International Journal of Statistics and Applied Mathematics 2023; 8(1): 95-106

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2023; 8(1): 95-106

https://www.mathsjournal.com
Received: 22-11-2022
Accepted: 24-12-2022

S Abirami
Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India

N Vijayasankar
Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India

Corresponding Author:
S Abirami
Department of Statistics, Annamalai University, Annamalai Nagar, Tamil Nadu, India

New modification of three parameters Sujatha distribution and its SPC applications

S Abirami and N Vijayasankar

DOI: https://doi.org/10.22271/maths.2023.v8.i1b.934

Abstract
In this paper, a new probability model called an area biased three parameter Sujatha distribution has been studied. Various structural properties of this new distribution have been discussed and its model parameters are estimated using the maximum likelihood estimation. a new distribution is examined and analysed with real-life data set to discuss its significance.
Finally, the techniques of a New Modification of Three Parameter have been used to the Sujatha distribution to the application of Statistical Process Control to check the performance of the production Process. The main objective of this paper is to introduce a control chart using area biased three parameter Sujatha distribution in order to study the production system and monitor the same.

Keywords: Statistical process control, control limits, three parameter Sujatha distribution, survival analysis, order statistics, maximum likelihood estimation

1. Introduction

The concept of weighted probability distributions plays an important role in various research areas related to Reliability, Biomedicine, Ecology and Branching process for the improvement of proper statistical models. The concept of weighted probability models attracted a lot of researchers to contemplate and carry out research on this topic. Fisher (1934) [4] introduced the concept of weighted models in connection with his studies on how the methods of ascertainment can influence the form of distribution of recorded observations. Later, it has been modified by Rao (1965) [10] in a unified manner where the observations fall into non-experimental, non-replicated and non-random categories. The weighted distribution reduces to length biased as the weight function considers only the length of units of interest. More generally, when the sampling mechanism selects units with probability proportional to measure of the unit size, the resulting distribution is called size biased. Size biased distributions are special case of more general form known as weighted distributions. The concept of length biased sampling was introduced by Cox (1969) [3] and Zelen (1974) [16]. Patil and Rao (1978) [9] studied weighted distributions and size biased sampling with applications to wild life population and human families. Van Deusen (1986) [15] arrived at size biased distribution theory independently and applied it in fitting assumed distributions to data arising from horizontal point sampling. Various authors have studied different probability models and illustrated their applications in different fields. Jayakumar and Elangovan (2019) [6] studies area biased Ailamujia distribution with applications in bladder cancer data. Hassan, Akhter and Para (2019) [5] discussed on weighted quasi exponential distribution with properties and applications. Sharma *et al.* (2017) [13] presented the length and area biased Maxwell distribution. Mudasir and Ahmad (2018) [8] study the characterization and estimation of length biased Nakagami distribution. Bashir and Rasul (2016) [1] introduced the Poisson area biased Lindley distribution and its applications on biological data. Rao and Pandey (2021) [11] studied the parameter estimation of length biased weighted Frechet distribution via Bayesian approach.

The three parameter Sujatha distribution is a newly introduced continuous lifetime model proposed by John and Aniefiok (2020) [7] and the proposed distribution is a modification of two parameter Sujatha distribution. Its several statistical properties such as shape of the pdf, hazard rate function, moment generating function, order statistics and raw moments have been derived and discussed. The parameters are also estimated by using the method of maximum likelihood estimation. Tesfay and Shanker (2018) [14] introduced the two parameter Sujatha distribution studied its various statistical properties and estimate its parameters through method of moments and method of maximum likelihood estimation.

2. Area Biased Three Parameter Sujatha (ABTPS) Distribution

The probability density function of three parameter Sujatha (TPS) distribution is given by

$$f(x;\theta,\lambda,\alpha)=\frac{\theta^2}{2(\theta^2+\lambda+\alpha)}(\theta\alpha x^2+2\lambda x+2\theta)e^{-\theta x}\ ;x>0,\theta>0,\alpha>0,\lambda>0 \tag{1}$$

and the cumulative distribution function of three parameter Sujatha distribution is given by

$$F(x;\theta,\lambda,\alpha)=\left(1-\left(1+\frac{\theta^2x^2\alpha+2\theta x}{2(\theta^2+\lambda+\alpha)}\right)e^{-\theta x}\right);x>0,\theta>0,\lambda>0,\alpha>0 \tag{2}$$

If X be the non-negative random variable with probability density function $f(x)$ and $w(x)$ be its non-negative weight function, then the probability density function of weighted random variable X_w is given by

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))},\ x>0.$$

Where $w(x)$ be the non - negative weight function and $E(w(x))=\int w(x)f(x)dx<\infty.$

For various weighted models, we have different choices of weight function *w(x)*. When $w(x) = x^c$, the resulting distribution is termed as weighted distribution. In this paper, we have to find the area biased version of three parameter Sujatha distribution. So consequently, we will take c = 2 in weights x^c in order to obtain the area biased three parameter Sujatha distribution and its pdf is given by

$$f_a(x)=\frac{x^2f(x)}{E(x^2)} \tag{3}$$

$$\text{Where } E(x^2)=\int_0^\infty x^2f(x;\theta,\lambda,\alpha)dx$$

$$E(x^2)=\frac{24\alpha+12\lambda+4\theta^3}{\theta^2(2(\theta^2+\lambda+\alpha))} \tag{4}$$

