International Journal of Statistics and Applied Mathematics 2023; 8(3): 26-35

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2023; 8(3): 26-35

https://www.mathsjournal.com
Received: 02-03-2023
Accepted: 12-04-2023

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Design of geometric moving average control chart to monitor poisson mean for an unstabilized production process

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DOI: https://doi.org/10.22271/maths.2023.v8.i3a.975

Abstract
Control chart is one of the important tools of Statistical Process Control to monitor the quality of goods or to improve the production process by identifying the assignable causes of variations. In this article we present a newly developed control chart to Monitor Poisson mean through Geometric Moving Average through EWMA Statistic. In a novice production process, the variations are inevitable and fluctuate between control or in-control process. Hence to suite the need of the un-stabilized process and to monitor the target TPEWMA is developed. The newly designed TPEWMA control charts and its parameters are determined based on the Truncated Poisson Distribution. The comparison on EWMA control chart with newly designed TPEWMA control charts shows better detection in small shifts. The Average Run Length of control chart is the measure of efficiency of the control chart and is larger for an in-control quality process. But at the same time ARL's are smaller for small shifts. Hence one can give pressure on the producer to maintain the process through tightening the variable factor in a TPEWMA control chart.

Keywords: EWMA statistic, PEWMA, TPEWMA, poisson mean, average run length

1. Introduction

Statistical Process Control (SPC) comprises of Statistical techniques to monitor and control a production process. SPC tools and procedures can help to monitor process behaviour, discover issues in the systems, and find the prevalence of assignable causes of variations if any.

Process Control Charts was initially developed by Dr. Walter A. Shewhart in 1920. It is a type of chart which is used to study how the process changes over time. Control Charts are widely used to monitor the quality characteristics and to detect the process stability.

The measurable quality characteristic such as volume, dimension, weight, etc. can monitored through control chart for variables. The number of failures in production run, number of malfunction of disk in a computer, etc., can be monitored through the control charts for attributes.

Shewhart control charts are useful for long run and large shifts in the production process. The Cumulative sum (CUSUM) control chart and Exponentially Weighted Moving Average (EWMA) control charts are used to detect small process shifts. The CUSUM control chart was initially proposed by Page (1954). The EWMA control chart was developed by Roberts (1959) [1]. The EWMA control chart was developed for monitoring process mean in detecting small shifts. Basically, the EWMA control chart follows the normal distribution with mean μ and variance σ^2. Borror *et al.* (1998) [9] proposed an EWMA for a Poisson Mean (PEWMA). However there is no control charts in the literature to monitor the mean for an un-stabilized process. Hence in this paper, new control charts are developed for an un-stabilized environment.

In this section, 0 Truncated Poisson EWMA (0 TPEWMA), 0-1 Truncated Poisson EWMA (0-1 TPEWMA) and 0-2 Truncated Poisson EWMA (0-2 TPEWMA) control limits are derived and control charts are developed. Average run length (ARL) is derived for the newly developed charts and is compared with existing charts.

The content of this paper is as follows: Section 2 contains the relevant literature review of control charts. Section 3 consists of methodology of the research and a matlab program is written to determine the parameters. Sections 4 contain the findings and are given in the tables and graphs. The control limits are derived and control charts of TPEWMA are given. Section 5 contains the comparisons with other types of control charts and the final Section comprises of the results and conclusion.

2. Review of literature

S.W. Roberts (1959) [1] introduced the control chart based on geometric moving averages to monitor the process mean of a normal distribution and is more sensitive for detecting small shifts. Cox (1961) [2] predicted a mean square error exponentially weighted moving average control chart. Hunter (1986) [3] developed an exponentially weighted moving average control chart for continuous process control. Crowder (1987) [4] determined Average Run Length (ARL) for EWMA scheme with tables and graphs.

Gan (1990) [5] analysed EWMA control chart based on the observations from the Poisson distributions. The three modified EWMA control chart are Ceil (CEWMA), Round (REWMA), Floor (FEWMA) Control charts. The CEWMA control chart is designed to access the smaller increments in Poisson data more effective when a process means shifts can move towards the larger value. Likewise, the FEWMA control chart is designed to access the smaller decrements is more effective when the shifts move towards the smaller value. The REWMA is an approximation of unmodified EWMA. Lucas and Saccucci (1990) [6] have studied the properties and performance of EWMA schemes and conclude that EWMA is more efficient for detecting process mean. They also developed a False Initial Response (FIR) for EWMA control chart.

