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Linearly immutable continuously time series modeled bivariate stochastic processes with vector values: Distinguishing features

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Abstract
Certain observations are assumed to be missed while studying finite continual extended Fourier transformations of time series with precisely stable (i+j) vector values. This is assumed to be the case. This is because the procedure requires studying extended finite Fourier transforms in a standardized manner. The goal is to get as close to an exact interpretation of the results as possible with the data at hand. The results will be put to use in decision-making, which is why this is being done. As a result of this new data, the continuously Fourier transformation will take a starring role in the findings. Asymptotic moments are currently receiving a lot of consideration from researchers all over the world. Case studies on the topic of electrical energy will be used to test our theoretical concepts.

Keywords: Autocovariance, continuously fixed time process, power spectrum, spectrum density, tapered data

1. Introduction

We examine the statistical properties of the linearity relationship between X(t) and Y(t) as represented by the extended finite Fourier transformation, following proposals by D.R. Brillinger (1967) [3], M. Rosenblatt, (1967) [3]; D.R. Brillinger (2001) [4], Ghazal and Farag (2005) [8], Teamah (2004) [1] and Elhassain (2014) [2] and Ghazal, *et al*. (2005) [8]; For a brief overview, of the format of the article, consider the following: (Part1) is an introductory section., Part (2) explored the Approximate Attributes of process as observed, investigated the Approximating aspects of the process as unobserved in Part (3), and in Part (4) we implement our theoretic ideas into practical application, Our method was applied to a study of Average monthly energy imports and exports of the General Electric Corporation between January 2011 and December 2020.

2. The Observed Process's Approximate Attributes

Presume of a fixed sequence that is a vector of values $(i+j)$

$$\Re(t) = [X(t) \quad Y(t)]^T \tag{2.1}$$

$t = 0, \pm 1, \pm 2, \ldots$, $X(t)$ i- valued-vector and $Y(t)$ j- valued-vector. In a definition of the mean function, we suppose that the process (2.1) is a fixed $(i+j)$ valued- vector sequence with parameters $[X_r(t) \quad Ys(t)]^T, r = 1,2,\ldots,j, s = 1,2,\ldots,i$ and that its moments is valid, thus we may deduce the mean function as follow.

$$EX(t) = 0, EY(t) = 0 \tag{2.2}$$

With covariance

$$\begin{aligned} E\{[X(t+g) - \tau_x][X(t) - \tau_x]^T\} &= \tau_{xx}(g) \\ E\{[X(t+g) - \tau_x][Y(t) - \tau_y]^T\} &= \tau_{xy}(g) \\ E\{[Y(t+g) - \tau_y][Y(t) - \tau_y]^T\} &= \tau_{yy}(g) \end{aligned} \tag{2.3}$$

With density spectrums

$$f_{xx}(h) = \int_{-\infty}^{\infty} 1/_{(2\pi)} \sum_{g=-\infty}^{\infty} \tau_{xx}(g) Exp(-ihg)$$
$$f_{xy}(h) = \int_{-\infty}^{\infty} 1/_{(2\pi)} \sum_{g=-\infty}^{\infty} \tau_{xy}(g) Exp(-ihg) \quad (2.4)$$
$$f_{yy}(h) = \int_{-\infty}^{\infty} 1/_{(2\pi)}\, g \sum_{u=-\infty}^{\infty} \tau_{yy}(g) Exp(-ihg), for - \infty < h < \infty$$

That $\beta_a(t), a = 1,2,\ldots,i, (t \in R)$ exists for all t, which is independent on

$$\Re(t) P[\beta_a(t) = 1] = p_a, P[\beta_a(t) = 0] = q_a, \quad (2.5)$$

Take note that

$$E\{\beta_a(t)\} = P, \quad (2.6)$$

Independent data can be used successfully without caring about the results of another. For the modified series perception, considering

$$\delta(t) = \beta(t)\Re(t), \quad (2.7)$$

Where

$$\delta_a(t) = \beta_a(t)\Re_a(t), \quad (2.8)$$

and

$$\beta_a(t) = \begin{cases} 1, \text{if } X_a(t), Y_a(t) \text{are recorded} \\ 0, otherwise \end{cases}, \quad (2.9)$$

