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λ-method of C-rating models for combat readiness assessment

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Abstract
This paper reports on a new development of great importance to readiness managers and decision-makers. As the issues concerning the status of a Combat Unit via sub resources, considerations the λ-method is a multi-dimensional approach to readiness problems and it represents an extension of the C-rating technique as developed by (Frank *et al.*, 1968). We present a conceptual framework for this new direction and provide a mathematical justification for the concept. We establish a non-commitment relationship between the Φ-state and the λ-factor of sub resources which are the cardinal concepts of this new approach.

Keywords: Combat readiness, resource areas, sub-resources, Φ-state, criticality factor (λ-factor)

1. Introduction

The subject of combat readiness has been of immensed interest to readiness managers and defence researcher (see, for example, Shishko and Paulson, 1981; Barzily, Marlow and Zacks, 1974; etc.) [3, 1, 6]. However, because of the obvious restricted nature of the subject, only a limited number of literatures are available in the open with highly sensitive research effort remaining classified. By far, the one major aspect of the subject that has generated particular interest to researcher is that of assessment technique (see for example, Brazily, Marlow and Zacks 1979, Gaver and Mazundar, 1976; Mazundar, 1969; Zacks, 1976; Komet, 1976; Rich *et al.*, 1984; Bigelow, J.H., 1988; Tsail, C.L. *et al.*, 1992) [6, 5, 4, 6, 7, 8, 9, 10]. Generally the methods for assessment available in the literature range from personal judgments to sophisticated calculation; actual assessments have varied from "yes" or" no" to indices and complicated mathematical statistical and probability models (Barzity *et al.*, 1979). The C-rating methods of assessing naval fleet readiness via Subresources as reported by (Frank *et al.*, 1968) [2] is one of the most fundamental and most remarkable with respect to the method and the depths of investigation.

However, it does appear that this technique, suffers from:

a) Lack of continental basis.
b) An erroneous assumption that all the sub-resources for a particular fleet or ship will be fully required for all identified tasks.
c) Lack of clear answer to the question, "Are all the ships in the fleet required for every mission?"
d) An assumption that the state of resources should be that of "fully ready" for the ship to be fully ready for a mission.
e) The principles of the weakest link approach presuppose that all the sub-resources are equally important for each mission.

The current effort is motivated by the need to advance the efforts of previous researchers, particular, by extending the contributions of (Frank *et al.*, 1968) [2]. Our development attempts to address adequately those issues some previous researchers had failed to address. An elaborate and hypothetical example of its use is being sent elsewhere for publication. Similarly, two other related works namely; combat Readiness Assessment Models: A survey and framework for Combat Readiness data collection reporting and evaluation system (CRD

CERS) are being sent elsewhere also. The remainder of this paper is organized as followings: Some preliminaries in combat readiness. This section is followed by one which addresses the problem. The fourth section is devoted to the development of the conceptual framework of the λ-method. The last section provides an analytical justification for the method.

1.1 Some Preliminaries and Basic Concepts

The concept of readiness suggests the existence of some phenomenon such as a challenge or event, which requires some resource(s) to meet. The phenomenon can be regarded as the process of assuming a posture to meet an activity or task or a mission. The posture to be attained is indeed the required state of the system to meet the challenge. The desired posture has to be assumed before engaging the event (activity). In other words, the concept is essentially a pre-event phenomenon. Basically, various resources, human and (or) material, are ingredients that must be marshaled in some appropriate or desirable state to meet the challenge (event). Readiness of combat forces is widely accepted as the capability of such forces to perform the mission or function for which they are organized and designed. There are a number of positions taken by various contributions on this concept (for example, Rich *et al*., 1984 [8]; etc.). However, we are considering that of (Shishko and Paulson, 1981) [3] quite representative. The authors define the concept as, the ability of forces, units, weapon systems or equipment to deliver the output for which they were designed (including the ability to be deployed and employed without unacceptable delays). They add that sustainability is the “Staying Power” of forces, unit’s weapons systems, and equipment. Without the military realm, readiness is a pre-D day/(pre-hostilities) phenomenon. We shall regard sustainability as the ability to maintain continuous readiness, since even when an operation is on-stream, to sustain the operation; it will mean ensuring that the unit/system is continuously ready for each activity that makes up the entire operation. By definition, Readiness is the level of preparedness to perform a task or embark on a mission. It is the aggregate capacity to carry out this basic function, given an inventory of resources and their status. It is of different types.

1.2 Types of readiness

On a conceptual basis, there are three type of readiness. There are:

i. Perceived readiness (R_p).
ii. Expected readiness (R_e).
iii. Actual readiness (R_a).

During peacetime, the navy may for example, perceive a threat or task. The readiness of the navy to match this threat or perform the task will be known as perceived readiness. When an actual task has identified, there will be some expected level of readiness required to execute the task successfully. This type of readiness is referred to as expected readiness. In combat situations, it is usually quite a difficult to assess this precisely and more often than not, this normally contributes to the loss of battle by units. When a war or conflict is imminent, then an actual task has been identified. This situation normally gives raise to the need to re-evaluate your expected readiness because your perceived readiness may have been overtaken by developments. The resultant readiness following such a re-evaluation is what we call actual readiness.

1.3 Basic issues

Assessing the combat readiness of a naval fleet could turn out to be as complex as the prosecution of war itself. Conventionally, five basic issues are addressed when assessing readiness which we call the 5W-H of readiness assessment. They are:

i) What should be assessment?
ii) Who should do the assessment?
iii) How should the assessment be done?
iv) When should the assessment be done?
v) Where the assessment should be done?

A naval fleet is usually made up of ships of different types designed to carry out specific roles. Various resources are committed in the process of performing the roles. It has because customary to classify these resources and then use the classification as a basis for carrying out the required readiness assessment. We make the point here that generally, attention is focused on pre-hostility regime. Modification exists for handling hostility regime (sustained readiness) and post-hostility regime. Post-hostility assessment normally serves as input into the fine-tuning of readiness plans for subsequent campaigns.

1.4 Problem

Subresources are the foundation upon which mission (task) accomplishment depends on. The terms “task” and “mission” are used inter-changeably. However, note that in a given mission there might be one or more tasks to be carried out for the mission to be accomplished. And of course, for every task there might be as a number of activities. When we use task we shall be referring to a single task, while mission could mean a single task or more. The goal of the commander is for the fleet to accomplish successfully any identified task. Any further assumption (s) made subsequently will be clearly stated as appropriate.