By substituting equations (1) and (4) in equation (3), we will obtain the probability density function of area biased three parameter Sujatha distribution

$$f_a(x)=\frac{x^2\theta^4}{24\alpha+12\lambda+4\theta^3}(\theta\alpha x^2+2\lambda x+2\theta)e^{-\theta x} \tag{5}$$

and the cumulative distribution function of area biased three parameter Sujatha distribution can be obtained as

$$F_a(x)=\int_0^x f_a(x)dx$$

$$F_a(x)=\int_0^x \frac{x^2\theta^4}{24\alpha+12\lambda+4\theta^3}(\theta\alpha x^2+2\lambda x+2\theta)\ e^{-\theta x}dx$$

$$F_a(x) = \frac{1}{24\alpha + 12\lambda + 4\theta^3} \int_0^x x^2 \theta^4 (\theta \alpha x^2 + 2\lambda x + 2\theta) e^{-\theta x} dx$$

$$F_a(x) = \frac{1}{24\alpha + 12\lambda + 4\theta^3} \left(\theta^5 \alpha \int_0^x x^4 e^{-\theta x} dx + 2\lambda\theta^4 \int_0^x x^3 e^{-\theta x} dx + 2\theta^5 \int_0^x x^2 e^{-\theta x} dx \right) \tag{6}$$

Put $\theta x = t \quad \Rightarrow \theta dx = dt \quad \Rightarrow dx = \frac{dt}{\theta}$

When $x \to x, t \to \theta x,\ x \to 0, t \to 0$ and also $x = \frac{t}{\theta}$

After the simplification of equation (6), we will obtain the cumulative distribution function of area biased three parameter Sujatha distribution as

$$F_a(x) = \frac{1}{24\alpha + 12\lambda + 4\theta^3} \left(\alpha\gamma(5, \theta x) + 2\lambda\gamma(4, \theta x) + 2\theta^2 \gamma(3, \theta x) \right) \tag{7}$$

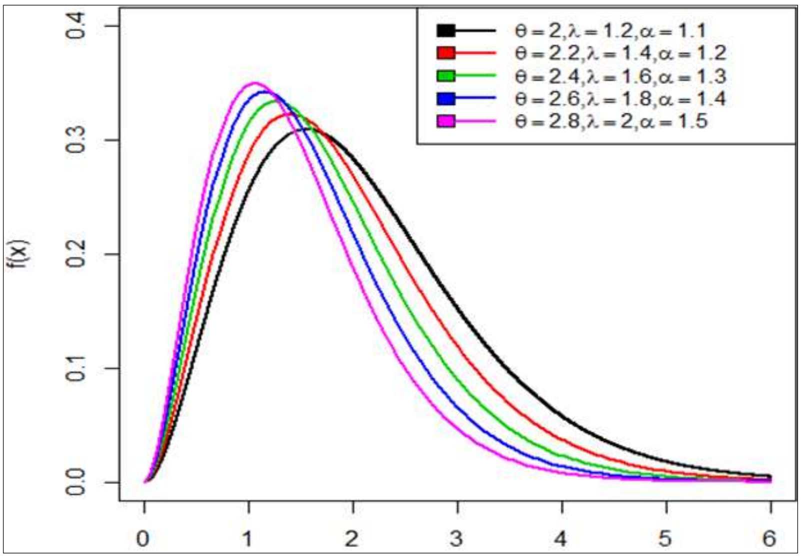

Fig 1: Pdf plot of area biased three parameter sujatha distribution

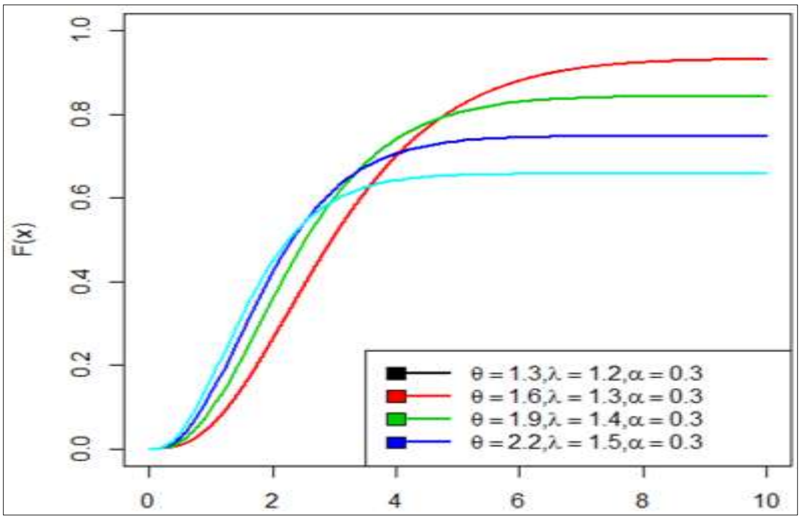

Fig 2: Cdf plot of area biased three parameter sujatha distribution

3. Survival Analysis

In this section, we will discuss the survival function, failure rate and reverse hazard rate functions of the area biased three parameter Sujatha distribution.