Assumption

The time interval (t) is limited in the data window $\ell_a^{(T)}(t)$ so that it has a restricted range, a finite variation, and vanishes between0 $and\ T-1$. Let

$$T^{-1} \int_0^T \ell_a^{(T)}(t)\, dt \underset{T\to\infty}{\longrightarrow} \int_0^1 \ell_a(g)\, dg \ , \qquad a = \overline{1,i}$$

$$\gamma_{a_1,\ldots,a_k}^{(T)}(h) = \int_0^T \left[\prod_{r=1}^{N} \ell_{ar}^{(T)}(t)\right] exp\{-iht\}\, dt$$

3. Approximating Aspects of the Unobserved Procedure

Theorem 3.1. [7]

If we assume that the fixed stochastic procedure is represented by$X_a(t), Y_a(t)$, $a = 1,2,\ldots, min(i,j)$, that missing data points are represented by $\delta_a(t) = \beta_a(t)\Re_a(t), a = 1,2,\ldots. min(i,j)$, We obtain the following if $\beta_a(t)$ is a Bernoulli sequence of stochastic process that satisfies (2.8) and (2.9).

$$E\{\delta_a(t)\} = 0, \quad (3.1)$$

$$Cov\{\delta_{a_1}(t_1), \delta_{a_2}(t_2)\} = P_{a_1 a_2} \begin{bmatrix} \tau_{xx}(g) & \tau_{xx}(g)K(h)^T \\ K(h)\tau_{xx}(g) & K(h)\tau_{xx}(g)K(h)^T \end{bmatrix}, \quad (3.2)$$

Lemma 3.1.

Specifically, if we set $\omega_a^{(T)}(h), a = 1,\ldots., min(j,i)$ similar to

$$\omega_a^{(T)}(h) = \left[2\pi \int_0^T \left(\ell_a^{(T)}(t)\right)^2\right]^{-1/2} \int_{-\infty}^{\infty} \ell_a^{(T)}(t)\delta_a(t)\, exp\{-iht\}\, dt, for h \in R \quad (3.3)$$

Then the dispersion of $\omega_a^{(T)}(h)$ is thus determined to be as follows:

$$D\omega_a^{(T)}(h) =$$
$$P_{aa} \times \begin{bmatrix} \int_{-\infty}^{\infty} f_{aa}(h-\psi) \times \zeta_{aa}(\psi)d\psi & \int_{-\infty}^{\infty} f_{aa}(h-\psi)K(h)^T \times \zeta_{aa}(\psi)d\psi \\ \int_{-\infty}^{\infty} K(h)f_{aa}(h-\psi) \times \zeta_{aa}(\psi)d\psi & \int_{-\infty}^{\infty} K(h)f_{aa}(h-\psi)K(h)^T \times \zeta_{aa}(\psi)d\psi \end{bmatrix} \quad (3.4)$$

Where

$$\zeta_{aa}^{(T)}(x) = \left[\int_0^T (2\pi)(\ell_a^{(T)}(t)dt\right]^{-1} \left|\partial_a^{(T)}(x)\right| \ ,$$
$$\partial_a^{(T)}(x) = \int_0^T \ell_a^{(T)}(t)\, exp(-ixt)dt, x \in R$$

