

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2023; 8(2): 101-108
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<https://www.mathsjournal.com>
Received: 17-01-2023
Accepted: 19-02-2023

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Controlling and enhancing performance using QM-window in queuing models

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DOI: <https://doi.org/10.22271/math.2023.v8.i2b.959>

Abstract

The Queuing phenomenon is considered as one of the most frequently observed occurrences in different service industries. It has experienced a significant development in recent years as a result of economic development and population growth, which has increased the pressure on these sectors to produce high-quality services that meet customer requirements, satisfy consumers, and maximize available resources. All previous challenges and variables surrounding service sectors have necessitated the search for a scientific technique that contributes to enhancing performance and overcoming challenges and obstacles related to service supply. Queuing models are regarded as one of the most significant scientific tools for resolving various waiting phenomena related with actual service delivery. Especially in the sector of health care services in Libya where the main challenge that almost every major hospital faces is waiting in line. It may be time to wait long reflection for incompetence in hospital operations. In this regard, this study provided an overview of queuing models, emphasizing their role and significance in monitoring and improving performance using QM-Window software, which is one of the specialised programs in data science that facilitates obtaining performance indicators and recognizing through them the representative model of the service and its acceptability or proposing an alternative model.

Keywords: Queues models, arrival rate, service rate, performance measures, QM-window

1. Introduction

Administrative and service decisions no longer use intuition, guessing, or trial and error. Instead, a new scientific methodology uses advanced methods to make the most accurate and logical conclusions. The theory of queuing is unique in its application areas. It predicts processes in systems with waiting phenomena. In overcrowded hospitals, waiting is an everyday occurrence. This study will apply queuing theory to Al-Saraya Hospital in Al-Khums, Libya. The reality of the institution under study will be established to identify some of the difficulties and attempt to discover solutions utilizing QM-Window software to provide hospital patients with satisfactory service. To monitor and enhance health services, this research defines the queue model in the institution under review and identifies the causes of waiting. This study estimates the service supply time in Al-Saraya Hospital in Al-Khoms, Libya and examines the queuing model to determine its acceptability for the institution. Abdominal clinical departments affect multi-specialty hospitals' ability to provide high-quality medical care. Hospitals use the abdominal department as a source of profit to invest in technology and reduce inpatient losses. Despite the crucial nature of abdominal clinical department, they do not address patient complaints about long waiting periods, which are usually caused by visible lines. So, hospitals in developing countries have a wonderful potential to restructure their systems and adopt best methods and technologies with improved processes to better serve their patients. The abdominal clinical department should be developed for efficiency and throughput because it connects the hospital to the community. The article's structure and progression follow these goals. We examine abdominal department work flow. Second, we'll run a time analysis in the abdominal department. Finally, we'll use the data we collected to draw conclusions, discuss our findings, and accept the study's limitations.

2. Literature Review

Several attempts have been to apply queuing theory to estimate service quality control using various queue models and its applications. (Hsu & Tapiero, 1989) ^[1], used the MG1 queue model was used to perform a study of a manufacturing facility, and it was discovered that the Quantity and Quality of difficulties in the planning of manufacturing processes are interconnected. Park (2001) ^[2] discovered, healthcare provider, consultation, and patient variables all play a role in determining wait times. (Bae & Kim, 2010) ^[3], did a comparative study between MG1 and GM1 queue models to identify the service quality for impatient customers. (Satanaryana *et al.*, 2015) ^[4], used the UCL which is (Upper Control Limit) and LCL which is (Lower Control Limit) parameters were used to estimate the performance of the MM1 queue model, which was used to determine the range of services available at the airport. According to the authors, using queue models to optimize the servers can increase the system's quality and efficiency. (Kuzu, 2015) ^[5], performed a comparison of conventional and queue approaches for implementing queue management in ticketing systems. According to the authors, implementing queue management improves service quality significantly. Since (Aziati & Hamdan, 2018) ^[6] Research of hospital wait times with a focus on doctor-patient appointments in general hospitals found that healthcare provider, consultation, and patient features were the main determinants of wait times. (Yadav & Sohani, 2019) ^[7], utilized the MM1 queue model to estimate the performance of the food chain's services, and described how as the number of servers grows, so does the system's service quality. The "Queuing model as a Method of Queue resolution in Nigeria Banking Sector" is the subject of research by Anichebe (2019) ^[8].