MacGregor and Harris (1993) [7] have suggested that exponentially weighted moving variance (EWMV) and the mean squared deviation (EWMS) in the alternative case for monitoring process variance. Rhoads, Montgomery & Mastrangelo (1996) [8] proposed a new Fast Initial Response (FIR) scheme for EWMA which is compared with Lucas and Saccucci (1990) [6] FIR. Borror, Rigdon and Champ (1998) [9] developed a Control chart based on Poisson data and derived an ARL using Markov Chain approach. They also conclude than the LCL for Poisson EWMA control chart (PEWMA) is usually positive, hence the downward changes in the process mean can be detected. Steiner (1999) [10] developed time varying control limits for a EWMA control chart. Khoo, Michael, and Sim (2005) [11] developed a Robust EWMA control chart for easier detection of the outliers. Shu (2008) [12] designed an Adaptive EWMA control chart for monitoring process variance. Naveed *et al* (2018) [13] designed a control chart which is based on EWMA but the statistic of EWMA is extended. This control chart can detect the quick shifts in the process mean.

Amitava Mitra, Kang Bok Lee, Subhabrata Chakraborti (2019) [14], proposed an Adaptive EWMA control chart for monitoring process mean for both smaller and larger shifts. Arslan *et al* (2022) [15] developed an Improved Adaptive EWMA (IAEWMA) control chart for efficient monitoring of process mean shift. The performance comparison tools are used to show that IAEWMA is more outperforms than some other control charts.

3. Methodology

The mean of the production process is monitored through Poisson data and the truncated Poisson variate is augmented with EWMA Statistic. The new control limits are derived by finding the mean and variance of TPEWMA.

3.1 Zero Truncated Poisson EWMA Control Chart

Let X_1, X_2, X_3 ………be the sequence of independent and identical random variables following the zero truncated Poisson distribution. To monitor the process mean let us consider the statistic of geometric moving average control chart which was proposed by Roberts (1959) [1],

$$Z_t = \lambda X_t + (1-\lambda) Z_{t-1} \tag{1}$$

Where the λ be a constant which ranges from $0 < \lambda \leq 1$. Then $Z_0 = \mu_0$ when the starting value is the process target (i.e., i=1). Equation (1) is also called as EWMA Statistic.

Then the mean and variance of the zero truncated Poisson EWMA is,

$$\mathrm{E}(Z_t) = \frac{\mu_0}{(1-e^{-\mu_0})} \tag{2}$$

and

$$\mathrm{Var}(Z_t) = \left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{{\mu_0}^2+\mu_0}{(1-e^{-\mu_0})} - \frac{{\mu_0}^2}{(1-e^{-\mu_0})^2}\right] \tag{3}$$

For large values of t the variance is approximately reduces to,

$$\mathrm{Var}(Z_t) \approx \left[\frac{\lambda}{(2-\lambda)}\right]\left[\frac{{\mu_0}^2+\mu_0}{(1-e^{-\mu_0})} - \frac{{\mu_0}^2}{(1-e^{-\mu_0})^2}\right] \tag{4}$$

Hence, the control limits for zero truncated Poisson EWMA control chart are,

$$\mathrm{UCL} = \frac{\mu_0}{(1-e^{-\mu_0})} + k\sqrt{\left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{{\mu_0}^2+\mu_0}{(1-e^{-\mu_0})} - \frac{{\mu_0}^2}{(1-e^{-\mu_0})^2}\right]}$$

$$\text{CL} = \frac{\mu_0}{(1-e^{-\mu_0})}$$

$$\text{LCL} = \frac{\mu_0}{(1-e^{-\mu_0})} - \text{k}\sqrt{\left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{\mu_0{}^2+\mu_0}{(1-e^{-\mu_0})} - \frac{\mu_0{}^2}{(1-e^{-\mu_0})^2}\right]}$$