Proof.
Using equation (3.3) we have

$$D\omega_a^{(T)}(h) =$$

$$= p_{a_1a_2}\begin{bmatrix} \int_{-\infty}^{\infty} f_{a_1a_2}(u) \times \zeta_{a_1a_2}(h_1-u,h_2-u)du & \int_{-\infty}^{\infty} f_{a_1a_2}(u)K(h)^T \times \zeta_{a_1a_2}(h_1-u,h_2-u)du \\ \int_{-\infty}^{\infty} K(h)f_{a_1a_2}(u) \times \zeta_{a_1a_2}(h_1-u,h_2-u)du & \int_{-\infty}^{\infty} K(h)f_{a_1a_2}(u)K(h)^T \times \zeta_{a_1a_2}(h_1-u,h_2-u)du \end{bmatrix}$$

When $a_1 = a_2 = a, a = 1,2,\dots,min(i,j)$, and $h_1 = h_2 = h, h \in R$, via Substitution $h - u = \psi$, Hence, we get equation (3.4).

Theorem 3.2. If $\zeta_{aa}{}^{(T)}(x)$, $a = 1,\dots,min(i,j), x \in R$ is limited and continually function at the point $x = h, h \in R$, then the spectral density function $f_{aa}(x), a = 1,\dots,min(i,j)$, $x \in R$ is also limited and continuously at this point then.

$$\lim_{T\to\infty} D\omega_a{}^{(T)}(h) = \begin{bmatrix} f_{aa}(h)) & f_{aa}(h)K(\upsilon)^T \\ K(\upsilon)f_{aa}(h)) & K(\upsilon)f_{aa}(h)K(\upsilon)^T \end{bmatrix}, a = 1,\dots,min(i,j) \quad (3.5)$$

Proof
To prove formula (3.5), we have to establish that

$$\lim_{T\to\infty} \left| D\omega_a{}^{(T)}(h) - p_{aa}\begin{bmatrix} f_{aa}(h)) & f_{aa}(h)K(\upsilon)^T \\ K(\upsilon)f_{aa}(h)) & K(\upsilon)f_{aa}(h)K(\upsilon)^T \end{bmatrix} \right| = 0, \quad (3.6)$$

Lemma (3.1) provides the following.

$$\left| D\omega_a{}^{(T)}(h) - p_{aa}\begin{bmatrix} f_{aa}(h)) & f_{aa}(h)K(\upsilon)^T \\ K(\upsilon)f_{aa}(h)) & K(\upsilon)f_{aa}(h)K(\upsilon)^T \end{bmatrix} \right| \le$$

$$\le p_{aa}\begin{bmatrix} \int_{-\infty}^{\infty} | f_{aa}(h-\psi)| & \int_{-\infty}^{\infty} | f_{aa}(h-\psi)K(\upsilon)^{(T)}| \\ \int_{-\infty}^{\infty} |K(\upsilon)f_{aa}(h-\psi)| & \int_{-\infty}^{\infty} |K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}| \end{bmatrix} -$$

$$- \begin{bmatrix} \int_{-\infty}^{\infty} | f_{aa}(h)| & \int_{-\infty}^{\infty} | f_{aa}(h)K(\upsilon)^{(T)}| \\ \int_{-\infty}^{\infty} |K(\upsilon)f_{aa}(h)| & \int_{-\infty}^{\infty} |K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}| \end{bmatrix} \eta_{aa}{}^{(T)}(\psi)d\psi \le$$

$$\le p_{aa}\begin{bmatrix} \int_{-\infty}^{\psi} | f_{aa}(h-\psi)| & \int_{-\infty}^{\psi} | f_{aa}(h-\psi)K(\upsilon)^{(T)}| \\ \int_{-\infty}^{\psi} |K(\upsilon)f_{aa}(h-\psi)| & \int_{-\infty}^{\psi} |K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}| \end{bmatrix} -$$

$$- p_{aa}\begin{bmatrix} \int_{-\infty}^{\psi} | f_{aa}(h)| & \int_{-\infty}^{\psi} | f_{aa}(h)K(\upsilon)^{(T)}| \\ \int_{-\infty}^{\psi} |K(\upsilon)f_{aa}(h)| & \int_{-\infty}^{\psi} |K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}| \end{bmatrix} \eta_{aa}{}^{(T)}(\psi)d\psi +$$