1.5 Description and Conceptual Framework

Our method (Lambda (λ) approach) for combat readiness assessment is based on two main concepts, namely; the prevailing (static) condition (PSC) of subresource which we refer to as the Φ-state of subresource and the criticality status of the sub-resources with respect to a specific task and this we call the λ–factor (Lambda factor). The two concepts are concomitant in the matrix of readiness assessment.

2. The Φ-state concept

Suppose that F_h is any given naval fleet, then F_h will comprise a number of ships. Suppose further that M is the total number of ships in F_h, F_h will most probably be made of ships of different types according to the roles they are expected to play (Frigates, Aircraft carries, submarines, Fast Attack Craft, Land Ship Thanks, etc.). Let S_h be any Man of War (Fighting ship) of $F_h (S_h \epsilon F_h)$, then S_h has four resource areas (RA) namely operation, Logistics, Manpower and Engineering. Each of these RA’s will be made up of a number of subresources, note various subresoures will normally be in some “functional” state. The state for a particular subresource at a given point in time (or defined interval) could be described in terms of its quality, quantity performance level, or some other characteristic for interest. This measure will be known as the Prescribed Performance Standard (PPS) for any given subreseource. The PPS at the time of evaluation is what refer to as the static condition (Φ-state) of the subresources. Obviously, for a given type or set of subresources, differentiated PSS apply depending on what is being measured or on the appropriate elements of 5W–H, with particular reference to the achievement of the desired objective.

2.1 Analysis of the Φ-Concept

We adopt (with minor modification) the performance measure used by Anyaeche and Oluleye (1990) in their Least Cost Maintenance Policy for a fleet of trucks. Accordingly, Let L_i (i = 1, 2, 3, …, N) be subresources in F_h. Two subresources, L_1 and L_2 are said to be in the same functional state if their Φ state measures are the same. Suppose that a Subresource can assume a Φ-state of Excellent, Good, Fair or Poor. Then, at any given time T, each L_i can be said to be in one and only one of the states namely, Φ_0, Φ_1, Φ_2, or Φ_3. This is a major foundation upon which we develop our work. Consider the i-th Subresource in F_h. This resource may be required on all or some of the ships. Ships of the same type will normally carry the same type of subresources. The resulting structure is a matrix G of dimension N x M with entries Φ_{ij} of $1 \le i \le N$ and $1 \le j \le m$. We may Φ_v, (v = 0, 1, 2, 3,) to mean that when V = 0, the state of the marine divers for examples on board the Man of war is Excellent when V =1, it means the marine diver is good, and so on. We remark that the choosing of PPS is carefully done through a rigorous and mindful process of analysis of the relevant factors. The process is executed by well-trained specialist and seasoned experts. Also, when i = 1 the problem reduces to that of a row-Vector (Φ_{11}, Φ_{12}, Φ_{13}, …, Φ_{1m}). Clearly, we asses in the case of status of the Subresource L_1 as it stand in the various ships. We call this a Single Subresource Assessment (SSA).
Similarly, j = 1, we have a column vector

$$\begin{bmatrix} \Phi_{11} \\ \Phi_{21} \\ \cdot \\ \vdots \\ \cdot \\ \Phi_{N1} \end{bmatrix},$$

Which case, we are assessing the status of the various subresources in ships j = 1. We remark that M < N. For a typical navy, M can be as small as 2 or as large as it could afford. Indeed M, is usually a function of affordability and conceived defence policy and posture of a nation.

Comment

Each subresource that is relevant to the accomplishment of a particular task, contributes to fleet readiness. Consider, for example, two subresources L_r and L_s ($r \neq s$). Suppose there are two tasks, T_g and T_k ($g \neq k$). Now L_r may be regarded as more important that L_s in accomplishing task T_g in terms of its envisaged contribution; however, the reverse may be the case when considering task, T_k, i.e. Ls may now become more relevant and important. We take the position that for any pair of resources, one may be more important than the other for a given task, T_k. Armed with this premise, we proceed to introduce the concept of resource criticality (RC) to mission accomplishment.

3. The Concept of Resource Criticality (RC)

The concept of resource criticality or the λ-position, is rooted in what we call the Resource Requirement Question (RRQ):
"Is the subresource in question required for the identified task or mission?"
In other words, we are interested in knowing whether or not a given subresources is a combat essential for the specified task. If the answer to the RRQ is "Yes", then we say that, that subresource is a readiness candidate (or indicator). Otherwise, we say that it is inconsequential in the matrix of readiness for the identified task regardless of its Φ-state standing, i.e. regardless of its PSC. Figure 1 shows a flow-chart for the determination of readiness candidates.
Now, when the subresource is a readiness candidate i.e. the answer to the Resource Requirement Question (RRQ) is "Yes", then what follows is the "Criticality Question" (CQ), namely "How critical is this subresource (readiness indicator) to the accomplishment of the identified task? It is pertinent to note that, what determines whether a resource is critical to the execution of a task or not, is the nature of the mission. The criticality vector is, therefore introduced only when a task/mission has been identified. Criticality therefore be meaningfully discussed in isolation to a task (i.e. the contextual factor). In reality, therefore, we cannot talk about combat readiness of troops or the fleet via subresources until there is a prescribed task either real or perceived. And, of course, a perceived task gives rise to a perceived readiness while real task gives rise to actual readiness.