3.2 Zero – One Truncated Poisson EWMA Control Chart:
Let X_1, X_2, X_3 ………be the sequence of independent and identical random variables following the zero - one truncated Poisson distribution. Then, consider the statistic from (1),

$$Z_t = \lambda X_t + (1-\lambda) Z_{t-1}$$

where the λ be the constant which ranges from $0 < \lambda \le 1$. Then $Z_0 = \mu_0$ when the starting value is the process target (i.e., i=1). Then the mean and variance of the zero-one truncated Poisson EWMA control chart is,

$$\text{E}(Z_t) = \frac{\mu_0[1-e^{-\mu_0}]}{[1-e^{-\mu_0}(1+\mu_0)]} \tag{5}$$

and

$$\text{Var}(Z_t) = \left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{\mu_0}{[1-e^{-\mu_0}(1+\mu_0)]}\left[\mu_0 + (1-e^{-\mu_0}) - \frac{\mu_0[1-e^{-\mu_0}]^2}{[1-e^{-\mu_0}(1+\mu_0)]}\right]\right] \tag{6}$$

For large values of t the variance is approximately reduces to,

$$\text{Var}(Z_t) \approx \left[\frac{\lambda}{(2-\lambda)}\right]\left[\frac{\mu_0}{[1-e^{-\mu_0}(1+\mu_0)]}\left[\mu_0 + (1-e^{-\mu_0}) - \frac{\mu_0[1-e^{-\mu_0}]^2}{[1-e^{-\mu_0}(1+\mu_0)]}\right]\right] \tag{7}$$

Hence, the control limits for truncated Poisson EWMA control chart are,

$$\text{UCL} = \frac{\mu_0[1-e^{-\mu_0}]}{[1-e^{-\mu_0}(1+\mu_0)]} + \text{k}\sqrt{\left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{\mu_0}{[1-e^{-\mu_0}(1+\mu_0)]}\left[\mu_0 + (1-e^{-\mu_0}) - \frac{\mu_0[1-e^{-\mu_0}]^2}{[1-e^{-\mu_0}(1+\mu_0)]}\right]\right]}$$

$$\text{CL} = \frac{\mu_0[1-e^{-\mu_0}]}{[1-e^{-\mu_0}(1+\mu_0)]}$$

$$\text{LCL} = \frac{\mu_0[1-e^{-\mu_0}]}{[1-e^{-\mu_0}(1+\mu_0)]} - \text{k}\sqrt{\left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{\mu_0}{[1-e^{-\mu_0}(1+\mu_0)]}\left[\mu_0 + (1-e^{-\mu_0}) - \frac{\mu_0[1-e^{-\mu_0}]^2}{[1-e^{-\mu_0}(1+\mu_0)]}\right]\right]}$$

3.3 Zero – Two Truncated Poisson EWMA Control Chart
Let X_1, X_2, X_3 ………be the sequence of independent and identical random variables following the zero - two truncated Poisson distribution. Then, consider the statistic from (1),

$$Z_t = \lambda X_t + (1-\lambda) Z_{t-1}$$

Where the λ be the constant which ranges from $0 < \lambda \le 1$. Then $Z_0 = \mu_0$ when the starting value is the process target (i.e., i=1) and k be the sigma limit which is usual in control limits.
Then the mean and variance of the zero - two truncated Poisson EWMA control chart is,

$$\text{E}(Z_t) = \frac{\mu_0 e^{-\mu_0}[e^{\mu_0}-1-\mu_0]}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]} \tag{8}$$

and

$$\text{Var}(Z_t) = \left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{\mu_0 e^{-\mu_0}}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}\left[[\mu_0 e^{\mu_0} + e^{\mu_0} - 1 - 2\mu_0] - \frac{\mu_0 e^{-\mu_0}[e^{\mu_0}-1-2\mu_0]^2}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}\right]\right] \tag{9}$$

For large values of t the variance is approximately reduces to,

$$\mathrm{Var}(Z_t) \approx \left[\frac{\lambda}{(2-\lambda)}\right]\left[\frac{\mu_0 e^{-\mu_0}}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}\left[[\mu_0 e^{\mu_0}+e^{\mu_0}-1-2\mu_0]-\frac{\mu_0 e^{-\mu_0}[e^{\mu_0}-1-2\mu_0]^2}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}\right]\right] \quad (10)$$