$$+ p_{aa}\begin{bmatrix} \int_{-\psi}^{\psi} | f_{aa}(h-\psi)| & \int_{-\psi}^{\psi} | f_{aa}(h-\psi)k(\upsilon)^{T}| \\ \int_{-\psi}^{\psi} | k(\upsilon)f_{aa}(h-\psi)| & \int_{-\psi}^{\psi} |k(\upsilon)f_{aa}(h-\psi)k(\upsilon)^{T}| \end{bmatrix} -$$

$$- p_{aa}\begin{bmatrix} \int_{-\psi}^{\psi} \left|f_{aa}(h)\right| & \int_{-\psi}^{\psi} \left|f_{aa}(h)K(\upsilon)^{(T)}\right| \\ \int_{-\psi}^{\psi} \left|K(\upsilon)f_{aa}(h)\right| & \int_{-\psi}^{\psi} \left|K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}\right| \end{bmatrix}\eta_{aa}{}^{(T)}(\psi)d\psi +$$

$$- p_{aa}\begin{bmatrix} \int_{\psi}^{\infty} \left|f_{aa}(h-\psi)\right| & \int_{\psi}^{\infty} \left|f_{aa}(h-\psi)k(\upsilon)^{T}\right| \\ \int_{\psi}^{\infty} \left|k(\upsilon)f_{aa}(h-\psi)\right| & \int_{\psi}^{\infty} \left|k(\upsilon)f_{aa}(h-\psi)k(\upsilon)^{T}\right| \end{bmatrix} -$$

$$- p_{aa}\begin{bmatrix} \int_{\psi}^{\infty} \left|f_{aa}(h)\right| & \int_{\psi}^{\infty} \left|f_{aa}(h)K(\upsilon)^{(T)}\right| \\ \int_{\psi}^{\infty} \left|K(\upsilon)f_{aa}(h)\right| & \int_{\psi}^{\infty} \left|K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}\right| \end{bmatrix}\eta_{aa}{}^{(T)}(\psi)d\psi$$

$B_1 + B_2 + B_3$

We'll clarify each one. In particular, we obtain since $f_{ab}(\psi)$is continuoue at $\Psi = h, a, b = 1, .., min(i, j)$, then we have

$$\mathrm{B}_2 = p_{aa}\begin{bmatrix} \int_{-\psi}^{\psi} \left|f_{aa}(h-\psi)\right| & \int_{-\psi}^{\psi} \left|f_{aa}(h-\psi)k(\upsilon)^{T}\right| \\ \int_{-\psi}^{\psi} \left|k(\upsilon)f_{aa}(h-\psi)\right| & \int_{-\psi}^{\psi} \left|k(\upsilon)f_{aa}(h-\psi)k(\upsilon)^{T}\right| \end{bmatrix} -$$

$$- p_{aa}\begin{bmatrix} \int_{-\psi}^{\psi} \left|f_{aa}(h)\right| & \int_{-\psi}^{\psi} \left|f_{aa}(h)K(\upsilon)^{(T)}\right| \\ \int_{-\psi}^{\psi} \left|K(\upsilon)f_{aa}(h)\right| & \int_{-\psi}^{\psi} \left|K(\upsilon)f_{aa}(h-\psi)K(\upsilon)^{(T)}\right| \end{bmatrix}\eta_{aa}{}^{(T)}(\psi)d\psi +$$

$$= p_{aa}\begin{bmatrix} \int_{-\psi}^{\psi} \left|f_{aa}(h-\psi) - f_{aa}(h)\right| & \int_{-\psi}^{\psi} \left|f_{aa}(h-\psi)k(\upsilon)^{T} - f_{aa}(h)k(\upsilon)^{T}\right| \\ \int_{-\psi}^{\psi} \left|k(\upsilon)f_{aa}(h-\psi) - f_{aa}(h)k(\upsilon)\right| & \int_{-\psi}^{\psi} \left|k(\upsilon)f_{aa}(h-\psi)k(\upsilon)^{T} - k(\upsilon)f_{aa}(h)k(\upsilon)^{T}\right| \end{bmatrix}$$

