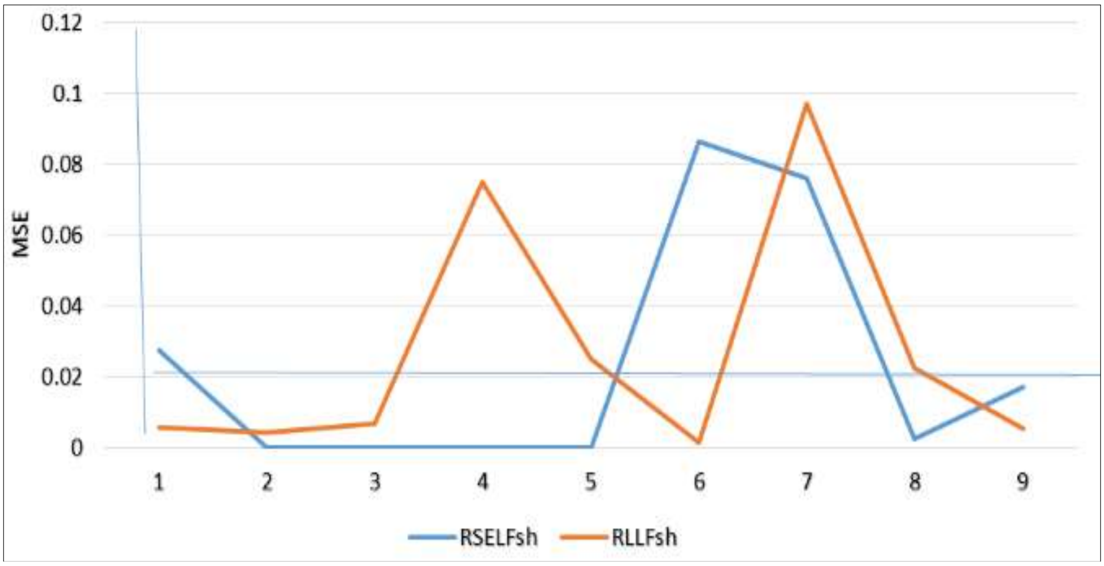

Fig 16: MSE of the estimate when n=25 $\Psi(\widehat{\beta})$= 0.01

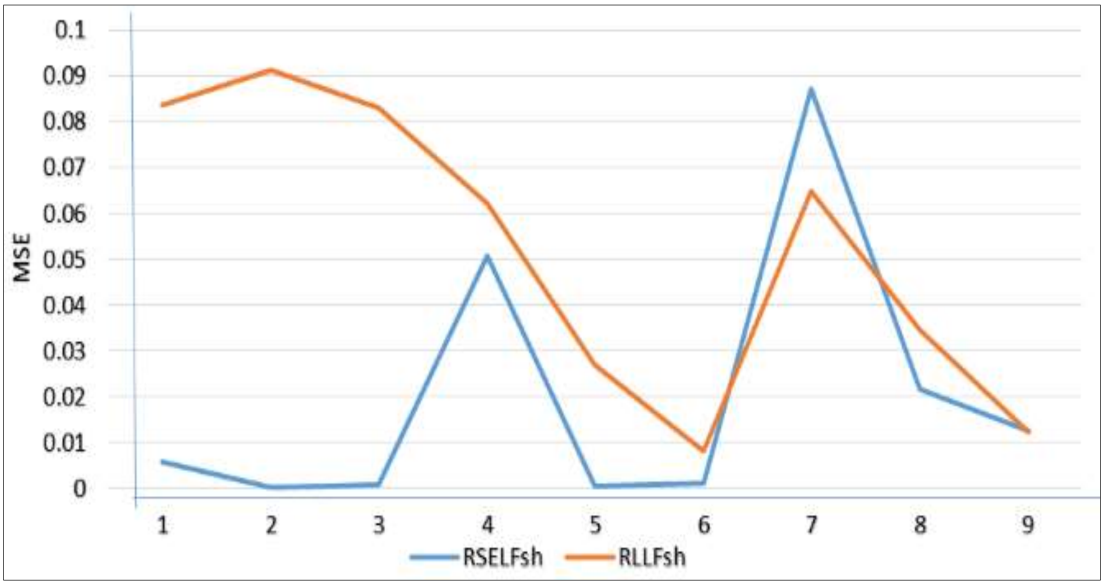

Fig 17: MSE of the estimate when n=50 $\Psi(\widehat{\beta})$= 0.99

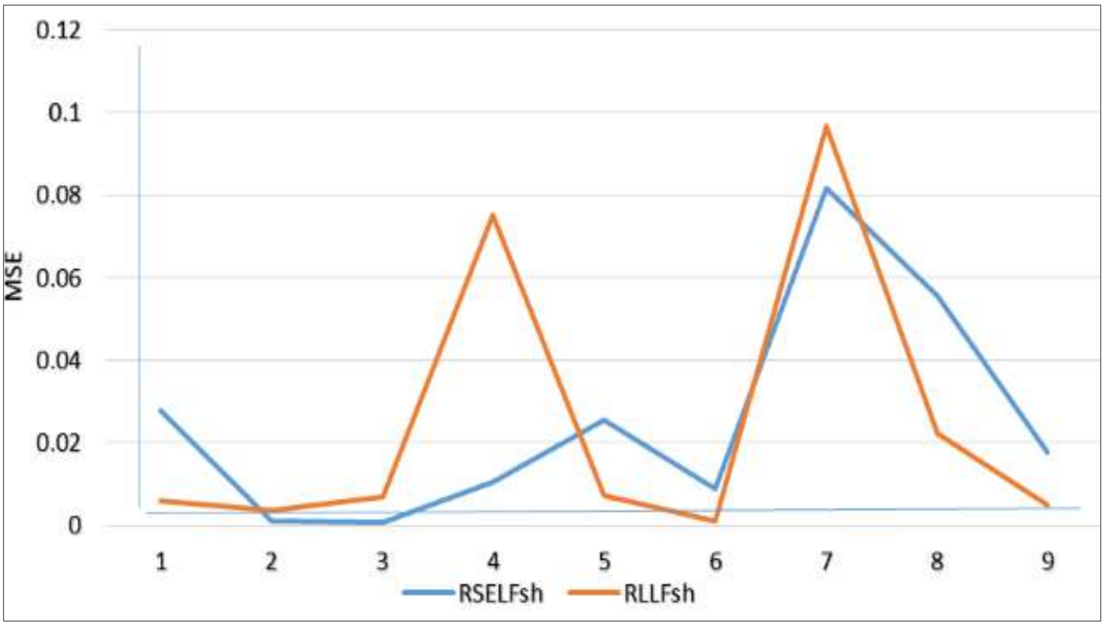

Fig 18: MSE of the estimate when n=50 $\Psi(\widehat{\beta})$= 0.01

5. Conclusions

In this paper we considered the shrinkage estimates of R based on record values using Classical and Bayesian Estimation method when the scale parameter λ which is common for all the distribution is known. From the numerical study conducted so far we can conclude that

1. When sample size increases bias and mean square error decreases.
2. In classical method bias and mean square error of estimate of $\hat{R}_{sh}$ is less than that of $\hat{R}_{Th}$ and $\hat{R}_{Ms}$. So $\hat{R}_{sh}$ is performs better than $\hat{R}_{Th}$ and
3. In Bayesian method of estimation bias and mean square error of estimate of $\hat{R}_{SELFsh}$ is less than that of $\hat{R}_{LLFsh}$. So the estimates under squared error loss function is performs better than that of Linex loss function.

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