3.1 Survival function

The survival function of area biased three parameter Sujatha distribution can be obtained as

$$S(x) = 1 - F_a(x)$$

$$S(x) = 1 - \frac{1}{24\alpha + 12\lambda + 4\theta^3}\left(\alpha\,\gamma(5,\,\theta x) + 2\lambda\gamma(4,\theta x) + 2\theta^2\gamma(3,\theta x)\right)$$

3.2 Hazard function

The hazard function is also known as failure rate or force of mortality and is given by

$$h(x) = \frac{f_a(x)}{1 - F_a(x)}$$

$$h(x) = \frac{x^2\theta^4(\theta\alpha x^2 + 2\lambda x + 2\theta)e^{-\theta x}}{(24\alpha + 12\lambda + 4\theta^3) - (\alpha\gamma(5,\,\theta x) + 2\lambda\gamma(4,\theta x) + 2\theta^2\gamma(3,\theta x))}$$

3.3 Reverse hazard function

The reverse hazard function is given by

$$h_r(x) = \frac{f_a(x)}{F_a(x)}$$

$$h_r(x) = \frac{x^2\theta^4(\theta\alpha x^2 + 2\lambda x + 2\theta)e^{-\theta x}}{(\alpha\gamma(5,\theta x) + 2\lambda\gamma(4,\theta x) + 2\theta^2\gamma(3,\theta x))}$$

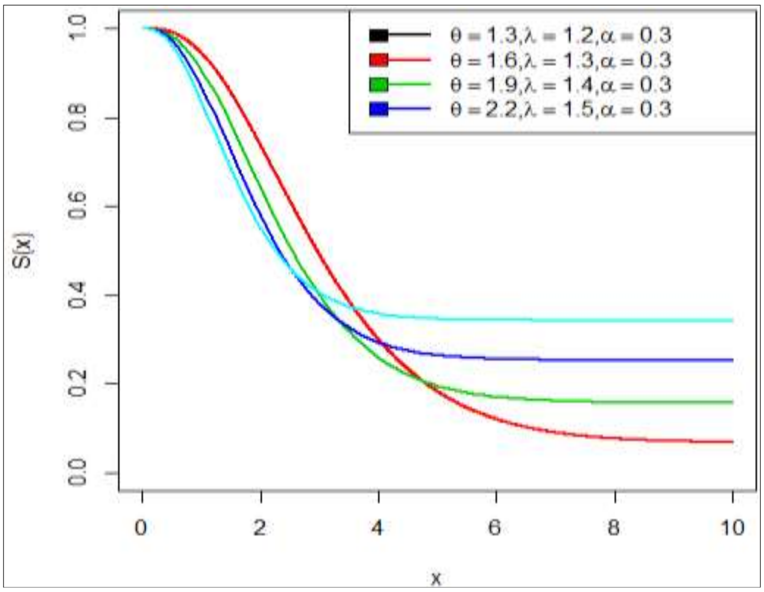

Fig 3: Survival plot of area biased three parameter sujatha distribution

4. Statistical Properties

In this section, we will discuss various structural properties of area biased three parameter Sujatha distribution which include moments, harmonic mean and moment generating function.

4.1 Moments

Let the random variable *X* following area biased three parameter Sujatha distribution with parameters *θ, λ* and *α,* then the r^{th} order moment of proposed model can be obtained as

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_a(x)dx$$

$$E(X^r) = \mu_r' = \int_0^\infty x^r \frac{x^2\theta^4}{24\alpha + 12\lambda + 4\theta^3}(\theta\alpha x^2 + 2\lambda x + 2\theta)e^{-\theta x}dx$$

$$E(X^r) = \mu_r' = \frac{\theta^4}{24\alpha + 12\lambda + 4\theta^3}\int_0^\infty x^{r+2}(\theta\alpha x^2 + 2\lambda x + 2\theta)e^{-\theta x}dx$$

$$E(X^r) = \mu_r' = \frac{\theta^4}{24\alpha + 12\lambda + 4\theta^3}\left(\theta\alpha\int_0^\infty x^{(r+5)-1}e^{-\theta x}dx + 2\lambda\int_0^\infty x^{(r+4)-1}e^{-\theta x}dx + 2\theta\int_0^\infty x^{(r+3)-1}e^{-\theta x}dx\right) \tag{8}$$

After the simplification of equation (8), we obtain

$$E(X^r) = \mu_r' = \frac{\alpha\Gamma(r+5) + 2\lambda\Gamma(r+4) + 2\theta^3\Gamma(r+3)}{\theta^r(24\alpha + 12\lambda + 4\theta^3)} \tag{9}$$