$B_2 \leq \Omega \int_{-\psi}^{\psi} \eta_{aa}{}^{(T)}(\psi)d\psi \leq \Omega \int_{-\infty}^{\infty} \eta_{aa}(\psi)d\psi\, \eta_{aa}{}^{(T)}(\psi)d\psi$

Assuming that $f_{ab}(\psi)$ is continuing at $\psi = h, a, b = 1, .., min(i, j)$, we've got
$B_2 \leq \Omega$. Now, B_2 is very small, so any Ω very small, so $B_2 = 0$. With a constant G that limits the value of $f_{aa}(h), a = 1, ..., min(i, j), h \in R$ to be limited, we have
$B_1 \leq 2G \int_{-\infty}^{-\psi} \eta_{aa}{}^{(T)}(\psi)d\psi \underset{T\to\infty}{\to} 0,$

In a similar manner, $B_3 \underset{T\to\infty}{\to} 0$. Therefore
$\left|D\omega_a{}^{(T)}(h) - p_{aa}\begin{bmatrix} f_{aa}(h) & f_{aa}(h)K(\upsilon)^{T} \\ K(\upsilon)f_{aa}(h) & K(\upsilon)f_{aa}(h)K(\upsilon)^{T} \end{bmatrix}\right| \underset{T\to\infty}{\to} 0$

The theorem has been proved, then.

4. Practical Study

4.1 Energy Import/Export Investigation

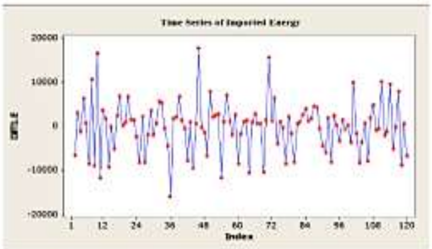
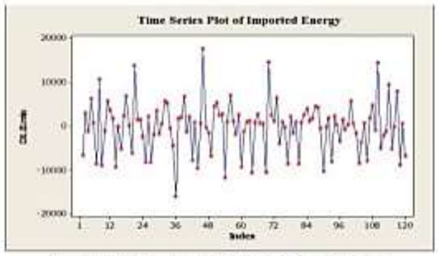
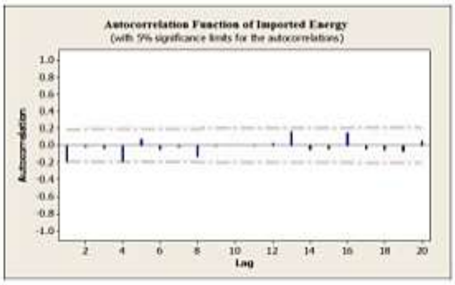
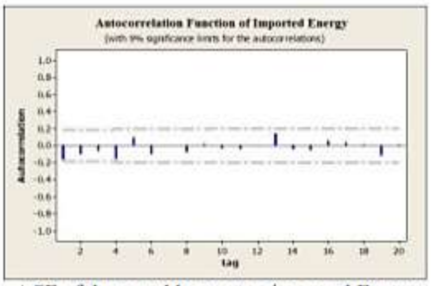
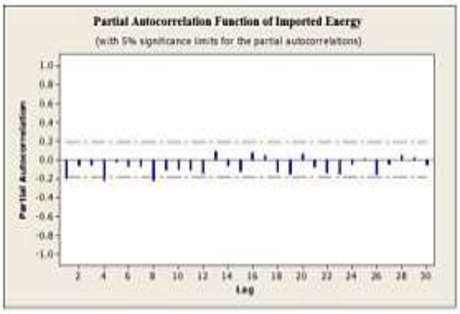
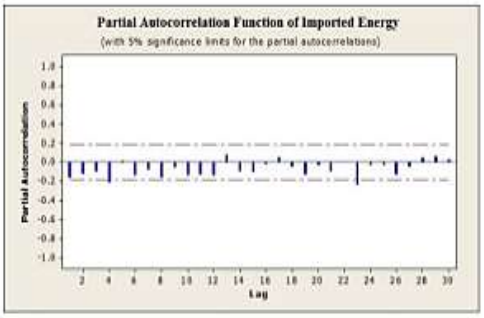
From January 2011 to December 2020, this analysis gives a monthly average of the Energy exported by General Electric Company and the Energy imported by the company.