3. Preliminary and Notations on Queuing Theory

The queuing model is an operations research probabilistic mathematical model that assists decision-making. This concept aims to fix challenges that develop when demand exceeds supply, causing disruptions and waiting queues outside channels or providers. This section shows the research procedure used to systematically construct components until the task is finished. Before implementing the method, we must understand queuing theory's core concepts: Arrival process, servicing and leaving procedures, number of service channels, queuing principles (such as first-in, first-out), queue capacity and number of clients being serviced are all aspects of a line that queue theory addresses. Many fields employ queuing theory to improve flow. This queuing model principle can improve medical facility wait times, helps analyses healthcare waiting lines, in most healthcare systems, it can be used for either emergency repairs or short-term facility and resource planning. This work uses a case study research approach because it provides for in-depth investigation of large quantity of data involving a finite number of instances or units throughout a brief time frame. This study applies queuing theory to hospital patient waiting for abdominal department clinics to reduce clinical waiting times. Subjectivity or objectivity affected research results. To understand where delays may arise, we wanted to know how long patients spent at each consultation phase in a typical clinic. Analyzing patients' wait times in line, at the registration channel, in transit, and in the doctor's waiting room. We then collected more investigation-related data.

3.1 Components of Queue Model

1. Population Size Calling Service.
2. Population Calling Service.
3. Personal Characteristic.
4. Arrival Distribution.
5. Service Distribution.
6. Number of Service channels.
7. Service Discipline.

3.2 Queue Model Characteristics

1. **The Wait Time of Customer in The Queue:** It's the maximum number of customers who could possibly need service at any given time, including both those currently waiting and those who have already received it. While the queue's size is unlimited, its length is also potentially unlimited.
2. **The Number of Queues:** In certain cases, like with traffic on a single highway (service channel), there will be only one queue. But, in other cases, like with telephone service, there will be several queues.
3. **Guidelines for Priority Queuing:** Describes the sequence in which customers are selected to receive services as follows:
 1. First In first Out (FIFO) OR First Come first Served (FCFS).
 2. Last In first Out (LIFO) OR Last Come first Served (LCFS).
 3. Service in Random Order (SIRO).
 4. Service in Priorities (SIP).

4. Characteristics of Service Channel

4.1 Methods for Service Supply

1. A waiting system with one queue, one service channel and one phase.
2. A waiting system with one queue and several service channels in one phase.
3. A waiting system with several queues and several service channels in one phase.
4. A waiting system with one queue in which service is delivered in multiple phases.
5. Waiting system, several queues, and numerous service channels.

4.2 Rate of Service Supply

It is the average number of persons who can be served in a limited time and may include:

1. **Fixed rate:** This means that each person who asks for a service will get it at a fixed time.
2. Service durations varies.

4.3 Exit From the System

- Can rejoin the queue of individuals requesting service.
- It may get into predicting poor possibilities for customers again.

4.4 The Goals of Applying Queuing Models

1. Find the average queue waiting time.
2. Analysis of production capacity.
3. Evaluate the service quality provided.
4. Examining the market's competitive status.
5. Rationalizing expenditures and decreasing costs.

5. The Main Probability Distributions Utilized in Queuing Models

5.1 Poisson distribution

It is known as the law of rare probabilities, and it is utilized in several random operations whose.

Items are generated in a particular time or spatial unit Poisson distribution formula in its most generic form is

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \dots \tag{1}$$

With parameter λ , and $(\lambda = np)$.