Putting r = 1, 2, 3 and 4 in equation (9), we will obtain the first four moments of area biased three parameter Sujatha distribution as

$$E(X) = \mu_1' = \frac{120\alpha + 48\lambda + 12\theta^3}{\theta(24\alpha + 12\lambda + 4\theta^3)}$$

$$E(X^2) = \mu_2' = \frac{720\alpha + 240\lambda + 48\theta^3}{\theta^2(24\alpha + 12\lambda + 4\theta^3)}$$

$$E(X^3) = \mu_3' = \frac{5040\alpha + 1440\lambda + 240\theta^3}{\theta^3(24\alpha + 12\lambda + 4\theta^3)}$$

$$E(X^4) = \mu_4' = \frac{40320\alpha + 10080\lambda + 1440\theta^3}{\theta^4(24\alpha + 12\lambda + 4\theta^3)}$$

$$\text{Variance} = \frac{720\alpha + 240\lambda + 48\theta^3}{\theta^2(24\alpha + 12\lambda + 4\theta^3)} - \left(\frac{120\alpha + 48\lambda + 12\theta^3}{\theta(24\alpha + 12\lambda + 4\theta^3)}\right)^2$$

$$S.D(\sigma) = \sqrt{\frac{720\alpha + 240\lambda + 48\theta^3}{\theta^2(24\alpha + 12\lambda + 4\theta^3)} - \left(\frac{120\alpha + 48\lambda + 12\theta^3}{\theta(24\alpha + 12\lambda + 4\theta^3)}\right)^2}$$

4.2 Harmonic mean

The harmonic mean for the proposed area biased three parameter Sujatha distribution can be obtained as

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{1}{x} f_a(x)dx$$

$$H.M = \int_0^\infty \frac{x\theta^4}{24\alpha + 12\lambda + 4\theta^3}(\theta\alpha x^2 + 2\lambda x + 2\theta)e^{-\theta x}dx$$

$$H.M = \frac{\theta^4}{24\alpha+12\lambda+4\theta^3}\int_0^\infty x(\theta\alpha x^2+2\lambda x+2\theta)e^{-\theta x}dx$$

$$H.M = \frac{\theta^4}{24\alpha+12\lambda+4\theta^3}\left(\theta\alpha\int_0^\infty x^{4-1}e^{-\theta x}dx+2\lambda\int_0^\infty x^{3-1}e^{-\theta x}dx+2\theta\int_0^\infty x^{3-2}e^{-\theta x}dx\right)$$

After simplification, we obtain

$$H.M = \frac{\theta(6\alpha+4\lambda+4\theta^2)}{24\alpha+12\lambda+4\theta^3}$$

4.3 Moment Generating Function and Characteristic Function

Let X be the random variable following area biased three parameter Sujatha distribution with parameters θ, λ and α, then the MGF of X can be obtained as

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_a(x)dx$$

$$M_x(t) = E(e^{tx}) = \int_0^\infty \left(1+tx+\frac{(tx)^2}{2!}+\ldots\right) f_a(x)dx$$

$$M_x(t) = E(e^{tx}) = \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!}x^j f_a(x)dx$$

$$M_x(t) = E(e^{tx}) = \sum_{j=0}^\infty \frac{t^j}{j!}\mu_j'$$

$$M_x(t) = E(e^{tx}) = \sum_{j=0}^\infty \frac{t^j}{j!}\left(\frac{\alpha\Gamma(j+5)+2\lambda\Gamma(j+4)+2\theta^3\Gamma(j+3)}{\theta^j(24\alpha+12\lambda+4\theta^3)}\right)$$

$$M_X(t) = \frac{1}{24\alpha+12\lambda+4\theta^3}\sum_{j=0}^\infty \frac{t^j}{j!\theta^j}\left(\alpha\Gamma(j+5)+2\lambda\Gamma(j+4)+2\theta^3\Gamma(j+3)\right)$$

Similarly, the characteristic function of area biased three parameter Sujatha distribution can be obtained as

$$\varphi_x(t) = M_X(it)$$

$$M_X(it) = \frac{1}{(24\alpha+12\lambda+4\theta^3)}\sum_{j=0}^\infty \frac{it^j}{j!\theta^j}\left(\alpha\Gamma(j+5)+2\lambda\Gamma(j+4)+2\theta^3\Gamma(j+3)\right)$$

5. Order Statistics

Order statistics play an important role in many practical as well as theoretical areas. Let $X_{(1)}, X_{(2)},\ldots,X_{(n)}$ be the order statistics of a random sample $X_1, X_2,\ldots, X_n$ drawn from a continuous population with probability density function $f_x(x)$ and cumulative distribution function $F_x(x)$, then the probability density function of r^{th} order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1-F_X(x)\right)^{n-r} \qquad (10)$$

By using the equations (5) and (7) in equation (10), we will obtain the probability density function of r^{th} order statistics of area biased three parameter Sujatha distribution as

$$f_{x(r)}(x)=\frac{n!}{(r-1)!(n-r)!}\left(\frac{x^2\theta^4}{24\alpha+12\lambda+4\theta^3}(\theta\alpha x^2+2\lambda x+2\theta)e^{-\theta x}\right)$$

$$\times\left(\frac{1}{24\alpha+12\lambda+4\theta^3}\left(\alpha\gamma(5,\theta x)+2\lambda\gamma(4,\theta x)+2\theta^2\gamma(3,\theta x)\right)\right)^{r-1}$$

$$\times\left(1-\frac{1}{24\alpha+12\lambda+4\theta^3}\left(\alpha\gamma(5,\theta x)+2\lambda\gamma(4,\theta x)+2\theta^2\gamma(3,\theta x)\right)\right)^{n-r}$$

