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Heat transfer flow through a porous medium two long vertical plates with constant heat flux at one plate

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Abstract
In this paper analysis is made when one of the plates of the channel is subjected to constant heat flux and the other is maintained at constant temperature. Coupled non- liner equations are solved and expressions for velocity and temperature distributions and Nusselt number are obtained in terms of different parameters, It is being observed that the grashoff number is to increase the velocity distribution. It is interesting to note that when the plates are moving with same velocity but in opposite direction the effect of permeability parameter is to reduce the velocity of the flow in half of the channel width near the upper moving plate but in second half the process reverses.

Keywords: Nusselt number, temperature, grashoff number

1. Introduction
In fluids flowing past a heated or cooled body the transfer of heat takes places by conduction, convection and radiation. If the temperature of the body is not very high heat transfer due to radiation is negligible. Heat transfer problems are classified into two categories [1]. Forced convection, and [2] free convection. In forced convection fluid is made to flow past surface by an external agent and in free or natural convection warmer fluid next to the surface causes the flow due to the density. Difference resulting from the temperature variation throughout the fluid. In free convection the effect of gravity on the heated fluid of variable density essentially causes the motion. Case may arise when both free and forced convection take place in a fluid motion. 0 strach [1, 2] made notable contribution in natural heat transfer flows. He pointed out that Grashoff number plays a significant role in affecting the fluid flow. Eckert and Drack [3] and Welty, *et al*. [4] have discussed free and forced convection in detail. Soriano *et al*. [5] have discussed the importance of small Grashoff number in free convection flow heat transfer.
Looking the applications of free convection Vajravelu [6] considered a problem of heat transfer between two long vertical thin plates moving in opposite direction but with same velocity. The author reported that there is an increase of fluid velocity and temperature with Eckert number, Prandtl number and plate velocity. Kanwal and Sharma [7] extended the problem of Vajravelu [6] for porous plates channel and reported that Reynolds number related to suction velocity affects the skin-friction and rate of heat transfer at plates. Recently Sharma and Singh [8] extended the problem again by taking suction velocity as time function.
Flows through porous media are very much prevalent in nature and therefore the study of such flows has become quite important. When the flow velocity in the porous medium is not high the flow is governed by Darcy's law which does not contain convective acceleration of the fluid. When the permeability of the porous medium is high convective force plays an important role in fluid motion and so extended Darcy's law known as generalized Darcy's law must be employed. Several studies [9-12] have been made on the basis of generalized Darcy's law accompanied by convective term.
If the study is so be made through a medium made up of fibrous material and their permeability is not small. We have to take a body force model. In such a model the fluid Occupies almost all part of the porous medium and we may take that the viscous stress τ_{ij} is expressed in the same form as in a pure fluid:

$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \quad (1)$$

Then, with the neglect of compressibility of the fluid, the fundamental equations in the porous medium are given by

$$\text{Div } \vec{V} = 0 \quad (2)$$

$$\rho\,(\vec{V}\ \text{grad})\,\vec{V} = \vec{F} - \text{grad p} - \frac{\mu}{K}\vec{V} + \nabla^2 \mu\, \vec{V} \quad (3)$$

Where $\vec{F}$ is body forces, p is the pressure, σ the density and K is permeability of the medium.

Equations (2) and (3) known as Brinkman model [13]. Are important equations for the flow in porous media. Tam [14], Saffman [15] have given a rigorous theoretical proof using ensemble average and point force approximations. Rudriah *et al.* [16] have discussed a number of problems in their book.

Equation (3) is also allow the matching of velocities and tractions at the boundary between fluid and porous medium when the flow is across the permeable boundaries. Koplic *et al.* [17] and Kim and Russel [18] have investigated some problems using such boundary conditions.

In this paper an analysis of heat transfer flow between two long vertical plates when the gap between plates is filled with a medium of high porosity has been carried out with the condition that one of plates is subjected to constant heat flux and the other is maintained at constant temperature. The coupled non-linear equations are solved for velocity and temperature fields. The expressions for velocity, temperature and Nusselt number are obtained and the variations are shown in graphical and tabular forms. It is being observe that when the plates are moving with same velocity, the velocity profile is parabolic in nature but increment in permeability parameter $(\sigma = \frac{h}{\sqrt{K}})$ is to reduce the velocity as compared with free fluid flow.

It is interesting to note that when the plates are moving with same velocity but in opposite direction the effect of σ is to reduce the velocity of the fluid in half of channel width near the upper moving plate but in another half the process reverses.

2. Formulation of the problem and equations of motion

We consider a channel made up of two long vertical plates moving with different velocities U_1 and U_2. The plate moving with velocity U_1 is subjected to constant heat flux and the other is maintained at constant temperature. The gap between the plates is filled with a medium of high permeability.