5.2 Exponential Distribution

It is beneficial for studying the number of customers arriving within a given time, as well as the Interval times between two consecutive connections and it is also applied to the analysis of providing service times. The standard expression for an exponential distribution is

$$f(x) = \lambda e^{-\lambda x} \quad x \geq 0, \quad \lambda > 0, \tag{2}$$

6. Mathematical Models of The Most Popular Queuing Systems

6.1 The Simple Queue Model

The Poissonion arrival and Exponential Service Time in a Single-Channel Queuing Model (M/M/1), This model's problem arises from waiting queues with a single service channel, where the time of arrival to the service channel is random, the time of random arriving following Poisson distribution, and the time of service supply fits the exponential distribution, where customers arrive individually and form a single row, and the service is provided to them in a single phase. The model can be expressed as follows:

M: Refers to the rates of arrival that are distributed according to the Poisson process, and denoted by λ .

M: Refers to the rate of service supply following an exponential distribution, indicated by μ .

1: Refers to a service supply channel.

FIFO: First In first out.

∞ : The queuing model implies an unrestricted maximum allowable quantity and customer source.

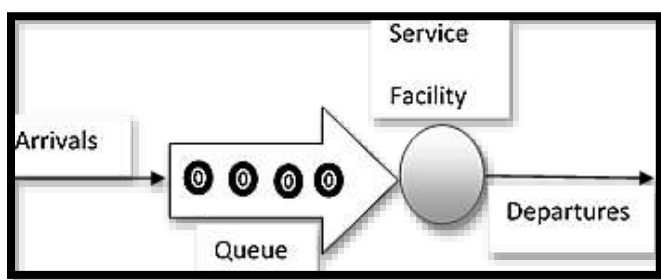


Fig 1: Shows a Single Channel Waiting Queue

6.1.1 Model Assumptions

1. Providing service on a first-in, first-out basis to arriving customers, (FIFO).
2. A consumer who joins a queue remains in it until service is provided.
3. Customer arrivals are independent, with a constant rate of arrival.

4. The customer arrival rate follows Poisson distribution with λ -rate.
5. Rate of providing service times follows an exponential distribution with rate μ .
6. The rate of arrival, λ , is smaller than the time of service $(\lambda < \mu)$.

6.1.2 Model Form

$$L_s = \frac{\lambda}{(\mu - \lambda)} \tag{3}$$

Where, L_s is the mean number of system consumers.

$$W_s = \frac{1}{(\mu - \lambda)} \tag{4}$$

Where, W_s is the average time a customer's spend in a system.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{5}$$

Where, L_q is the mean number of customers in the queue.

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}, \tag{6}$$

Where, W_q is the average time a customer spends in the queue.

$$p = \frac{\lambda}{\mu}, \tag{7}$$

Where, P is the Service supply coefficient (the mean number of customers that obtaining the service in one unit of time)

$$P_0 = 1 - \frac{\lambda}{\mu}, \tag{8}$$

Where, P_0 is the slack time (the system is not busy).

6.2 Multi-Channel Service Queuing Model with Exponential Service Times and Poisson Customers (M/M/S)

In a system with numerous channels for queuing customers, each customer is served by one of several servers. To clarify our argument, let's say all of the customers in line decide to go to the first available server. These channels' service times

are distributed independently and identically, following $\frac{1}{\mu}$ as the mean of an exponential distribution. Poisson distribution with rate λ describes the incoming data. All customers will form a single line to wait and be directed to the nearest service channel. Let's assume once more that customers follow the Poisson distribution and that service providing times follow the exponential distribution. in a multiple-channel system, all customers are given equal priority and all servers are supposed to operate at the same rate. In addition to those already stated for the single-channel model, the additional assumptions also hold.