Hence, the control limits for truncated Poisson EWMA control chart are,

$$\mathrm{UCL} = \frac{\mu_0 e^{-\mu_0}[e^{\mu_0}-1-\mu_0]}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}+k\sqrt{\left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{\mu_0 e^{-\mu_0}}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}\left[[\mu_0 e^{\mu_0}+e^{\mu_0}-1-2\mu_0]-\frac{\mu_0 e^{-\mu_0}[e^{\mu_0}-1-2\mu_0]^2}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}\right]\right]}$$

$$\mathrm{CL} = \frac{\mu_0 e^{-\mu_0}[e^{\mu_0}-1-\mu_0]}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}$$

$$\mathrm{LCL} = \frac{\mu_0 e^{-\mu_0}[e^{\mu_0}-1-\mu_0]}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}-k\sqrt{\left[\frac{\lambda[1-(1-\lambda)^{2i}]}{(2-\lambda)}\right]\left[\frac{\mu_0 e^{-\mu_0}}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}\left[[\mu_0 e^{\mu_0}+e^{\mu_0}-1-2\mu_0]-\frac{\mu_0 e^{-\mu_0}[e^{\mu_0}-1-2\mu_0]^2}{[1-e^{-\mu_0}\left(1+\mu_0+\frac{\mu_0{}^2}{2}\right)]}\right]\right]}$$

4. Comparison with control charts

The control limits and control charts for 0TPEWMA, 0-1TPEWMA, 0-2TPEWMA are shown in the following tables and figures. Let us consider the example data from the below given Table 1.a

Table 1a: Observation data*

I	X_i	I	X_i
1	9.45	16	9.37
2	7.99	17	10.62
3	9.29	18	10.31
4	11.66	19	8.52
5	12.16	20	10.84
6	10.18	21	10.90
7	8.04	22	9.33
8	11.46	23	12.29
9	9.2	24	11.50
10	10.34	25	10.60
11	9.03	26	11.08
12	11.47	27	10.38
13	10.51	28	11.62
14	9.4	29	11.31
15	10.08	30	10.52

*__Source:__ Montgomery, D. C. (2009), "Introduction to Statistical Quality Control", Sixth Edition, Wiley India, New Delhi

For the various value of k and λ and the target value μ_0 = 10 the control limits and control charts for 0 TPEWMA, 0-1 TPEWMA, and 0-2 TPEWMA are calculated and shown in tables 1.1 to 3.1

Table 2: Control limits 0 TPEWMA, 0-1 TPEWMA & 0-2 TPEWA

When k=3 & λ=0.5						
Z_i	0 TPEWMA		0-1 TPEWMA		0-2 TPEWMA	
	UCL	LCL	UCL	LCL	UCL	LCL
9.945	14.743	5.258	14.737	5.272	14.700	5.345
9.750	15.303	4.698	15.296	4.713	15.253	4.793
9.704	15.434	4.567	15.426	4.583	15.382	4.664
9.899	15.466	4.535	15.459	4.550	15.413	4.632
10.125	15.474	4.527	15.467	4.542	15.421	4.624
10.131	15.476	4.525	15.469	4.540	15.423	4.622
9.922	15.476	4.525	15.469	4.540	15.424	4.622
10.076	15.477	4.524	15.469	4.540	15.424	4.622
9.988	15.477	4.524	15.469	4.540	15.424	4.622
10.023	15.477	4.524	15.469	4.540	15.424	4.622