Let axis of x' be taken along one of the plates in the upward direction and Y' - axis normal to it. We further assume that the plates are very long as compared to distance h between the plates. Let u' and v' are the velocity components in the directions of x' *and* y' − *axis* respectively. Since the plates are solid the momentum and energy equations can be written as:

$$-\rho g - \frac{\partial p}{\partial x'} + \mu \frac{d^2u'}{dy'^2} - \frac{\mu}{K} u' = 0 \quad (4)$$

$$\frac{\partial p}{\partial y'} = 0 \quad (5)$$

$$\frac{k}{\rho c_p}\frac{d^2T}{dy'^2} + \frac{\mu}{\rho c_p}\left(\frac{du'}{dy'}\right)^2 = 0 \quad (6)$$

Where g is acceleration due to gravity, k' is permeability of the medium and other symbols have their usual meanings. The term $-\rho g$ in equation (4) represents the fluid element in negative direction of x' for small temperature differences the density ρ in the buoyancy term is considered to vary with temperature whereas the density appearing elsewhere in these equations is considered constant. Here pressure gradient term $-\frac{\partial p}{\partial x'}$ in the x' direction result from change in elevation is not zero. From the equation of state, we have

$$\frac{\rho - \rho_c}{\rho} = -\beta\,(T - T_c) \quad (7)$$

Where ρ_c the reference density is when the temperature of the fluid is $T_{c'}$, β is coefficient of volume expansion and T is temperature distribution. On using well known boussinesq approximation.

$$\frac{\partial}{\partial x'}(p - p_c) = 0 \quad (8)$$

Where p_c is reference pressure corresponding to the density ρ_c, equations (4) to (6) reduce to the form?

$$\nu \frac{d^2u'}{dy'^2} - \frac{\nu u'}{k} + g\beta\,(T - T_C) = 0 \quad (9)$$

$$\frac{k}{\rho c_p}\frac{d^2T}{dy'^2} + \frac{\nu}{C_p}\left(\frac{du'}{dy'}\right)^2 = 0 \quad (10)$$

The boundary conditions are:

$$y' = 0:\ u' = U_1,\ \frac{dT}{dy'} = -\frac{g}{k} \quad (11)$$

$$y' = h:\ u' = U_2,\ T = T_0$$

Let us assume the following non-dimensional quantities

$$u = \frac{u'h}{\nu},\ y = \frac{y'}{h},\ \theta = \frac{T - T_c}{q\, h/k}$$

$$R_1 = \frac{U_1 h}{\nu},\ R_2 = \frac{U_2 h}{\nu}$$

$$G = \frac{g\beta h^4 q}{k\nu^2}\ \text{(Grashoff number)}$$

$$\sigma = \frac{h}{\sqrt{k}}\ \text{(Permeability parameter)}$$

$$Pr = \frac{\rho \nu c_p}{k}\ \text{(prandt1 number)} \quad (12)$$

$$E = \frac{k}{qh^3}\frac{\nu^2}{c_p}\ \text{(Eckert number)}$$

Equation (9) and (10) with the help of (12) reduce to

$$u'' + G\theta - \sigma^2 u = 0 \quad (13)$$

$$\theta'' + E\ pr\ (u')^2 = 0 \quad (14)$$

with prime denotes differential with respect to y.

The boundary conditions reduce to

$$Y = 0:\ u = R_1,\ \frac{d\theta}{dy} = -1$$

$$Y = 1:\ u = R_2,\ \theta = 0 \quad (15)$$

3. Method of solution

For incompressible flow, the Eckert number is small and so following Soundalgekar (19) physical variables u and θ are expend in E as follows

$$u = u_0 + E\, u_1 + 0\,(E^2) \quad (16)$$

$$\theta = \theta_0 + E\,\theta_1 + 0\,(E)^2 \quad (17)$$

Substituting (16) and (17) in (13) to (15) and on solving for u and θ with the help of corresponding boundary condition, we get (1) Solution for $\sigma \neq 0$

$$u = c_1\, e^{\sigma y} + c_2\, e^{-\sigma \gamma} + \frac{G}{\sigma^2} + (l\text{-}y) + E\,[N\, e^{\sigma y} + Q\, e^{-\sigma y} + G\,(-\frac{1}{\sigma^2} + \frac{m}{\sigma^2}\, y) + Pr\,\{C_1C_2\,(y^2 + \frac{2}{y^2}) - \frac{2c_1G}{\sigma^4}\, y e^{\sigma y} - \frac{G^2}{2\sigma^6}\,(y^2 + \frac{2}{y^2}) - \frac{2c_2G}{\sigma^4}\, y\, e^{-\sigma y} + \frac{G_1^2}{12\sigma^2}\, e^{2\sigma y} + \frac{C_2^2}{12\sigma^2}\, e^{-2\sigma y}\}] \quad (18)$$