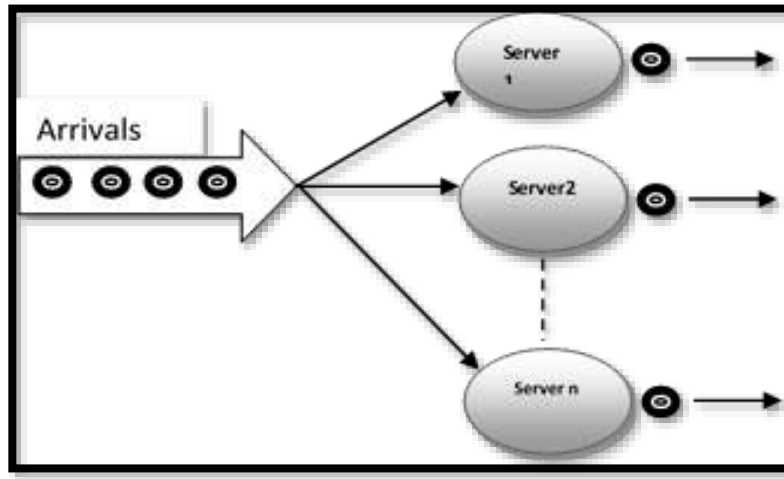


Fig 2: Multiple-Channel Waiting Queue

6.2.1 Model Assumptions

1. The same conditions for using the (M/M/1) model, except for the condition of a waiting queue, where the rate of arrival λ is less than the service supply rate multiplied by the number of service channel ($\lambda < s\mu$), where s the number of service channel is.
2. The queue has several service channels.

6.2.2 Model Form

1. The probability that there are (0) Customer in the system (the system is not busy).

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=s-1} \frac{1}{n} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{s} \left(\frac{\lambda}{\mu} \right)^s \frac{s\mu}{s\mu - \lambda}} \tag{9}$$

2. The probability that there are (n) units in the system given that

$$P_n = \frac{1}{n} \left(\frac{\lambda}{\mu} \right)^n P_0 \quad n < s$$

$$P_n = \frac{1}{s s^{n-s}} \left(\frac{\lambda}{\mu} \right)^n P_0 \quad n \geq s \tag{10}$$

3. The system's estimated average number of customers (L_s)

$$L_s = \frac{\lambda \mu \left(\frac{\lambda}{\mu} \right)^s}{(s-1)(s\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu} \tag{11}$$

4. The average number of customers expected per queue (L_q)

$$L_q = \frac{P_0 \left[\left(\frac{\lambda}{\mu} \right)^s \lambda \mu \right]}{(s\mu - 1)(s\mu - \lambda)^2} \tag{12}$$

5. Customers' average wait time in the system (W_s)

$$W_s = \frac{W_q}{\left(\frac{1}{\mu} \right)} \tag{13}$$

6. Average waiting time for customers in the queue (W_q)

$$W_q = W_s - \frac{1}{\lambda} = \frac{L_s}{\lambda} \tag{14}$$

7. Coefficient of utilization (system busy) (ρ)

$$\rho = \frac{\lambda}{s\mu} \tag{15}$$

7. Statistical Study of the Waiting System in Al-Saraya Hospital

Attributed to the various mathematical models that describe the waiting phenomena in terms of the probability distributions following each of the arrival and service supply times, it is required to define these distributions beforehand.

7.1 Determines The Total Observing Duration

The expected number of patients seen in the internal clinic (abdominal clinics) at Al-Saraya Hospital was calculated over a three-week period beginning on November 5, 2022 and ending on November 25, 2022, which corresponds to the hospital's official working days. The formula for this calculation is established out in the table below.

Table 1: Determination of Total and Partial Observing Durations Throughout the Study's Time Frame

Working Days of the Week	All The Days Except Wednesday And Friday
Study-Designated "Working Days"	Saturday-Monday-Thursday
Official Working Hours	From 5pm-11pm
Study-Designated Hours	From 6 pm-9pm
Observed Time in Hours	3 hours
Observed Time in Minutes	180 minutes
Observed Duration in Minutes	10 minutes
The Maximum Number of Observed Periods Per Day	18 periods per day
The Maximum Number of Observed Periods Per Week	54 periods per week
The Maximum Number of Observed Durations Within Three Weeks	162 periods per three weeks