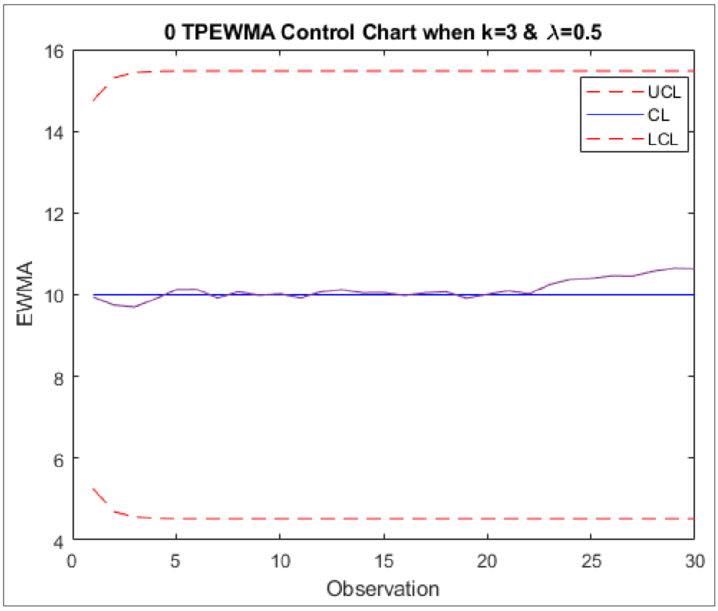

Fig 1: Control chart for 0 TPEWMA

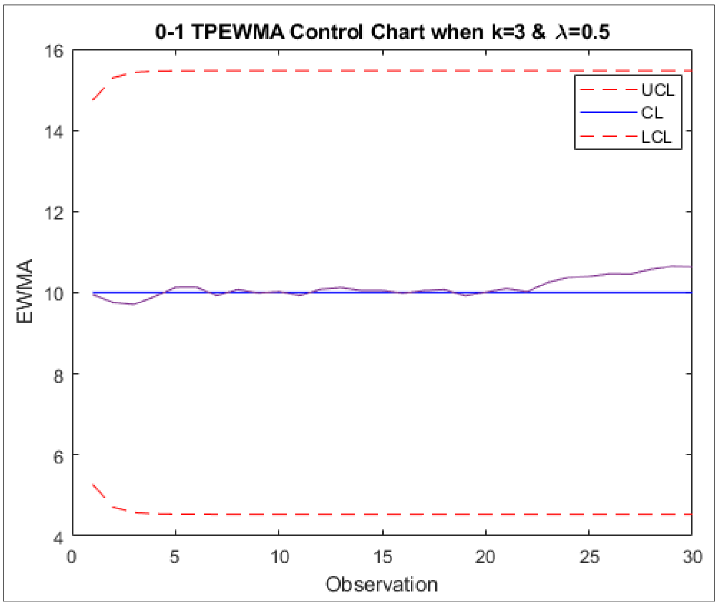

Fig 2: Control chart for 0-1 TPEWMA

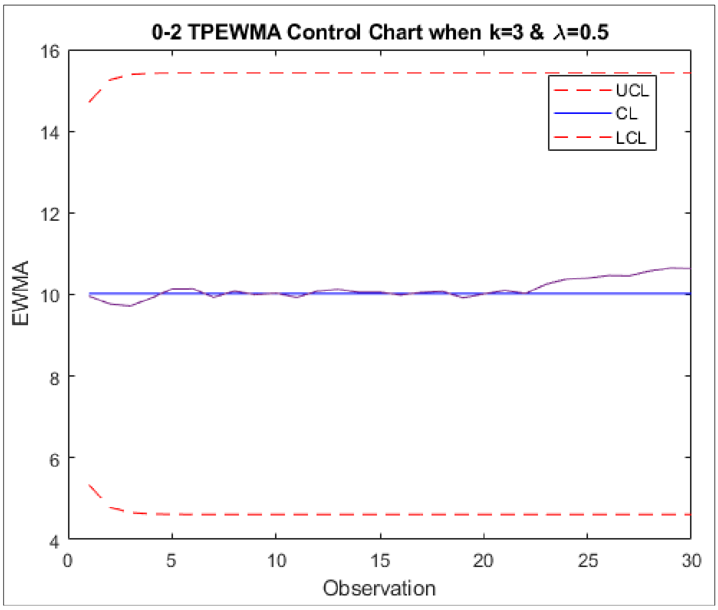

Fig 3: Control chart for 0-2 TPEWMA

Table 3: Control limits 0 TPEWMA, 0-1 TPEWMA & 0-2 TPEWMA

When k=1 & λ=0.25						
Z_i	0 TPEWMA		0-1 TPEWMA		0-2 TPEWMA	
	UCL	LCL	UCL	LCL	UCL	LCL
9.945	10.791	9.210	10.793	9.216	10.802	9.243
9.750	10.988	9.012	10.991	9.019	10.997	9.048
9.704	11.084	8.917	11.086	8.923	11.091	8.954
9.899	11.134	8.867	11.136	8.873	11.141	8.905
10.125	11.161	8.840	11.163	8.846	11.168	8.878
10.131	11.176	8.825	11.178	8.831	11.183	8.863
9.922	11.185	8.816	11.186	8.823	11.191	8.855
10.076	11.189	8.811	11.191	8.818	11.195	8.850
9.988	11.192	8.809	11.194	8.815	11.198	8.847
10.023	11.194	8.807	11.195	8.814	11.200	8.846