Therefore, the probability density function of higher order statistic $X_{(n)}$ of area biased three parameter Sujatha distribution can be obtained as

$$f_{x(n)}(x)=\frac{nx^2\theta^4}{24\alpha+12\lambda+4\theta^3}(\theta\alpha x^2+2\lambda x+2\theta)e^{-\theta x}$$

$$\times\left(\frac{1}{24\alpha+12\lambda+4\theta^3}\left(\alpha\gamma(5,\theta x)+2\lambda\gamma(4,\theta x)+2\theta^2\gamma(3,\theta x)\right)\right)^{n-1}$$

and the probability density function of first order statistic $X_{(1)}$ of area biased three parameter Sujatha distribution can be obtained as

$$f_{x(1)}(x)=\frac{nx^2\theta^4}{24\alpha+12\lambda+4\theta^3}(\theta\alpha x^2+2\lambda x+2\theta)e^{-\theta x}$$

$$\times\left(1-\frac{1}{24\alpha+12\lambda+4\theta^3}\left(\alpha\gamma(5,\theta x)+2\lambda\gamma(4,\theta x)+2\theta^2\gamma(3,\theta x)\right)\right)^{n-1}$$

6. Bonferroni and Lorenz Curves

The bonferroni and Lorenz curves are also termed as classical or income distribution curves are applied in different fields like reliability, medicine, insurance and demography. The bonferroni and Lorenz curves are given by

$$B(p)=\frac{1}{p\mu_1'}\int_0^q xf(x)dx$$

and $L(p)=\frac{1}{\mu_1'}\int_0^q xf(x)dx$

Where $\mu_1'=\frac{120\alpha+48\lambda+12\theta^3}{\theta(24\alpha+12\lambda+4\theta^3)}$ and $q=F^{-1}(p)$

$$B(p)=\frac{\theta(24\alpha+12\lambda+4\theta^3)}{p(120\alpha+48\lambda+12\theta^3)}\int_0^q\frac{x^3\theta^4}{24\alpha+12\lambda+4\theta^3}(\theta\alpha x^2+2\lambda x+2\theta)e^{-\theta x}dx$$

$$B(p)=\frac{\theta^5}{p(120\alpha+48\lambda+12\theta^3)}\int_0^q x^3(\theta\alpha x^2+2\lambda x+2\theta)e^{-\theta x}dx$$

$$B(p)=\frac{\theta^5}{p(120\alpha+48\lambda+12\theta^3)}\left(\theta\alpha\int_0^q x^{6-1}e^{-\theta x}dx+2\lambda\int_0^q x^{5-1}e^{-\theta x}dx+2\theta\int_0^q x^{4-1}e^{-\theta x}dx\right)$$

After the simplification of above equation, we obtain

$$B(p)=\frac{\theta^5}{p(120\alpha+48\lambda+12\theta^3)}\left(\theta\alpha\,\gamma(6,\theta q)+2\lambda\gamma(5,\theta q)+2\theta\gamma(4,\theta q)\right)$$

$$L(p)=\frac{\theta^5}{(120\alpha+48\lambda+12\theta^3)}\left(\theta\alpha\,\gamma(6,\theta q)+2\lambda\gamma(5,\theta q)+2\theta\gamma(4,\theta q)\right)$$

7. Maximum Likelihood Estimation and Fisher's Information Matrix

In this section, we will discuss the method of maximum likelihood estimation to estimate the parameters of area biased three parameter Sujatha distribution and also studies its Fisher's information matrix. Let $X_1, X_2,\ldots,X_n$ be a random sample of size n from the area biased three parameter Sujatha distribution, then the likelihood function is given by

$$L(x)=\prod_{i=1}^{n} f_a(x)$$

$$L(x)=\prod_{i=1}^{n}\left(\frac{x_i^{\,2}\,\theta^4}{24\alpha+12\lambda+4\theta^3}(\theta\alpha x_i^{\,2}+2\lambda x_i+2\theta)e^{-\theta x_i}\right)$$

$$L(x)=\frac{\theta^{4n}}{(24\alpha+12\lambda+4\theta^3)^n}\prod_{i=1}^{n}\left(x_i^{\,2}(\theta\alpha x_i^{\,2}+2\lambda x_i+2\theta)e^{-\theta x_i}\right)$$

The log likelihood function is given by

$$\log L=4n\log\theta-n\log(24\alpha+12\lambda+4\theta^3)+2\sum_{i=1}^{n}\log x_i+\sum_{i=1}^{n}\log(\theta\alpha x_i^{\,2}+2\lambda x_i+2\theta)-\theta\sum_{i=1}^{n}x_i \tag{11}$$

By differentiating the log likelihood equation (11) with respect to parameters θ, λ and α. we must satisfy the normal equations as

$$\frac{\partial\log L}{\partial\theta}=\frac{4n}{\theta}-n\left(\frac{12\theta^2}{24\alpha+12\lambda+4\theta^3}\right)+\sum_{i=1}^{n}\left(\frac{\alpha x_i^{\,2}+2}{(\theta\alpha x_i^{\,2}+2\lambda x_i+2\theta)}\right)-\sum_{i=1}^{n}x_i=0$$

$$\frac{\partial\log L}{\partial\lambda}=-n\left(\frac{12}{24\alpha+12\lambda+4\theta^3}\right)+\sum_{i=1}^{n}\left(\frac{2x_i}{(\theta\alpha x_i^{\,2}+2\lambda x_i+2\theta)}\right)=0$$

$$\frac{\partial\log L}{\partial\alpha}=-n\left(\frac{24}{24\alpha+12\lambda+4\theta^3}\right)+\sum_{i=1}^{n}\left(\frac{\theta x_i^{\,2}}{(\theta\alpha x_i^{\,2}+2\lambda x_i+2\theta)}\right)=0$$

The above likelihood equations are too complicated to solve it algebraically. Therefore, we use R and wolfram mathematics for estimating the required parameters of the proposed distribution.