7.2 The Statistical Study of Arrival Phenomenon

As the process of patients arriving irregularly and at unequal intervals and cannot be predicted in advance, the study of the phenomenon of arrival at the service channel is of great importance in the theory of queues models. The reason for determining the probability distribution is to which the phenomenon of patients arriving at service channel is subjected, we tracked the arrival of these patients over a period of time. It covered three weeks, and 80 observing periods were selected at random from an expected total of 344 observing periods. In order to compute the arrival rate, it is possible to calculate the arrival rate (λ), which is the average number of patient entering the system over an expected 10-minute period (λ), the next table will be utilized,

Table 2: Comprehensive Enumeration of the Number of Arrivals Requesting the Service

Number of arrivals	0	1	2	3	Summation
Observed frequency	12	11	84	55	162
Summation	0	11	168	165	344

The arrival rate (λ) is calculated from the following formula:

$$\lambda = \frac{\sum_{i=1}^4 f_i x_i}{\sum_{i=1}^4 f_i} = \frac{344}{162} = 2.12 \text{ persons/period}$$

After determining the arrival rate, the chi-square test was used to evaluate the distribution of the phenomenon of patient's arrival to service channel, where the following hypothesis was tested,

H₀: Patient's arrival to service channel, follows Poisson distribution.

H₁: Patient's arrival to service channel, doesn't follow Poisson distribution.

We used the following formula for determining the statistic of the test.

$$T = \frac{\sum_{i=1}^r (O_i - E_i)^2}{E_i}$$

Where,

O_i : is the Observed value,

E_i : is the expected value.

$$E_i = np_i$$

$$p_i = \frac{e^{-\lambda} \lambda^x}{x!}$$

Where (λ) indicates the previously determined arrival rate of patients, the value of the test statistic was $T = 3.05$, whereas the tabular chi-square value was 3.25, As the value of the test statistic is less than the value of the tabular chi-square, the null hypothesis has been accepted, indicating that the arrival phenomena follows the Poisson distribution.

7.3 The Statistical Study of Service Providing Time

Randomness characterizes service performance times because they are not fixed and vary amongst patients. To determine the probability distribution to which service performance times are liable, we will follow the same steps as in the statistical study of the arrival phenomenon, where the service time is calculated from the moment the patient enters until the moment he exits, 80 service duration time were randomly selected, as shown in the following table:

Table 3: Particular Time Duration of Service in Minutes

7	5	6	18	7	8	5	6	5	16
9	8	8	6	8	3	4	3	2	6
7	9	4	2	7	8	6	5	4	4
12	6	4	8	7	4	5	11	12	6
9	8	8	4	5	4	8	6	6	6
6	8	4	4	4	3	7	5	7	8
8	8	6	7	4	6	3	7	7	8
8	4	8	8	8	5	6	8	8	5

The data were processed to determine the average providing service time as shown in table (4).

Table 4: Determining the Average Providing Service Time

Interval of Service duration	Observed frequency	Midpoint of the Intervals	Frequency of the Midpoint
2-4.18	19	3.0925	58.784
4.18-6.37	23	5.2775	121.371
6.37-8.55	30	7.4625	223.865
8.55-10.74	3	9.647	28.935
10.74-12.925	3	11.832	35.496
12.925-15.11	0	14.0175	0
15.11-2.317	2	16.2025	32.405
Summation	80		500.82

Through the previous table, we can calculate the average providing service time as follows:

$$\mu = \frac{\sum_{i=1}^7 x_i f_i}{\sum_{i=1}^7 f_i} = \frac{500.86}{80} = 6.26 \text{ Minutes/person}$$

After determining the average providing service time, the chi-square test was conducted to assess the distribution of the service time's phenomena, where we tested the following two hypotheses,

H_0 : The distribution of providing service times follows an exponential distribution

H_1 : The distribution of providing service times doesn't follow an exponential distribution

The statistic were determined using

$$T = \frac{\sum_{i=1}^r (o_i - E_i)^2}{E_i}$$

$$E_i = np_i$$

$$p_i = \mu e^{-\mu x}$$

The test statistic was $T = 0.921$, while the tabular chi-square value is $\chi^2_{1-\alpha, K} = \chi^2_{0.05, 5} = 1.1455$, and since $T < \chi^2_{1-\alpha, K}$, we accept the null hypothesis, hence the arrival times are distributed exponentially.