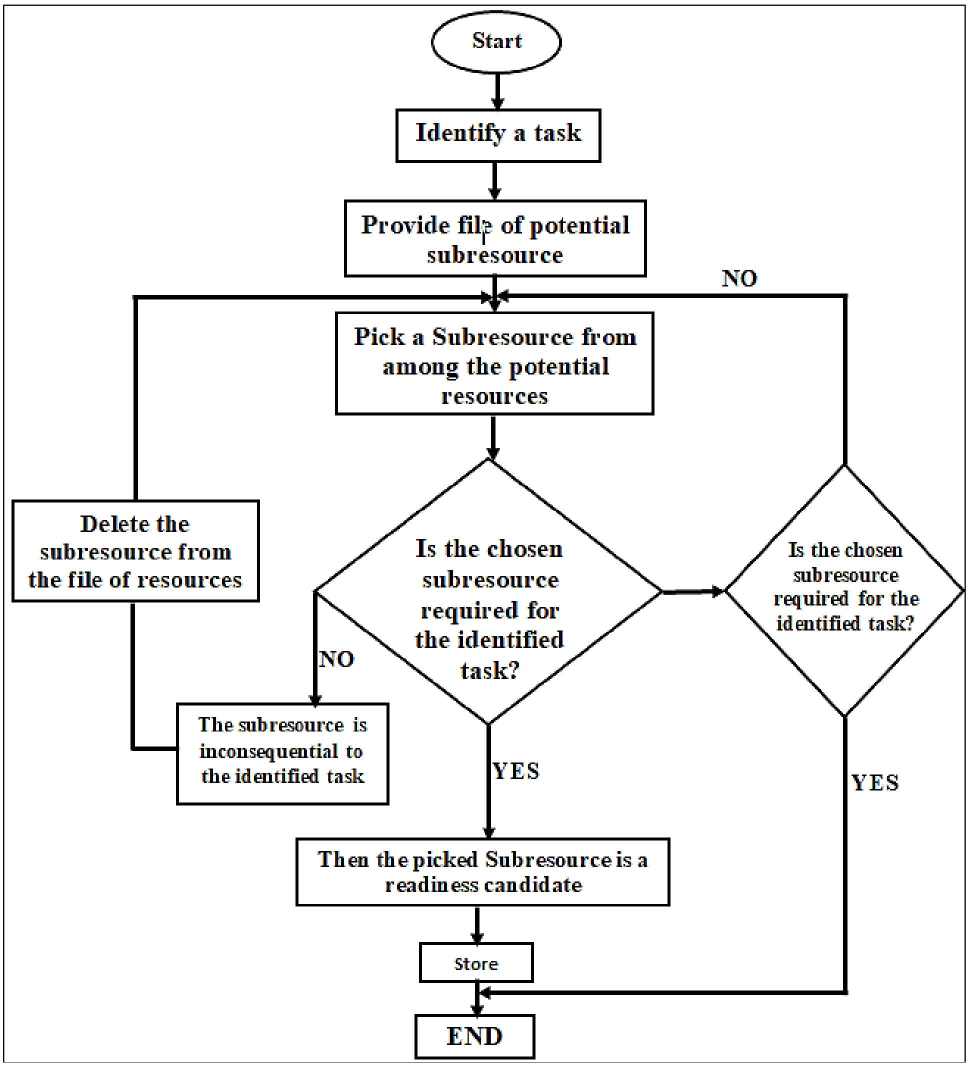

Fig 1: Identifying a readiness candidate

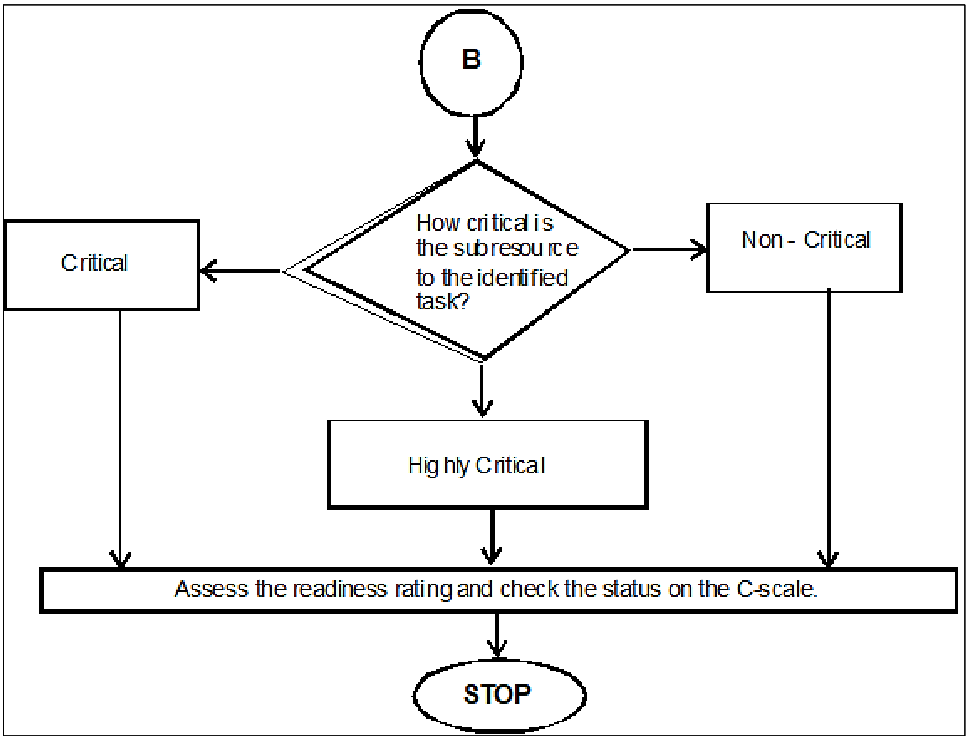

Fig 2: Determining the criticality of sub resources

3.1 The λ-Factor
For the purpose of analysis, the criticality of a subresource, L_i, will need to be determined. Let L_i, M, N, F_h, be as already defined. Then observe that for any type of subresources L_i (I = 1, 2, 3, …, N), its λ-factor with respect to a set of different missions may assume various critically levels. For example, a subresource can be highly critical for the attainment of a mission whereas the same subresource may be inconsequential or non-critical for the execution of another mission. We classify the various levels as, highly critical, critical, non-critical and inconsequential with the associated λ–factors as λ_0, λ_1, λ_2 and λ_3 respectively. For numerical analysis, we are at liberty to choose values for the various levels of Lambda. The relation $\lambda_0 < \lambda_1 < \lambda_2 < \lambda_3$ must be preserved. In practice, λ be between unity and zero so as to maintain a decreasing damaging effect or increasing undamaging effect as the case may be. Suppose we have types of subresources L1 and L2. Suppose further that each has mission damaging effect (MDE) of 0.8 and 0.5, respectively, with respect to mission M_k say, then if the λ-factor of the subresource L_1, is β, and that of L2 is $\mu < \beta$ in terms of undamaging effect means L1 is a more critical resource compared to L2 with respect to mission M_k.

The flow chart for determining criticality level of a subresource is shown figure 2 below. The chart follows the process of identifying a readiness candidate.

The event that L_1 is inconsequential is not reflected because it would have been fathomed. Further, the chart of determining the readiness candidate dovetails into the criticality question by following the source requirement question. Basically, having identified your criticality levels, when the criticality question is asked the answer is reduced to one of the set levels. The criterion for branching is determined by the experts based on policy imperatives. (We treat this in Section III to this study).