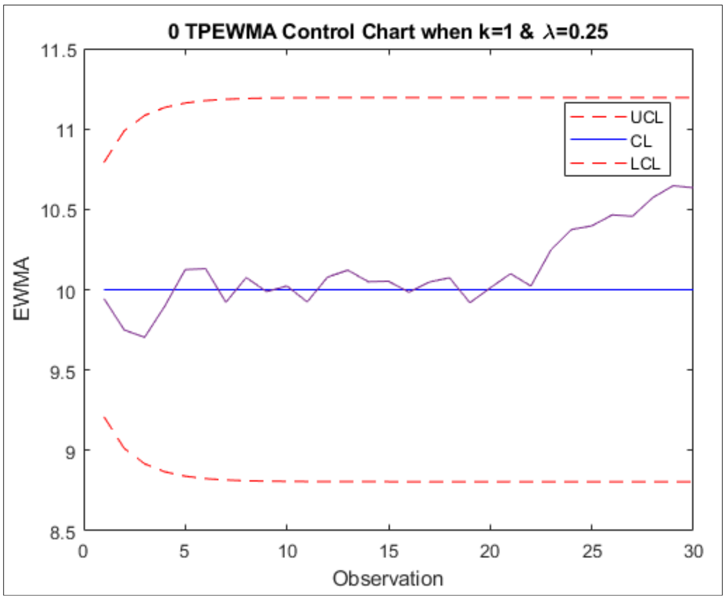

Fig 4: Control chart for 0 TPEWMA

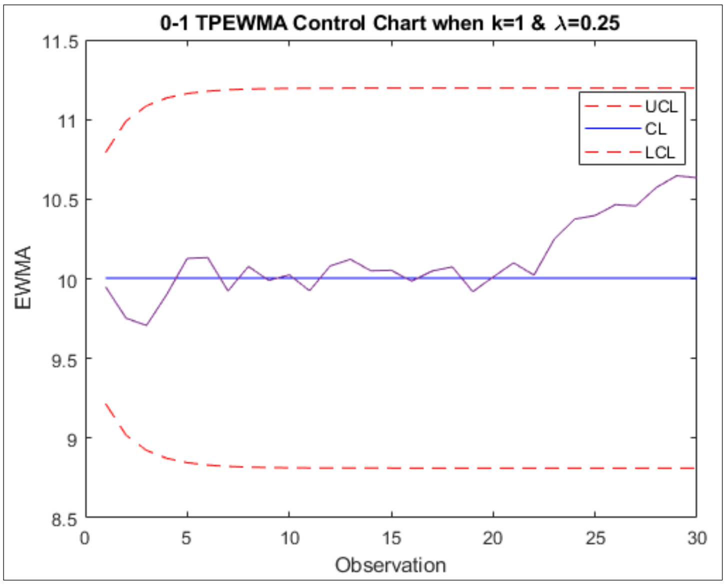

Fig 5: Control chart for 0-1 TPEWMA

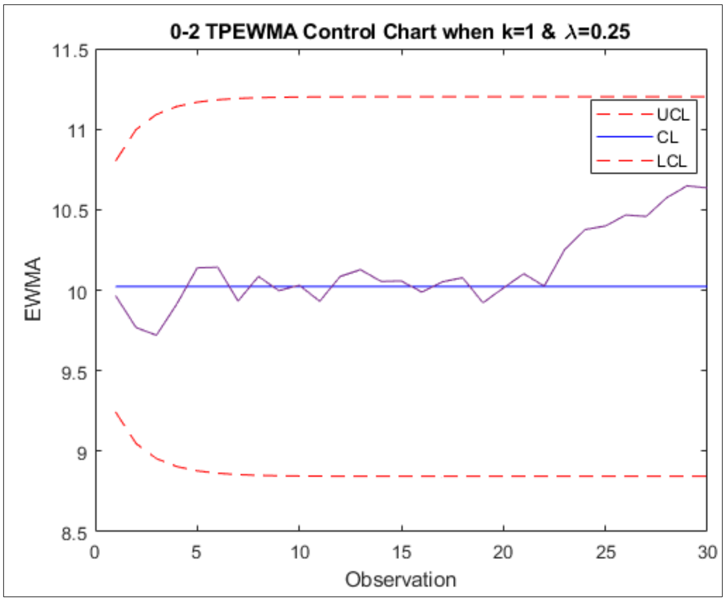

Fig 6: Control chart for 0-2 TPEWMA

Table 4: Control limits 0 TPEWMA, 0-1 TPEWMA & 0-2 TPEWMA

When k=0.5 & λ=0.1						
Z_i	0 TPEWMA		0-1 TPEWMA		0-2 TPEWMA	
	UCL	LCL	UCL	LCL	UCL	LCL
9.945	10.159	9.842	10.162	9.847	10.179	9.867
9.750	10.213	9.788	10.217	9.792	10.233	9.813
9.704	10.249	9.752	10.252	9.757	10.268	9.778
9.899	10.274	9.727	10.278	9.731	10.293	9.753
10.125	10.293	9.708	10.297	9.712	10.311	9.734
10.131	10.308	9.693	10.311	9.698	10.326	9.720
9.922	10.319	9.682	10.322	9.687	10.337	9.709
10.076	10.328	9.673	10.331	9.678	10.346	9.700
9.988	10.335	9.666	10.338	9.671	10.353	9.693
10.023	10.340	9.661	10.344	9.665	10.358	9.688