7.4 Identifies the Features of the Patient Waiting Queue Model

After a statistical analysis of arrival and service times, the following are the major characteristics of the patient waiting queue model in the Abdonomial Clinic at Al-Saraya Hospital,

1. A Poisson distribution with parameter $\lambda = 2.12$ patients/period is used to model the arrival probability distribution.
2. The service providing time probability distribution is an exponential distribution with the parameter $\mu = 6.26$ mins/person.
3. The Service priority in the service organization (Al Saraya Hospital) is first-in-first-out (FIFO).
4. The number of service applicants is unlimited.
5. The service channel options are infinite.

This means that the model that best represents the waiting queue at the investigated service channel is $(M / M / 1)(FIFO / \infty / \infty)$.

7.5 Indicators of Performance in Abdominal Clinic at Al Saraya Hospital

Indicators of performance were derived using the QM-Window software after the arrival time and service performance time were calculated as $\lambda = 2.12$ and $\mu = 6.26$ respectively, as follows,

Table 5: Outputs of QM-Window

Parameter	Value	Parameter	Value	Minutes	Seconds
M/M/1 (exponential service times)		Average Server Utilization	0.34		
Arrival rate (λ)	2.12	Average Number of patients in the queue (L_q)	0.17		
Service rate (μ)	6.26	Average Number of patients in the System (L_s)	0.51		
Number of Servers	1	Average waiting Time in the queue (W_q)	0.08	4.91	294.49
		Average Number of patients in the System (W_s)	0.24	14.49	869.57
		Probability (% of time) System is empty (P_0)	0.66		

The Following Can Be Observed from the Preceding Table

1. Coefficient of Utilization $P=0.34$.
2. The mean number of patients in the queue, $L_q= 0.17$ patient.
3. The average number of patients in the system, $L_s= 0.51$ patient.
4. Average waiting time for patient in queue, $W_q=0.08$ hour.
5. Average time spent by customers waiting in system, $W_s=0.24$ hours.

We can also calculate probabilities when:

1. Number of Patient in the system (N) = Number of service channels(K), $N=K$
2. Number of Patient in the system (N) is less than Number of service channels(K)
 $N \leq K$
3. Number of Patient in the system (N) is More than

- Number of service channels(K)
 $N \geq K$
- $0 \leq K \leq 9$

Table 6: The probabilities for the Number of Service Channels (K) Various Cases

K	Prob. (N =K)	Prob. (N ≤ K)	Prob. (N ≥ K)
0	0.66	0.66	0.34
1	0.22	0.89	0.11
2	0.08	0.96	0.04
3	0.03	0.99	0.01
4	0.01	1	0
5	0	1	0
6	0	1	0
7	0	1	0
8	0	1	0
9	0	1	0

The following figures explain table (6)