4.1.1 Energy Import Analytics

We will compare the results we get using our model of time process with missed values to the results from the standard procedure, when all values are recorded.

Assuming the data$X_a(t), (t = (1,2,\ldots,T]$, which is the average monthly Energy that is imported, where all data of the series are recorded, and recorded with some missed, we can write the findings as$\zeta_a(t) = \beta_a(t)X_a(t), a = 1,2,\ldots,i$, such that $X_a(t), (t = 0, \pm 1, \ldots\ldots)$ is a fixed i-valued-vector process. Table (1) compares Findings for the normal Scenario $\beta = 1, \zeta_a(t) = X_a(t)$ and that results when some values are missed at random ($\beta = 0$.)

Table 1: Investigation of Imported Energy Findings both with and without missed data

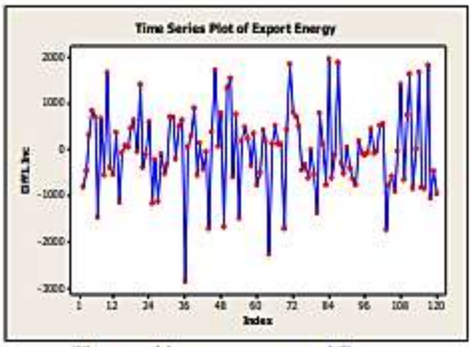
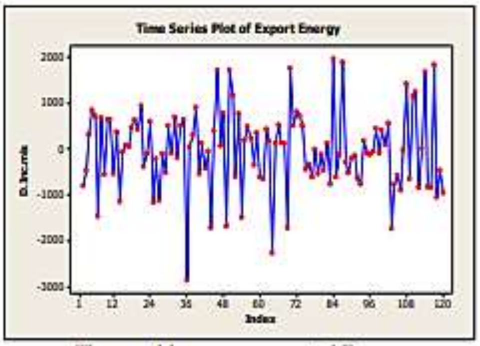
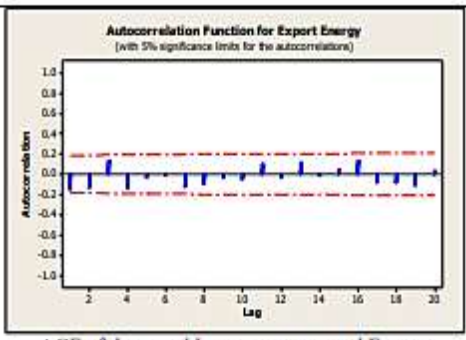
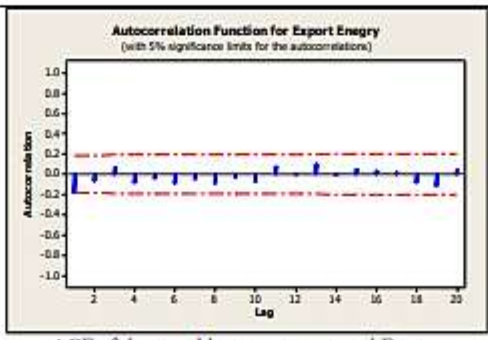
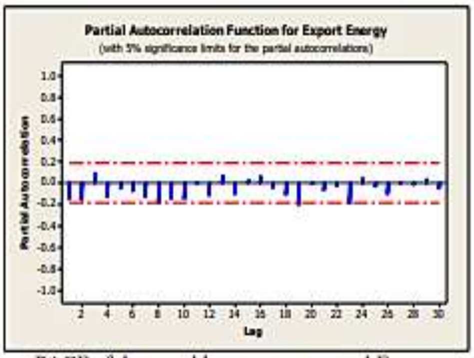
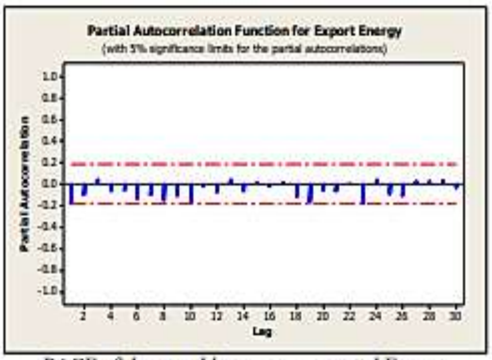
without missing data	with missing data
 The monthly average imported energy	 The monthly average imported energy
 ACF of the monthly average imported Energy	 ACF of the monthly average imported Energy
 PACF of the monthly average imported Energy	 PACF of the monthly average imported Energy
ARIMA Model: imported **Energy without missing data** *ARIMA*(1,1,1) Final Estimates of Parameters Type Co-ef SE Co-ef T P AR 1 0.6234 0.0821 8.19 0.000 MA 1 0.8981 0.0100 47.94 0.000 Residuals: SS = 4021137281 , MS = 27791227, DF = 116 Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 22 36 48 Chi-Square 8.75 20.0 31.2 44.96 DF 9 21 33 45 P-Value 0.450 0.605 0.501 