$$\theta = 1 - y + E\,[L + My + Pr\,\{C_1C_2\, y^2\, \sigma^2 + \frac{2c_1G}{\sigma^3}\, e^{\sigma y} - \frac{G^2}{2\sigma^4}\, y^2 - \frac{2G_2G}{\sigma^3}\, e^{-\sigma y} - \frac{C_1^2}{4}\, e^{2\sigma y} - \frac{C_2^2}{4}\, e^{-2\sigma y}\}] \quad (19)$$

Where

$$C_1 = R_2\, e^{-\sigma} - C_2\, e^{-2\sigma}$$
$$C_2 = \frac{1}{e^{2\sigma - 1}}\,(R_1\, e^{2\sigma} - R_2\, e^{\sigma} - \frac{G}{\sigma^2}\, e^{2\sigma})$$
$$L = -M - Pr\,[C_1C_2\,\sigma^2 + \frac{2c_1G}{\sigma^3}\, e^{\sigma} - \frac{2c_1G}{\sigma^3}\, e^{-\sigma} - \frac{2c_2G}{\sigma^3}\, e^{-\sigma} - \frac{G^2}{2\sigma^4} - \frac{C_1^{2\sigma}}{4}\, e^{-2\sigma} - \frac{C_2^2}{4}\, e^{-2\sigma}]$$
$$M = -Pr\,\sigma\,[\frac{2c_1G}{\sigma^3} + \frac{2c_2G}{\sigma^3} - \frac{C_1^2}{2} - \frac{C_2^2}{2}]$$
$$N = \frac{G}{e^{2\sigma - 1}}\,[(-\frac{1}{\sigma^2} + Pr\,(\frac{2c_1c_2}{\sigma^2} - \frac{G^2}{\sigma^8} + \frac{C_1^2}{12\sigma^2} + \frac{C_2^2}{12\sigma^2}) - e^{\sigma}\,\{\frac{1}{\sigma^2} + \frac{M^2}{\sigma^2} + Pr\,(C_1C_2\,(1 + \frac{2}{\sigma^2}) - \frac{2c_1G}{4}\, e^{\sigma} - \frac{G^2}{2\sigma^6}(1 + \frac{2}{\sigma^2}) - \frac{2C_2G}{\sigma^4}\, e^{-\sigma} + \frac{C_1^2}{12\sigma^2}\, e^{2\sigma} + \frac{C_2}{12\sigma^2}\, e^{-2\sigma})\}]$$
$$Q = -N - G\left[\frac{1}{\sigma^2} + Pr(\frac{2c_1c_2}{\sigma^2} - \frac{G^2}{\sigma^8} + \frac{C_1^2}{12\sigma^4} + \frac{C_2^2}{12\sigma^4})\right] \quad (20)$$

2. Solution for $\sigma = 0$

$$u = C'_1 + C'_2 y - \frac{Gy^2}{2}\,(1 - \frac{y}{3}) + E\, D_2 y - \frac{pr\, E\, Gy^2}{4}\,[D_1 - \frac{y^2}{6}\,\{c'^2_2 + 2c'_{2G}\,(\frac{y^2}{30} - \frac{y}{5}) + G^2\,(\frac{y^4}{280} - \frac{y^3}{35} + \frac{y^2}{15})\}] \quad (21)$$

$$\theta = 1 - y + \frac{Pr\, E\, D_1}{2} - pr\, E\,[\frac{C_2^2\, y^2}{2} + 2c'_2 G\,(\frac{Y^4}{24} - \frac{Y^3}{6}) + G^2\,(\frac{Y^6}{120} - \frac{Y^5}{20} + \frac{Y^4}{12})] \quad (22)$$

Where

$$C'_1 = R_1$$
$$C'_2 = R_2 - R_1 + \frac{G}{3}$$
$$D_1 = C'^2_2 - \frac{C'_2G}{2} - \frac{G^2}{12}$$
$$D_2 = \frac{G\, Pr}{4}\,[D_1 - \frac{Y^2}{6}\,(C'^2_2 - \frac{GC'_2}{3} + \frac{G^2}{24})] \quad (23)$$

3.1 Nusselt number

The Nusselt number is given by

$$Nu = \frac{gh}{k\,(T - T_c)} \quad (24)$$

with the help of non-dimensional quantities (12), the non-dimensional form of Nusselt number Nu is

$$Nu = \frac{1}{\theta(0)} \quad (25)$$

Where

$$\theta(0) = \theta_0(o) + E\,\theta_1\,(o)$$

Table 1: Temperature distribution for different values of σ and G ((Pr = 0.7, E = 0.01)