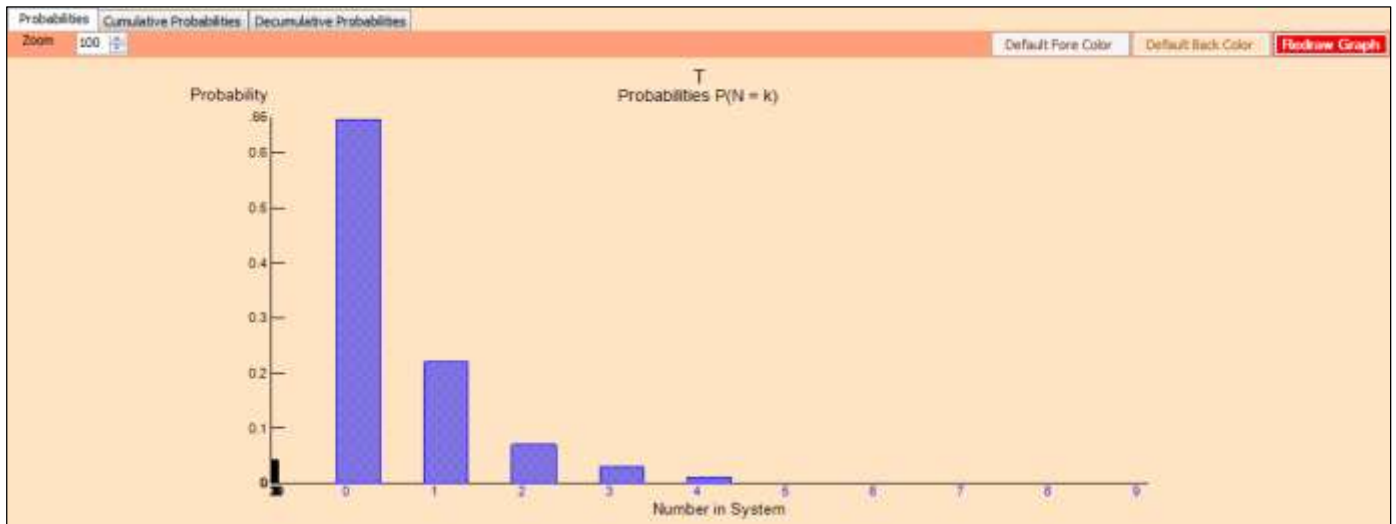


Fig 3: Number of Patient in The System (N) = Number of Service Channels (K), $N=K$

The graphic shows that the probability decreases from 0 to 9, indicating that the greater the number of service channels, the less probable there will be N patients.

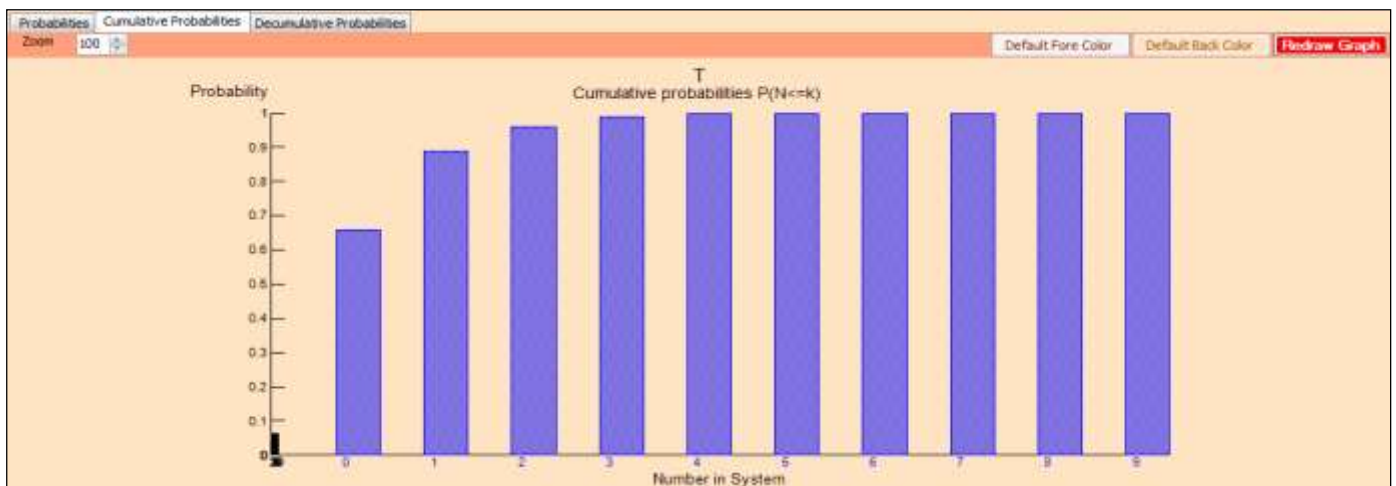


Fig 4: Number of Patient in the System (N) Is Less than Number of Service Channels (K), $N \leq K$

We can see from the previous graphic that the probability goes from 0 to 9, implying that the more service channels there are, the more likely it is that the number of patients N will be less than the number of service channels.