*Extension work of table 1.1 to 3.1 will be show in Ph.D. thesis

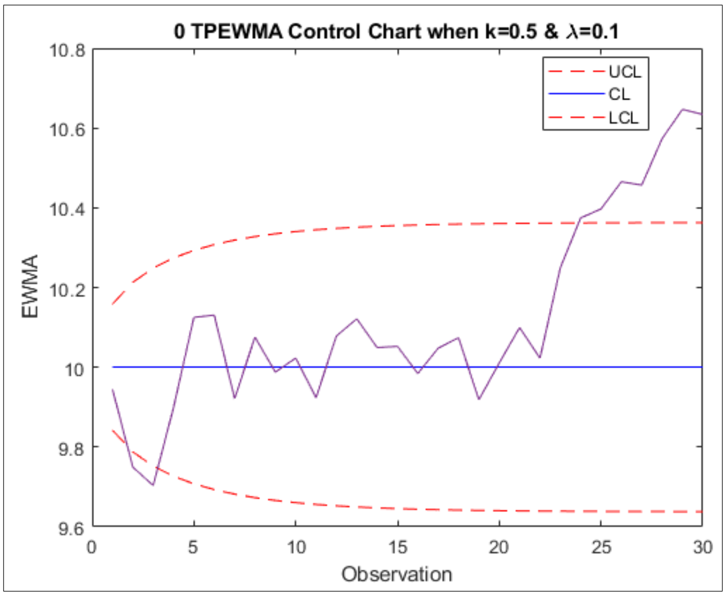

Fig 7: Control chart for 0 TPEWMA

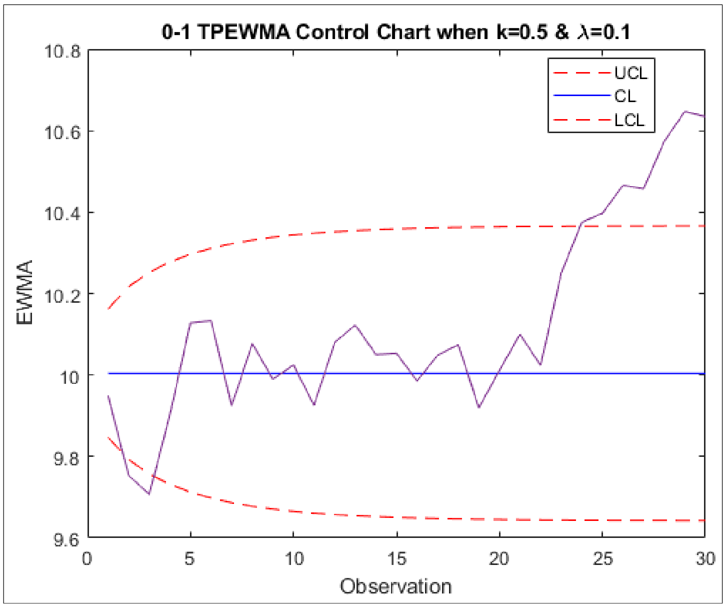

Fig 8: Control chart for 0-1 TPEWMA

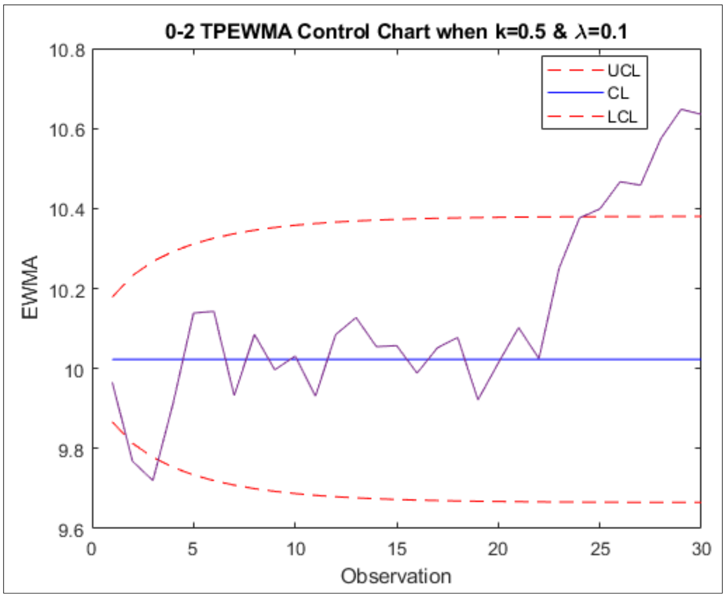

Fig 9: Control chart for 0-2 TPEWMA

5. Calculation of ARL

A study is conducted to examine the performance of the 0 TPEWMA, 0-1 TPEWMA and 0-2 TPEWMA control charts using the ARL. Comparison of ARL of three control charts with PEWMA is shown in the following table with various values of k and λ and the target value $\mu_0 = 10$.

Table 5: Comparison of PEWMA, 0 TPEWMA, 0-1 TPEWMA and 0-2 TPEWMA

λ = 0.95					
k	**UCL**	**PEWMA**	**0 TPEWMA**	**0-1 TPEWMA**	**0-2 TPEWMA**
3.5	21	1429.29	1428.40	1420.44	1384.86
3	19	289.49	289.34	287.94	281.70