3.2 The Dynamics of the Critical Factor
The critical factor is not a static phenomenon. It may change from one task to another and from time to time. It may also differ from one subresource to another. Basically, it is function of time or the mission to be embarked upon. Suppose our identified mission is $\mathcal{M}_k$, and the λ–depending on the mission can be expressed as

$$\lambda = f(\mathcal{M}_k)$$

A subresource L_i with a certain criticality λ_q (q, a measure of state) with respect to mission $\mathcal{M}_k$, may have criticality λ_r(r ≠ q) (r, and measure of state) with respect to mission $\mathcal{M}_k^*$ ($\mathcal{M}_k \neq \mathcal{M}_k^*$). The λ–factor may, therefore, change from one mission to another or even at various stages of the campaign.

3.3 Combat Readiness Ratings Concept (C-Ratings)
The Combat Readiness Level (Rating) connotes the degree of preparedness of the fleet to carry out the mission. Readiness is here defined as the capacity and capability of the ship to perform effectively the prescribed task through the application of available relevant subresources. We shall adopt for discussion the grading system introduced by (Frank *et al*., 1968) [2]. For easy reference, it is summarized in the following table 1 below.

Table 1: C-rating grades

C-Rating	Readiness
C-1	Fully Ready
C-2	Substantially Ready
C-3	Marginally Ready
C-4	Not Ready

This table means that a ship graded as C-1 is fully ready to face the challenge. Similarly a C-4 ship is not ready in the sense that it lacks the appropriate capacity to execute the task at hand. The next section which is largely based on these concepts, examines the C-rating phenomenon as an input-output system.

3.4 Input-Output Process
The Setup: The Lambda (λ) approach for determining the readiness of a fleet is examined and treated as analogous to an input-output system. This treatment is largely based on a 3-factor scale analysis, namely (and in that order)

- The Φ-Scale
- The λ-Scale
- The C-Scale

The Φ scale provides the input data which is analogous to the computer input component, while the λ-scale examines the data, and based on this a produced an output showing the readiness of the subresources to accomplish the task. This module forms what we call the "Readiness Central Processing Chamber" (RCPC). This is analogous to a computer Central Processing Unit. The C- scale provides the media for reading out the readiness level and this is analogous to the output of a computer (Screen, Printout). Indeed these scales arise naturally from the concepts developed in the proceeding We shall now, examine these models in some detail in what follows.

Assumptions

At the time of assessment the Φ-state of a subresource is unchanging. The various Φ-states of subresources have through some prescribed standards been determined.

The Φ-Scale and Grades

We shall adopt the following grading system for simplicity and the sake of precision. For the present discussion and without loss of generality, the table 2 below applies.

Table 2: Fractional deficiency of subresources

Φ-State	Fractional Deficiency
Φ_0	0
Φ_1	0.2
Φ_2	0.5
Φ_3	0.1

This establishes a Φ-State as shown in figure 3 below.

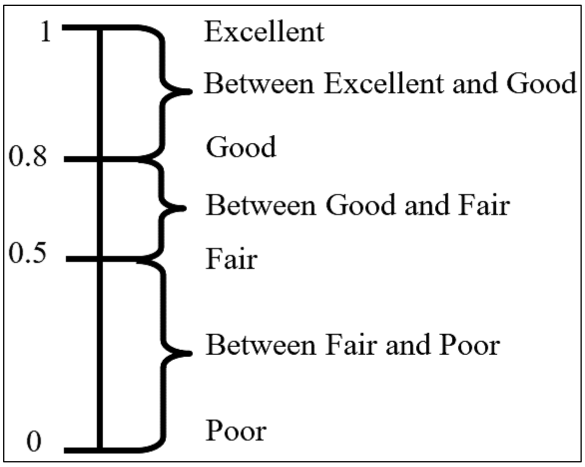

Fig 3: Φ-Scale

The import of what we have in figure 3 is that, a subresource L_i from F_h, whose status of interest has been measured and has been found to have a deficiency of 0.2 is said to be in state Φ_1. By the-scale, its PSC is need as good. In a similar fashion a subresource with a fractional deficiency of "0" is said to be in excellent condition i.e. it is in Φ_0-state, while a "1.0" deficiency implies that the Φ-state of the corresponding subresource is Φ_3, meaning it is in a poor state.

The λ-Scale and Grade

The λ-Scale is divided into 4, three operational and one non-operational. The scales are: λ_0-Scale: λ_1-Scale: λ_2-Scale: λ_3-Scale. This categorization of the λ-scale is highly significant and fundamental to the concept of the criticality factor and the λ-method as a whole. When a task is identified, a subresource is picked and examined by asking the resource fundamental question and taking it through the criticality flow-chart, the possible outcomes can be regarded as events. The events are mutually exclusive. Also, recall that a λ-value for a particular subresource L_i, say, may change as the campaign progresses. This is consistent with the dynamic nature of the λ-factor. It also suggests that the task needs to be continuously assessed and re-assessed as the exercise or campaign progresses. Our next point also considerably important. Having assessed the criticality of a subresource, we then establish that the subresource is fully ready, substantially ready marginally ready or not ready for the identified task. Observe the progressive development we have adopted. Before now we have been concerned with the prevailing (static) condition of the subresourecs as assuming one of the Φ-states, namely Φ_0, Φ_1,Φ_2, Φ_3. We have also been concerned about how critical the subresource is to the accomplishment of the identified task. Subsequently, we have presumed that the subresource would assume one of the critical levels, namely λ_0, λ_1,λ_2, λ_3. Again, observe no mention of whether the subresource has one of C-1, C-2, C-3, or C-4 readiness rating. This is not by omission but is indeed part of the novelty in this current effort.

Now, recall that this particular subresource of interest would have been in some Φ-state with respect to some laid down standards. Further, for an identified task a subresource may be in Φ_2-state i.e it is rated as fair and considered non-critical to the identified task, then this subresource might be regarded as fully ready for the identified task. Similarly, a subresource considered good but highly critical might be found to be only marginally ready for the identified task. The whole concept is that you do not conclude your judgement about the readiness of your subresource until you have assessed the criticality of the subresource to the ACTUAL TASK. When the task is perceived then the readiness obtained will be perceived readiness. Your subresource with their Φ-states get "processed" in the *"Readiness Central Processing Chamber"*. This chamber, which we also call the *Lambda Chamber* (λ-chamber) consists of the various Lambda Scales λ_0, λ_1,λ_2, λ_3. In brief, highly critical subresources are assessed for their C-rating value on the highly critical scale i.e., the λ_0-scale, while critical subresources take their assessment of the critical scale i.e, the λ_1-scale. Similarly, for non-critical and inconsequential subresources. This wise, a highly critical subresource with a better Φ-state compared to that of a non-critical subresource might after all have the same C-rating. The picture will be clearer as we progress with the development.