In order to obtain the confidence interval, we use the asymptotic normality results. We have that if $\hat{\beta}=(\hat{\theta},\,\hat{\lambda},\,\hat{\alpha})$ denotes the MLE of $\beta=(\theta,\lambda,\,\alpha)$. We can state the result as follows:

$$\sqrt{n}(\hat{\beta}-\beta) \rightarrow N_3(0, I^{-1}(\beta))$$

Where $I^{-1}(\beta)$ is Fisher's Information matrix.i.e.

$$I(\beta) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial \theta \partial \lambda}\right) & E\left(\frac{\partial^2 \log L}{\partial \theta \partial \alpha}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \lambda \partial \theta}\right) & E\left(\frac{\partial^2 \log L}{\partial \lambda^2}\right) & E\left(\frac{\partial^2 \log L}{\partial \lambda \partial \alpha}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \alpha \partial \theta}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha \partial \lambda}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) \end{pmatrix}$$

$$E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) = -\frac{4n}{\theta^2} - n\left(\frac{(24\alpha+12\lambda+4\theta^3)(24\theta)-(12\theta^2)^2}{(24\alpha+12\lambda+4\theta^3)^2}\right) - \sum_{i=1}^{n}\left(\frac{(\alpha x_i^2+2)^2}{(\theta\alpha x_i^2+2\lambda x_i+2\theta)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \lambda^2}\right) = n\left(\frac{144}{(24\alpha+12\lambda+4\theta^3)^2}\right) - \sum_{i=1}^{n}\left(\frac{4x_i^2}{(\theta\alpha x_i^2+2\lambda x_i+2\theta)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = n\left(\frac{576}{(24\alpha+12\lambda+4\theta^3)^2}\right) - \sum_{i=1}^{n}\left(\frac{(\theta x_i^2)^2}{(\theta\alpha x_i^2+2\lambda x_i+2\theta)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \theta \partial \lambda}\right) = n\left(\frac{144\theta^2}{(24\alpha+12\lambda+4\theta^3)^2}\right) - \sum_{i=1}^{n}\left(\frac{(\alpha x_i^2+2)(2x_i)}{(\theta\alpha x_i^2+2\lambda x_i+2\theta)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \theta \partial \alpha}\right) = n\left(\frac{288\theta^2}{(24\alpha+12\lambda+4\theta^3)^2}\right) + \sum_{i=1}^{n}\left(\frac{(\theta\alpha x_i^2+2\lambda x_i+2\theta)(x_i^2)-(\alpha x_i^2+2)(\theta x_i^2)}{(\theta\alpha x_i^2+2\lambda x_i+2\theta)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \lambda \partial \alpha}\right) = n\left(\frac{288}{(24\alpha+12\lambda+4\theta^3)^2}\right) - \sum_{i=1}^{n}\left(\frac{(2x_i)(\theta x_i^2)}{(\theta\alpha x_i^2+2\lambda x_i+2\theta)^2}\right)$$

Since β being unknown, we estimate $I^{-1}(\beta)$ by $I^{-1}(\hat{\beta})$ and this can be used to obtain asymptotic confidence intervals for θ, λ and α.

8. Application

In this section, we have used a real life data set in area biased three parameter Sujatha distribution to show that the area biased three parameter Sujatha distribution provides a better fit over three parameter Sujatha, two parameter Sujatha and Sujatha distributions. The real life data set is given below as.

The following data set collected by Balakrishnan is related with behavioral science which represents the General rating of Affective symptoms for preschoolers (GRASP) measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to the scale. The study conducted by authors in a city located at the south part of chile has allowed collecting real data corresponding to the scores of GRASP scale of children with frequencies which are given in parenthesis in table 1 as.

Table 1: Data regarding the GRASP scale of children reported by Balakrishnan *et al*. (2010) [2]

19(6)	20(15)	21(14)	22(9)	23(12)	24(10)
25(6)	26(9)	27(8)	28(5)	29(6)	30(4)
31(3)	32(4)	33	34	35(4)	36(2)
37(2)	39	42	44		

In order to estimate the unknown parameters along with the model comparison criterion values, R software technique is applied. In order to compare the area biased three parameter Sujatha distribution with three parameter Sujatha, two parameter Sujatha and Sujatha distributions, we consider the criterions *AIC* (Akaike Information Criterion), *BIC* (Bayesian Information Criterion), *AICC* (Akaike Information Criterion Corrected) and *-2logL*.The better distribution is which corresponds to the lesser values of *AIC*, *BIC*, *AICC* and *-2logL*. For calculating the criterions *AIC*, *BIC*, *AICC* and *-2logL*, following formulas are used.

$$AIC = 2k - 2\log L, \qquad BIC = k\log n - 2\log L \qquad \text{and} \qquad AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

Where k is the number of parameters in the statistical model, n is the sample size and $-2\log L$ is the maximized value of log-likelihood function under the considered model.