0.432	**ARIMA Model: imported Energy with missing data** *ARIMA*(1,1,1) Final Estimates of Parameters Type Co-ef SE Co-ef T P AR 1 0.5486 0.0798 7.80 0.000 MA 1 0.9014 0.0100 73.15 0.000 Residuals: SS = 3901621742 , MS = 27576233, DF = 116 Modified Box-Pierce (Ljung-Box) Chi-Square statistic Lag 12 24 36 48 Chi-Square 5.98 18.5 29.4 35.5 DF 9 21 33 45 P-Value 0.721 0.803 0.628 0.794

4.1.2 Analytical Exported of Energy

In this study, we will compare the results produced using the standard methods, in which all data is observed, with those obtained using a fixed-time process model with partial missed of data.

With a fixed j- valued-vector time process, $Y_a(t)$, $(t = (1,2,\ldots,T]$ and a stochastically independent Bernoulli sequence, $\beta_a(t)$, the results can be written as $\varpi_a(t) = \beta_a(t)Y_a(t), a = 1,2,\ldots,j$, an average monthly energy exports with complete records is denoted by $Y_a(t)$, where $a = 1,2,\ldots,j$. Table 2 compares the results for the two scenarios where some data are missing at random $(\beta = 0)$ and results for the standard case$(\beta = 0)$.

Table 2: Analyzing the Differences between Outcomes for Exported Energy with and Without Missed Data

without missing data	with missing data
 The monthly average exported Energy	 The monthly average exported Energy
 ACF of the monthly average exported Energy	 ACF of the monthly average exported Energy
 PACF of the monthly average exported Energy	 PACF of the monthly average exported Energy

ARIMA Model: The exported Energy with missing data

ARIMA(1,1,1)

Final Estimates of Parameters

Type	Co-ef	SE Co-ef	T	P
AR 1	0.7536	0.0615	8.98	0.000
AM 2	0.9014	0.0151	47.47	0.000