Remark: We remark that the λ_3-scale confirms any subresource in that category inconsequential and is, therefore dropped from further analysis. We, however, make reference to λ_3-subresource because we need the record for future task or periodic readiness analysis. Ideally, we refer to it as the "O"-scale. Thus, regardless of the readiness position of a λ_3 subresource, it is regarded as fully ready and the ship goes to battle with the subresource in its current status. The four λ-scales are in figures 4, 5, 6 and 7 below.

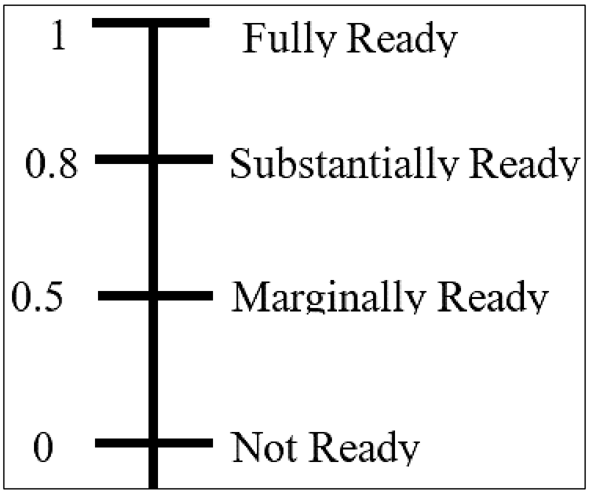

Fig 4: λ_0-Scale (Highly Critical)

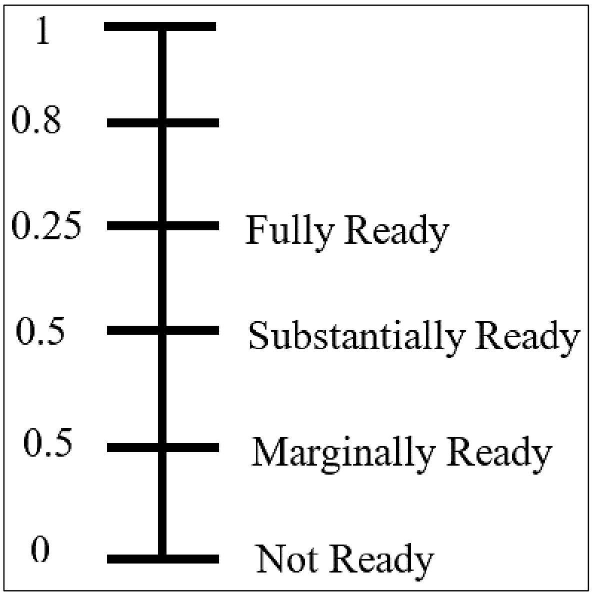

Fig 5: λ_1-Scale (Highly Critical)

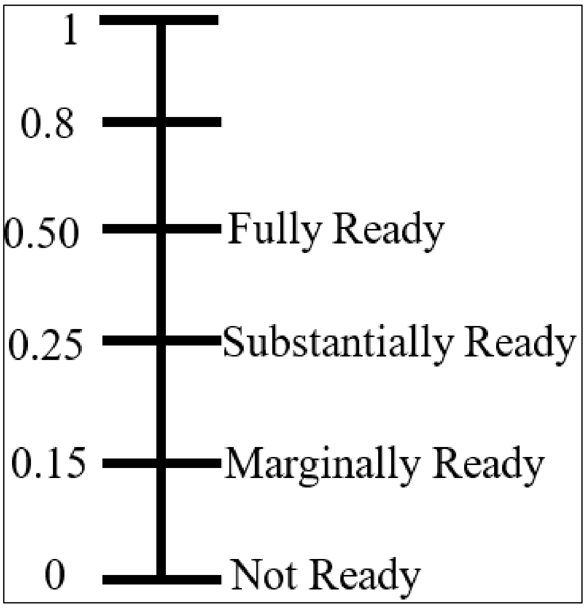

Fig 6: λ_2-Scale (Non-critical)

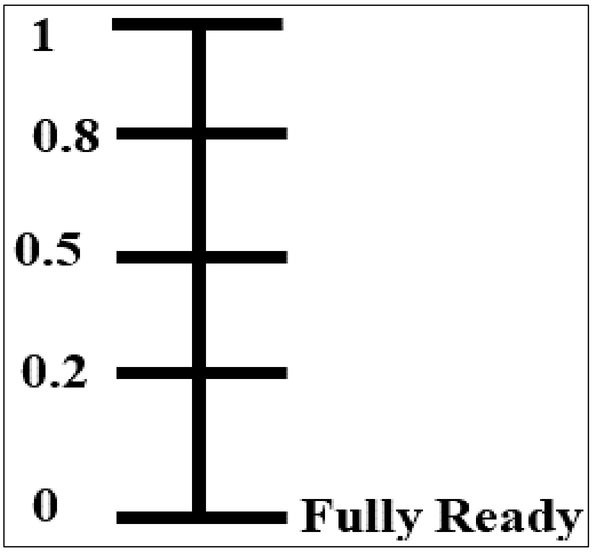

Fig 7: λ_3-Scale (Inconsequential)

The C-Scale and Grades

The corresponding C-scale rating or fractional unreadiness is as shown in table 3 below.

Table 3: Fractional Unreadiness

C-Rating	Fractional Un readiness	Symbol Used
C-1	0	α
C-2	0.2	β
C-3	0.5	y
C-4	1.0	δ

The C-scale is the decision scale. After "processing" the subresources in the readiness central processing chamber (RCPC), the outcome is read out on the C-scale. When evaluating the readiness of a ship we adopt a system whereby our evaluation is done via unreadiness of the various subreasources and the result read out on the C-scale.

The C-scale takes on values ranging from 0.0 to 1.0 as shown in figure 8 below.