Case 1: $R_1 = 0.5$, $R_2 = -0.5$, G =1.0						
σ	y→	0.0	0.2	0.4	0.6	0.8	1.0
0.0	θ→	1.0086	0.8085	0.6082	0.4077	0.2028	0
3.0	θ→	1.0036	0.8025	0.6024	0.4017	0.2007	0
Case 2: $R_1 = 0.5$, $R_2 = 0.5$, G = 1.0							
σ	y→	0	0.2	0.4	0.6	0.8	1.0
0.0	θ→	1.0002	0.8000	0.6000	0.4000	0.2000	0
3.0	θ→	1.0010	0.8011	0.6009	0.4006	0.2003	0
Case 3: $R_1 = 1.0$, $R_2 = 1.0$, $\sigma = 0$							
G	y→	0.0	0.2	0.4	0.6	0.8	1.0
1.0	θ→	1.0006	0.8006	0.6006	0.4004	0.2002	0
3.0	θ→	1.0008	0.8008	0.6007	0.4004	0.2002	0
Case 4: $R_1 = 0.5$, $R_2 = 0.5$, $\sigma = 3.0$							
G	y→	0.0	0.2	0.4	0.6	0.8	1.0
1.0	θ→	1.0010	0.8011	0.6009	0.4009	0.2003	0
5.0	θ→	1.0036	0.8033	0.6026	0.4015	0.2009	0

Table 2: Nusselt number Nu at y = 0 plate for different values of σ and G (Pr = 0.7, E = 0.01)

R_1	R_2	G	σ	Nu
0.5	-0.5	1.0	0.0	0.99147
0.5	-0.5	1.0	3.0	0.99641
0.5	0.5	1.0	0.0	0.99980
0.5	0.5	1.0	3.0	0.99902
1.0	0.5	1.0	0.0	0.99940
1.0	0.5	3.0	0.0	0.99920
0.5	0.5	1.0	3.0	0.99900
0.5	0.5	5.0	3.0	0.99641

4. Numerical discussions

Numerical calculations are made for velocity and temperature distributions. The velocity distribution is shown graphically while the temperature distribution is given in tabular form for different values of R_1, R_2, G and σ.

From the figure 1 It is being observed that when $R_1 > R_2$ the effect of G is to increase the velocity of the flow. In case when $R_1 = R_2$ the velocity distribution is parabolic in nature but increase of σ is to reduce the velocity as compared with free flow.

It is interesting to note that when the plates are moving with same velocity but in opposite direction the effect of σ is to reduce the velocity in half of the channel near the upper moving plate but in the second half the process reverses.

From the tables for temperature distribution, it is observed that θ decreases with y for each case i.e. when $R_1 = R_2$ or $R_1 \neq R_2$. It is to be noted here that when $R_1 = - R_2$, θ decreases with while for $R_1 = R_2$ the effect reverses. From cases 3 and 4 it is concluded that G increases the temperature in the fluid.

From the table 2 for Nusselt number, it is being observed that Nusselt number.

(a) Increases with permeability when plates are moving with same velocity but in opposite direction.

(b) Decreases with permeability and Grashoff number when the plates are moving with same velocity and in same direction.

(c) Increase with Grashoff number when the plates are moving with different velocity in same direction.

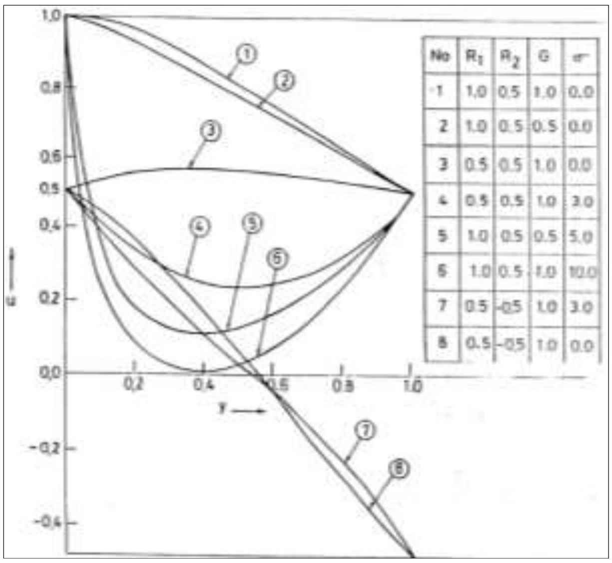

Fig 1: Velocity u plotted against y for different values of R_1, R_2, and G (E= 0.001, Pr = 0.7)

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