Residuals: SS = 89431624, MS = 638140, DF = 116

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	8.1	19.95	25.89	44.8
DF	9	21	33	45
P-Value	0.515	0.522	0.720	0.622

ARIMA Model: The exported Energy without missing data

ARIMA(1,1,1)

Final Estimates of Parameters

Type	Co-ef	SE Co-ef	T	P
AR 1	0.7656	0.0589	98.69	0.000
AM 1	0.8902	0.0093	67.98	0.000

Residuals: SS = 90135160, MS = 682338, DF = 116

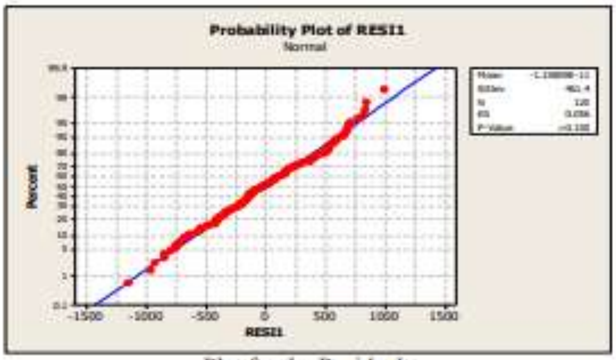
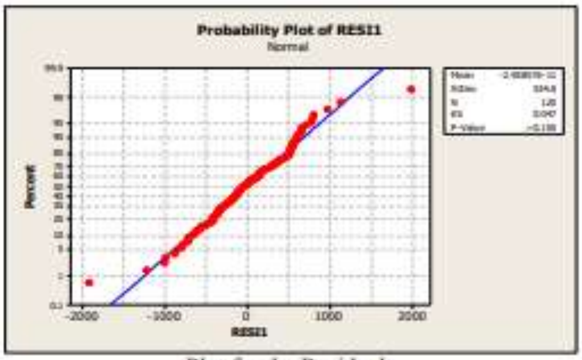
Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	12.1	25.98	35.89	51.1
DF	9	21	33	45
P-Value	0.202	0.205	0.317	0.258

4.1.3 Regression Model Analysis of Energy Imports and Exports

In this analysis, we will evaluate the two scenarios listed in the table below using our outcomes from monthly averages of energy imports and exports are analyzed using a regression model where the outcomes from model with some missed of observations and the classical results from the same model when all observations are available is shown through table (3).

Table 3: Missing Data in Regression Analysis: A Comparison to Complete Data

Without missing data	With missing data
The regression model is Exported Energy = 3260 + 0.190 imported Energy Predictor Co-ef SE Co-ef T P Constant 3260 1242 2.49 0.013 imported Energy 0.1901 0.003589 30.29 0.000 S = 459.317 R-Sq = 88.5% R-Sq= 88.4% Analysis of Variance Source DF SS MS F P Regression 1 186005060 186005060 917.73 0.000 Residual Error 118 24520185 223663 Total 119 210525245 Durbin-watson statistic = 1.69868	The regression model is Exported Energy = 2413 + 0.189 imported Energy Predictor Co-ef SE Co-ef T P Constant 2413 1374 1.49 0.038 imported Energy 0.189 0.006423 24.83 0.000 S = 645.097 R-Sq = 82.8% R-Sq(adj) = 82.7% Analysis of Variance Source DF SS MS F P Regression 1 167904350 178004361 608.05 0.000 Residual Error 118 29140020 297304 Total 119 197044370 Durbin-watson statistic = 1.57860
 Plot fpr the Residuals	 Plot fpr the Residuals

4.1.4 Conclusion

1. The findings of missing data time process analysis were identical to those obtained from regular time series analysis.
2. As with missed data, the results of the investigated for both the X(t) and Y(t) regression models are equivalent since they met the analytical, numerical, and least squares constraints for a regression model between standard time series.

5. References

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