Table 2: Comparison and Performance of fitted distributions

Distributions	MLE	S.E	-2logL	AIC	BIC	AICC
Area Biased Three Parameter Sujatha	$\hat{\theta} = 0.41290486$ $\hat{\lambda} = 0.00100000$ $\hat{\alpha} = 0.00100000$	$\hat{\theta} = 0.04208235$ $\hat{\lambda} = 0.05601785$ $\hat{\alpha} = 0.02559747$	93.45522	99.45522	102.7283	99.6552
Three Parameter Sujatha	$\hat{\theta} = 0.3514456$ $\hat{\lambda} = 0.3035788$ $\hat{\alpha} = 0.1131776$	$\hat{\theta} = 0.1611022$ $\hat{\lambda} = 0.9210912$ $\hat{\alpha} = 0.5684602$	118.2962	124.2962	127.5693	124.4962
Two Parameter Sujatha	$\hat{\theta} = 0.38692451$ $\hat{\alpha} = 8.87374520$	$\hat{\theta} = 0.08446987$ $\hat{\alpha} = 11.5743688$	118.4926	122.4926	124.6746	122.5917
Sujatha	$\hat{\theta} = 0.47186742$	$\hat{\theta} = 0.05761019$	120.3453	122.3453	123.4363	122.3780

From table 2 given above, it can be easily seen that the area biased three parameter Sujatha distribution have the lesser AIC, BIC, AICC and -2logL values as compared to three parameter Sujatha, two parameter Sujatha and Sujatha distributions. Hence, it can be concluded that the area biased three parameter Sujatha distribution leads to a better fit over three parameter Sujatha, two parameter Sujatha and sujatha distributions.

9. Control Limits for Area Biased Three Parameter Sujatha (ABTPS) Distribution

$$UCL = \frac{120\alpha + 48\lambda + 12\theta^3}{\theta(24\alpha + 12\lambda + 4\theta^3)} + 3\sqrt{\frac{720\alpha + 240\lambda + 48\theta^3}{\theta^2(24\alpha + 12\lambda + 4\theta^3)} - \left(\frac{120\alpha + 48\lambda + 12\theta^3}{\theta(24\alpha + 12\lambda + 4\theta^3)}\right)^2}$$

$$CL = \frac{120\alpha + 48\lambda + 12\theta^3}{\theta(24\alpha + 12\lambda + 4\theta^3)}$$

$$LCL = \frac{120\alpha + 48\lambda + 12\theta^3}{\theta(24\alpha + 12\lambda + 4\theta^3)} - 3\sqrt{\frac{720\alpha + 240\lambda + 48\theta^3}{\theta^2(24\alpha + 12\lambda + 4\theta^3)} - \left(\frac{120\alpha + 48\lambda + 12\theta^3}{\theta(24\alpha + 12\lambda + 4\theta^3)}\right)^2}$$

10. Numerical Example

The illustration of control limits is considered for illustrating the applications of the method proposed in the above. The Control limits using Area biased three parameter Sujatha distribution are obtained by using simulated data set for parameters α, λ *and* θ.

Table 3: Control limits for ABTPS distribution for $\alpha = 0.005$

λ	θ	CL	UCL	LCL
0.02	0.01	30.24077	543.9859	0
0.04	0.02	32.40461	443.2138	0
0.05	0.03	28.8121	352.6561	0
0.07	0.04	34.58765	349.9275	0
0.09	0.05	40.37282	347.3999	0
0.1	0.06	39.68298	317.1739	0

Table 4: Control limits for ABTPS distribution for $\alpha = 0.01$

λ	θ	CL	UCL	LCL
0.02	0.01	69.12115	758.6186	0
0.04	0.02	60.48614	549.6192	0
0.05	0.03	50.41555	422.4855	0
0.07	0.04	55.1138	399.3745	0
0.09	0.05	60.25442	383.1516	0
0.1	0.06	57.69681	344.863	0
0.2	0.07	135.9973	454.4706	0

Table 5: Control limits for ABTPS distribution for $\alpha = 0.02$

λ	θ	CL	UCL	LCL
0.02	0.01	190.0819	1148.721	0
0.04	0.02	138.2492	726.8152	0
0.05	0.03	108.0225	532.4548	0
0.07	0.04	106.9661	465.7864	0
0.09	0.05	108.6576	418.5224	0
0.1	0.06	100.9245	364.9979	0

Table 6: Control limits for ABTPS distribution for $\alpha = 0.03$

λ	θ	CL	UCL	LCL
0.02	0.01	342.847	1397.184	0
0.04	0.02	183.4955	795.2987	0
0.05	0.03	122.9798	553.7688	0
0.07	0.04	86.28172	432.0635	0
0.09	0.05	51.93717	324.3923	0
0.1	0.06	26.39635	232.9412	0

From table 1- 4, that for the stable label of the parameter α. the LCL becomes negative, it may be rounded to 0. The Area Biased Three Parameter Sujatha Distribution control chart are shown in Fig.1 for $\alpha = 0.02$, λ = 0.07 and θ = 0.04.