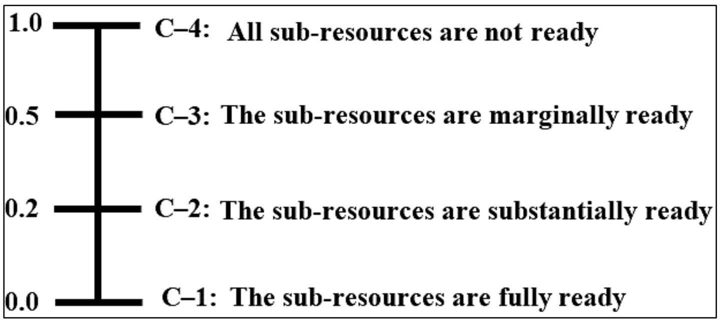

Fig 8: C-scale

Though there is liberty as to what scale to adopt, we believe that the best method for scaling subresources in terms of state, criticality or readiness should depend more on a given perception of the risk you can accommodate.

Note: In a situation where the Φ-scale falls between two states, for example between good and fair, the superior rating should be used. However, on the final scale (the decision scale) i.e. the C-scale figure can then be rounded up using the normal arithmetic procedure. This is to minimize loss of values at the disaggregate level of analysis.

Summary of the conceptual framework

So far, what we tried to establish in this chapter can be summarized in the following schematic representation as shown in figure 9, 10 and 11.

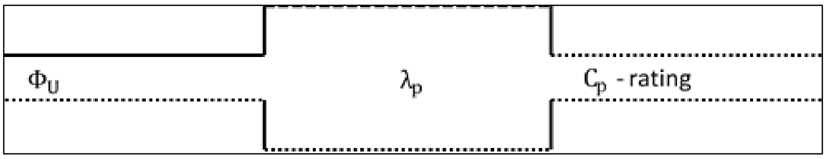

Fig 9: Input-Output Process Based on Perceived Task

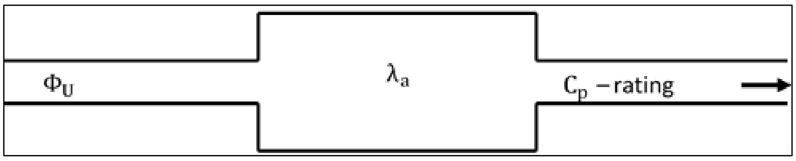

Fig 10: Input-Output Process Based on Actual Task

Where
Φ_u = Φ-scale when assessment is based on perceived task.
λ_p = criticality scale based on perceived task.
C_p = perceived combat readiness.
Φ_v = Φ–state based on actual task.
λ_a = λ–value (criticality) based on actual task.
C_A = the actual C-rating.

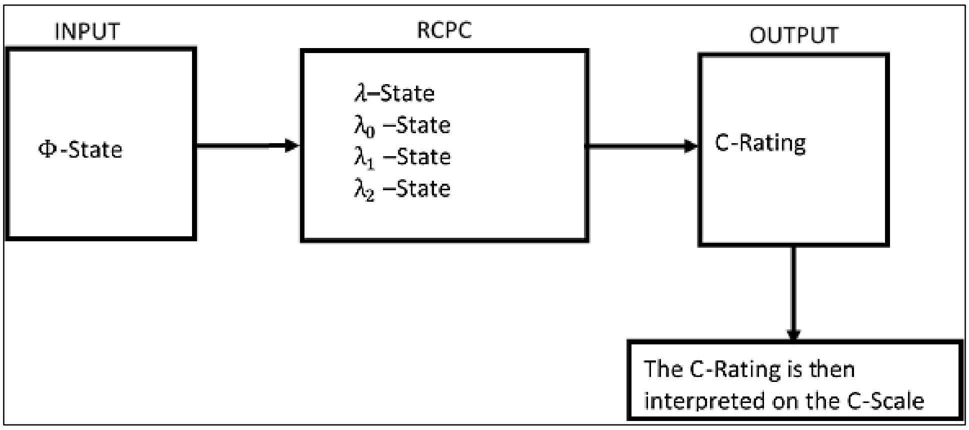

Fig 11: Input-Output Logic Chain

4. Analytical justification

Let F_h, S, L_i (i = 1, 2, 3, …N), T_k (k = 1, 2, 3, …, q)be as ready defined. Then, each subresouce $L_i \in S \; with \; S \in F_h$ would be in some particular $\Phi - state - \Phi_0, \Phi_1, \Phi_2 \; or \; \Phi_3$. Further, each of these L_i will be in some criticality level - $\lambda_0, \lambda_1, \lambda_2 or \; \lambda_3$ with respect to an identified task T_k. We are interested in determining the readiness of F_h for the mission T_k. S is a ship in the fleet.

The Development

We make the following assumption for the development in the sequel.

Assumption (1): Any Φ_3–subresource has no ad-verse effect on readiness, i.e. it does not increase the unreadiness of S and consequently that of F_h

Assumption (2): A Φ_3–subresource does not possess a visible deficiency (if it has a hidden deficiency, when discovered, the time and cost of bringing it up to the required standard is negligible compared to other subresources with poorer states). It is, therefore, considered to be fully ready (C-1) for the task regardless of its λ – factor.

Assumption (3): Any λ_3–subresource has a "temporary dormancy" effect on fleet readiness within the period of assessment. How, each of the subresources will either be highly critical, λ_0; critical, λ_1; non-critical, λ_2; or inconsequential, λ_3 to the success of task T_k. Consider any ship, S, as a sub resource in F_h. Evaluating the readiness of S with eject to the task T_k, 16 possible cases arise:

1. The state (Φ) of ship (S) is considered excellent and S is highly critical to task T_k.
2. The Φ of S is excellent and its λ – factor is critical to task T_k.
3. The Φ of S is excellent and its λ – factor is non-critical to the task T_k.
4. The Φ of S is excellent and its λ – factor is inconsequential to the task T_k.
5. The Φ of S is excellent and its λ – factor is highly critical to task T_k.
6. The Φ of S is excellent and its λ – factor is critical to task T_k.
7. The Φ of S is excellent and its λ – factor is non-critical to the task T_k.
8. The Φ of S is excellent and its λ – factor is inconsequential to the task T_k.
9. The Φ of S is excellent and its λ – factor is highly critical to task T_k.
10. The Φ of S is excellent and its λ – factor is critical to task T_k.
11. The Φ of S is excellent and its λ – factor is non-critical to the task T_k.
12. The Φ of S is excellent and its λ – factor is inconsequential to the task T_k.
13. The Φ of S is excellent and its λ – factor is highly critical to task T_k.
14. The Φ of S is excellent and its λ – factor is critical to task T_k.
15. The Φ of S is excellent and its λ – factor is non-critical to the task T_k.
16. The Φ of S is excellent and its λ – factor is inconsequential to the task T_k.