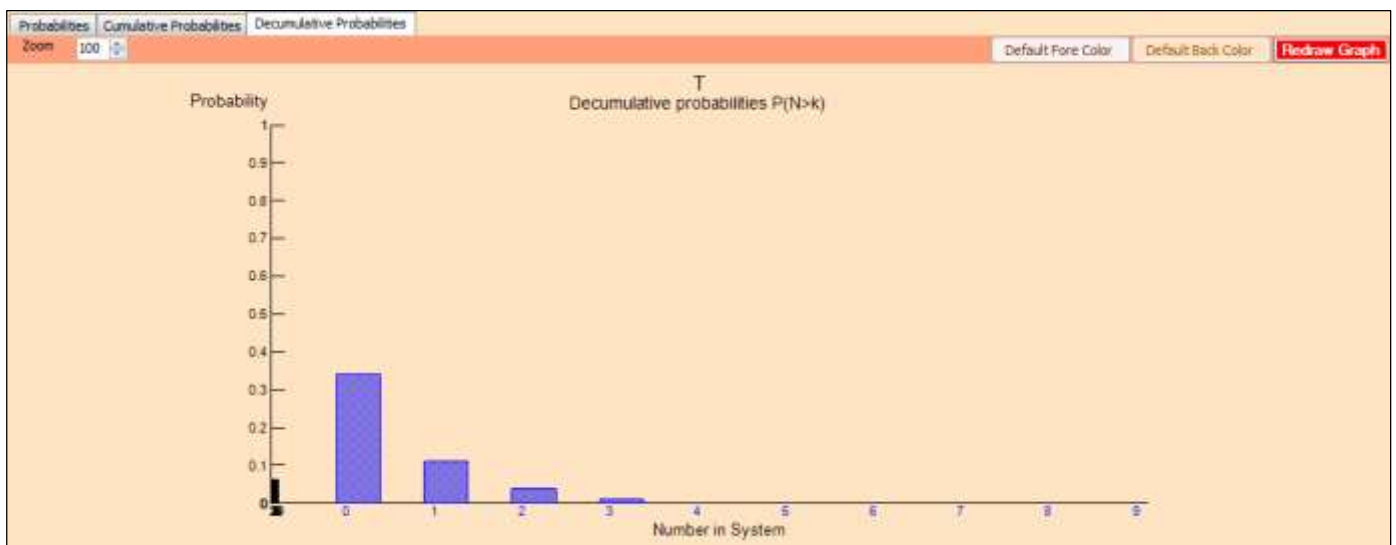


Fig 5: Number of Patient in the System (N) Is More than Number of Service Channels (K), $N \geq K$

We can see that the probability decreases from 0 to 9, indicating that the more service channels there are, the less likely it is that the number of patients, N, will be greater than the number of service channels K, implying that the problem

of forming waiting lines decreases as the number of service channels increases, $N \geq K$.

8. Results

From Our Statistical Study We Can Get The Following

1. Coefficient of Utilization $P = 0.34$, It means that the probability that the system (the abdominal clinic at Al Saraya Hospital) is busy is equal to 0.34, implying that the abdominal clinic at Al Saraya Hospital is operational 34% of the time, indicating that there is no serious congestion in the clinic during normal working hours.
2. The average number of patients in the waiting line is 0.17, which is a small amount and indicates that the patient will not have to wait long for receiving service.
3. There is no overcrowding in the healthcare system as a whole, as the average number of patients is 0.51 person.
4. The average time spent in queue is 0.08 hours, which is a suitable time period to deliver the service, indicating that the service is supplied to the patient without difficulty.
5. The time utilized by the system is 0.24, hours, which is not a substantial value and is not comparable with the capabilities of the department.

9. Recommendations

1. All institutions which are experiencing the problem of congestion must implement queuing models technology in order to track and enhance service performance quality.
2. Staff training in government agencies emphasizes how to utilize the queuing method to achieve optimal arrival to provide services with high quality.
3. Practicing study into the efficacy of operations research techniques in solving common service sector challenges like linear programming, staffing shortages, and transportation inefficiencies.

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