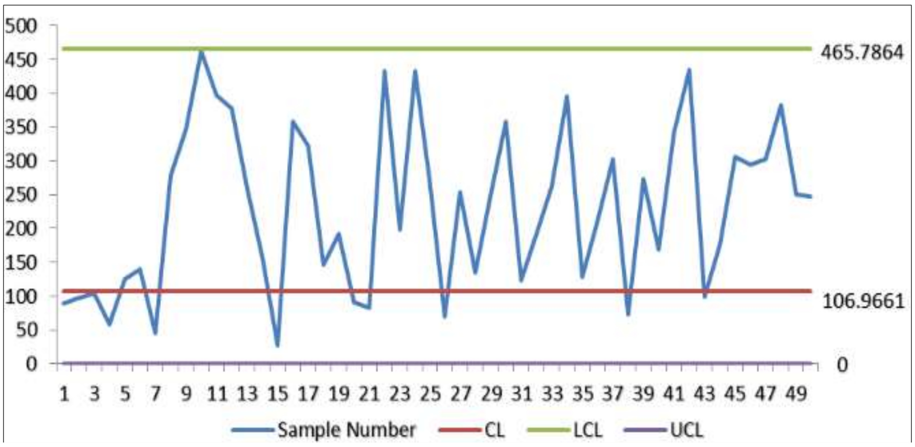

Fig 4: Control chart ABTPSD with α=0.02, λ=0.07 and 0=0.04.

It is also pointed out that, the explanations of the process control are essential or else the process will be in control. That is, depending on the manufacturing products, the manufacturing Engineers would set the parameter values based on the type of data they are dealing with as shown in table 1-4, the greater parameter values, the greater the control limits.

11. Conclusion

The present study deals with a new distribution termed as area biased three parameter Sujatha distribution. The proposed new distribution is generated by using the area biased technique to the baseline distribution. Its different statistical properties which include moments, harmonic mean, survival function, hazard rate function, reverse hazard rate function, order statistics, Bonferroni and Lorenz curves have been discussed. Its parameters have also been estimated by employing the method of maximum likelihood estimation. Finally, a new distribution has been examined and analysed by using a real-life data set and it is found from the result that the proposed area biased three parameter Sujatha distribution leads to a better fit than the three parameter Sujatha, two parameter Sujatha and Sujatha distributions.

12. References

1. Bashir S, Rasul M. Poisson area-biased Lindley distribution and its applications on biological data. Biometrics & Biostatistics International Journal. 2016;3(1):1-10. doi:10.15406/bbij.2016.03.00058
2. Balakrishnan N, Ramesh Gupta C, Debasis Kundu, Victor L, Antonio S. On mixture model based on Birnhaum-Saunders Distributions and associated inference. A study conducted by authors regarding the scores of the GRASP (General Rating of Affective

Symptoms for Preschoolers), in a city located at south part of the Chile; c2010.
3. Cox DR. Some sampling problems in technology, In New Development in Survey Sampling, Johnson, N. L. and Smith, H., Jr (eds.) New York Wiley-Inter Science; c1969. 506-527.
4. Fisher RA. The effects of methods of ascertainment upon the estimation of frequencies. Annals of Eugenics. 1934;6(1):13-25.
5. Hassan A, Akhter N, Para BA. A new generalization of one parameter quasi exponential distribution: Properties and Applications. International Journal of Scientific Research in Mathematical and Statistical Sciences. 2019;6(2):339-354.
6. Jayakumar B, Elangovan R. A new generalization of Ailamujia distribution with applications in bladder cancer data. International Journal of Scientific Research in Mathematical and Statistical Sciences. 2019;6(1):61-68.
7. John NB, Aniefiok II. A three - parameter Sujatha distribution with application. European Journal of Statistics and Probability. 2020;8(3):22-34.
8. Mudasir S, Ahmad SP. Characterization and estimation of the length biased Nakagami distribution Pakistan Journal of Statistics and Operation Research. 2018;14(3):697-715.
9. Patil GP, Rao CR. Weighted distributions and size biased sampling with applications to wildlife populations and human families. Biometrics. 1978;34(2):179-189. doi:10.2307/2530008
10. Rao CR. On discrete distributions arising out of method of ascertainment, in classical and Contagious Discrete, G.P. Patiled; Pergamum Press and Statistical publishing Society, Calcutta; c1965. p. 320-332.
11. Rao AK, Pandey H. Parameter estimation of length biased weighted Frechet distribution via Bayesian approach. Asian Journal of Probability and Statistics. 2021;11(1):42-51.
12. Core R, Team R. version 3.5.3. A language and environment for statistical Computing. R foundation for statistical computing, Vienna, Austria; c2019. URL http://www.R-project.org/.
13. Sharma VK, Dey S, Singh SK, Manzoor U. On length and area biased Maxwell distributions. Communications in Statistics – Simulation and Computation. 2017;47(5):1506-1528.
14. Tesfay M, Shanker R. A two-parameter Sujatha distribution. Biometrics & Biostatistics International Journal. 2018;7(3):188-197.
15. Van Deusen PC. Fitting assumed distributions to horizontal point sample diameters. Forest Science. 1986;32(1):146-148.
16. Zelen M. Problems in cell kinetic and the early detection of disease, in Reliability and Biometry, F. Proschan & R.J. Sering, eds, SIAM, Philadelphia. 1974;53(3):701-706.