Definition: We define $\Phi_i \Delta \lambda_j$ to mean, the sub resource under assessment is in the Φ_i and has critically vector λ_j (i, j = 0, 1, 2, 3) with respect to the identified mission.
Consequently, using this definition we summaries the 16 possible cases as blows: $\Phi_1, \Delta \lambda_0, \Phi_0 \Delta \lambda_1, \Phi_0 \Delta \lambda_2, \Phi_0 \Delta \lambda_3, \Phi 1 \Delta \lambda_0\ \Phi_1 \Delta \lambda_1, \Phi_1 \Delta \lambda_2, \Phi_1 \Delta \lambda_3, \Phi_2 \Delta \lambda_0, \Phi_0 \Delta \lambda_1, \Phi_2 \Delta \lambda_2, \Phi_2 \Delta \lambda_3, \Phi_3 \Delta \lambda_0\ \Phi_3 \Delta \lambda_1, \Phi_3 \Delta \lambda_2$ and $\Phi_3 \Delta \lambda_3$.

Definition: We define Z_{Fh} to be the total unreadiness of the various required Sub-resources/ship in our hypothetical fleet to per-form the task T_k.

Let fully ready (FR), substantially ready (SR), marginally ready (MR), and not ready (NR) be represented by α, β, γ, and σ, respectively. That is, the legend, α for C-1, for C-2; γ for C-3 and σ for C-4 holds. In effect, what we are saying is that a sub resource/ship can be in any Φ–state. Further-more, for every sub resource there will be a λ–factor with respect to the task. Now, after a sub resource is processed in the RCPC it will be found (from the C-scale) to admit one of the levels. Then we proceed as follows:-

The total unreadiness of the hypothetical fleet is, therefore, given by:
Z_{F_h} = α (N° of Required Φ_0 ships with criticality λ_0)
+ α (N° of Required Φ_0 ship with criticality λ_1)
+ α (N° of Required Φ_0 ship with criticality λ_2)
+ α (N° of Required Φ_0 ship with criticality λ_3)
+ β (N° of Required Φ_1 ship with criticality λ_0)
+ β (N° of Required Φ_1 ship with criticality λ_1)
+ β (N° of Required Φ_1 ship with criticality λ_2)
+ β (N° of Required Φ_1 ship with criticality λ_3)
+ ϒ (N° of Required Φ_2 ship with criticality λ_0)
+ ϒ (N° of Required Φ_2 ship with criticality λ_1)
+ ϒ (N° of Required Φ_2 ship with criticality λ_2)
+ ϒ (N° of Required Φ_2 ship with criticality λ_3)
+ ϒ (N° of Required Φ_3 ship with criticality λ_0)
+ ϒ (N° of Required Φ_3 ship with criticality λ_1)
+ ϒ (N° of Required Φ_3 ship with criticality λ_2)
+ ϒ (N° of Required Φ_3 ship with criticality λ_3) … (1)

Definition: Define NRS Φi ∇ λj = No of Required Φ_i ship/subresources with critcality λj, (i, j = 0, 1, 2, 3,). By above definition, equation (1) becomes:-

$$\lambda_j = \alpha\,(NRS\Phi_0 \nabla \lambda_0) + \alpha\,(NRS\Phi_0 \Delta \lambda_1) + \alpha\,(NRS\Phi_0 \Delta \lambda_2)$$
$$+ \alpha\,(NRS\Phi_0 \Delta \lambda_{3)} + \beta\,(NRS\Phi_1 \Delta \lambda_0) + \beta\,(NRS\Phi_1 \Delta \lambda_1)$$
$$+ \beta\,(NRS\Phi_1 \Delta \lambda_2) + \beta\,(NRS\Phi_1 \Delta \lambda_0) + \gamma\,(NRS\Phi_2 \Delta \lambda_0)$$
$$\gamma\,(NRS\Phi_2 \Delta \lambda_1) + \gamma\,(NRS\Phi_2 \Delta \lambda_2) + \gamma\,(NRS\Phi_2 \Delta \lambda_3)$$
$$+ \sigma(NR\Phi_3 \Delta \lambda_{0)} + \sigma\,(NR\Phi_3 \Delta \lambda_1) + \sigma\,(NR\Phi_3 \Delta \lambda_2) + \sigma\,(NR\Phi_3 \Delta \lambda_3) \quad \ldots (2)$$

Or

$$\lambda_j = \alpha\,[NRS\Phi_0 \Delta \lambda_0 + NRS\Phi_0 \Delta \lambda_1 + NRS\Phi_0 \Delta \lambda_2 + NRS\Phi_0 \Delta \lambda_3]$$
$$= \beta\,[NR\Phi_1 \Delta \lambda_1 + NR\Phi_1 \Delta \lambda_1 + NR\Phi_1 \Delta \lambda_2 + NR\Phi_1 \Delta \lambda_3]$$
$$= \gamma\,[NR\Phi_2 \Delta \lambda_0 + NR\Phi_2 \Delta \lambda_1 + NR\Phi_2 \Delta \lambda_2 + NR\Phi_2 \Delta \lambda_3]$$
$$= \sigma\,[NR\Phi_3 \Delta \lambda_0 + NR\Phi_3 \Delta \lambda_1 + NR\Phi_3 \Delta \lambda_2 + NR\Phi_3 \Delta \lambda_3] \quad \ldots (3)$$

Assumption 4.2.1, and 4.2.1, Equation (4.2.3) becomes

$$= \beta\,[NR\Phi_1 \Delta \lambda_0 + NR\Phi_1 \Delta \lambda_1 + NR\Phi_1 \Delta \lambda_2 + NR\Phi_1 \Delta \lambda_3]$$
$$= \gamma\,[NR\Phi_2 \Delta \lambda_0 + NR\Phi_2 \Delta \lambda_1 + NR\Phi_2 \Delta \lambda_2 + NR\Phi_2 \Delta \lambda_3]$$
$$= \sigma\,[NR\Phi_3 \Delta \lambda_0 + NR\Phi_3 \Delta \lambda_1 + NR\Phi_3 \Delta \lambda_2 + NR\Phi_3 \Delta \lambda_3] \quad \ldots (4)$$

Assumption 3, equation (4) becomes:

$$= \beta\,[NR\Phi_1 \Delta \lambda_0 + NR\Phi_1 \Delta \lambda_1 + NR\Phi_1 \Delta \lambda_2]$$
$$= \gamma\,[NR\Phi_2 \Delta \lambda_0 + NR\Phi_2 \Delta \lambda_1 + NR\Phi_2 \Delta \lambda_2]$$
$$= \sigma\,[NR\Phi_3 \Delta \lambda_0 + NR\Phi_3 \Delta \lambda_1 + NR\Phi_3 \Delta \lambda_2] \quad \ldots (5)$$

Definition: Let Q be the unreadiness fraction such that α, β, γ, σ, ∈ Q,
Such that

$$Q = \begin{cases} \alpha, if & No\ readiness\ is\ lost\ i.e. Unreadiness = 0 \\ \beta, if & 20\%\ readiness\ is\ lost\ i.e. Unreadiness = 0.2 \\ \gamma, if & 50\%\ readiness\ is\ lost\ i.e. Unreadiness = 0.5 \\ \sigma, if & 100\%\ readiness\ is\ lost\ i.e. Unreadiness = 1.00 \end{cases} \quad \ldots (6)$$

Then Equation (5) can be written as

$$Z_{F_r} = \sum_{\substack{v=\beta,\gamma,\sigma \\ i=1,2,3 \\ j=0,1,2}} Q_v[NR\emptyset_i \nabla\lambda_j] = \sum_{j=0}^{2}\{\beta[NR\emptyset_1\nabla\lambda_j] + \gamma[NR\emptyset_2\nabla\lambda_j] + \delta[NR\emptyset_3\nabla\lambda_j]\} \quad \ldots (7)$$

The mean Unreadiness $\bar{Z}_{F_h}$ is given by

$$\bar{Z}_{F_h} = \frac{\sum_v Q[NR\emptyset_i\nabla\lambda_j]}{Total\ number\ of\ required\ ships\ (TNRS)} \quad \ldots (8)$$

Let the numerator of (8) be denoted as $Z_{F_h}{}^*$. Suppose that the total number of ships in $F_h$is M but a total number of V ships (0 < V < M) are not required for the task. Then, M-V = total number of required ships for the identified task T_k say = TNRS
Denote equation 9 by M^*
Suppose further that of the M^* ships V of them are fully ready i.e. Unreadiness fraction = 0, then,
$M^* – V^*$ = No. of required but not fully ready ship … (10)

i.e. $M^* – V^*$ = No. of required ships for the accomplishment of the mission yet they still possess some level of deficiencies. Denote equation 10 by M^{**}.
Now, equation (8) can be written as

$$\bar{Z}_{F_h} = \frac{Z_{F_h}{}^*}{M-V} \quad \ldots (11)$$

Denote (11) by T^{**}, then

$$\bar{Z}_{F_h} = T^{**} \quad \ldots (12)$$

The mean readiness of $F_h$with respect to this identified Task is then given by:

$$\bar{Z}_{F_h} = 1 - T^{**} = R^* \quad \ldots (13)$$

The value of equation (13) i.e. R^* is then readout on the C-scale to know your readiness standing. A decision can be taken as to whether to embark on the campaign or not.
Observer that when V=0 it implies that All the ships are relevant to the task. In other words, none of the ships is inconsequential to the task i.e the number of $\lambda_3 - ships$ is zero. Whereas when V = M, it implies that all the ships in the fleet are not relevant/required for the task!

4. Conclusion Remarks on the Analytic Development

Also observe that this development apart from its novel approach considers a more realistic denominator for equation (8) i.e. M* (Total number of required ships for the identified task) instead of using M(Total number of ships in the fleet) as by previous investigators (See, for example, Frank, *et al.*, 1968) [2].
At this point, we shall repeat, but for the purpose of emphasis only, that, this development has been on the premise that, a subresource might be in a good or excellent condition ($\Phi -$state) yet its consequence for the fleet's mission success might be non-critical (λ_2) or even inconsequencial (λ_3). However, in some other cases the reverse might be the case i.e. the situation maybe such that the $\Phi -$ state is just fair (Φ_2) or even poor (Φ_3) while the consequence will be enormous. For example, the subresource might be critical or even highly critical to the fleet's mission success.
What the development in this work has tried to achieve is to completely examine the possibilities that can emerge.

It is interesting to observe how the development has neatly used the idea of having difference scales for the λ-Scales. We want to emphasize that, the number of $\lambda - scales$ will depend on how detailed the MHCOM wants to be. Again, the numerical values used are hypothetical but quite typical.

5. References

1. Barzily Z, Marlow WH, Zacks S. Survey of Approaches to Readiness. Naval Research Logistics Quarter (NRLQ). 1979;26(1):21-31.
2. Frank SA, Gruttke WB, Marlow WH, Mathis SJ. Readiness Measurement Via Sub-Resources C-Rating, Technical Paper Series T-216, Logistics Research Project. The Washington DC LD 196463; c1968.
3. Shishko R, Paulson M. Relating Resources to the Readiness and Sustainability of Combined Arms Units, RAND, R-2769-MRAL; c1981.
4. Mazunda M. Uniformly Maximum Variance Unbiased Estimates of Operational Readiness and Reliability in a two-state System. Naval Research Logistics Quarterly. 1969;16(2):199-206.
5. Caver DP, Mazumdar M. Statistical estimation in a problem of system reliability. Naval Research Logistics Quarterly. 1967;14(4):473-488.
6. Zacks S. Review of Statistical Problems and Methods in Logistics Research. Modern Trends in Logistics Research; c1976. p. 227-247.

7. Komet F Jr. Major Issues in Army Logistics. Modern Trends in Logistics Research, W.H Marlow-Ed, MIT Press; c1976.
8. Rich MD, Stanley WL, Anderson S. Improving U.S. Airforce Readiness and Sustainability. The RAND Publications, 31131/1-AF; c1984.
9. Bigelow JH. Related Selected Army Logistics Researches to Combat Performance Measures, The BAND, W-2765-A; c1988.
10. Tsail CL, Tripp R, Berman MB. The Vision Assessment System: Class IX Sustainment Planning. The RAND Corporation R-3969-A; c1992.