2.5	18	139.15	139.08	138.46	68.45
2	16	36.98	36.97	36.83	36.21
1.5	15	20.52	20.51	20.44	11.78
1	13	7.38	7.38	7.36	7.27
0.5	12	4.80	4.80	4.79	4.74

Table 6: Comparison of PEWMA, 0 TPEWMA, 0-1 TPEWMA and 0-2 TPEWMA

$\lambda = 0.5$					
k	**UCL**	**PEWMA**	**0 TPEWMA**	**0-1 TPEWMA**	**0-2 TPEWMA**
3.5	16	36.98	36.97	36.83	36.21
3	15	20.52	20.51	20.44	20.13
2.5	15	20.52	20.51	20.44	20.13
2	14	11.98	11.98	11.94	11.78
1.5	13	7.38	7.38	7.36	7.27
1	12	4.80	4.80	4.79	4.74
0.5	11	3.30	3.30	3.29	3.26

Table 7: Comparison of PEWMA, 0 TPEWMA, 0-1 TPEWMA and 0-2 TPEWMA

$\lambda = 0.1$					
K	**UCL**	**PEWMA**	**0 TPEWMA**	**0-1 TPEWMA**	**0-2 TPEWMA**
3.5	13	7.38	7.38	7.36	7.27
3	12	4.80	4.80	4.79	4.74
2.5	12	4.80	4.80	4.79	4.74
2	11	3.30	3.30	3.29	3.26
1.5	11	3.30	3.30	3.29	3.26
1	11	3.30	3.30	3.29	3.26
0.5	10	2.40	2.40	2.39	2.38

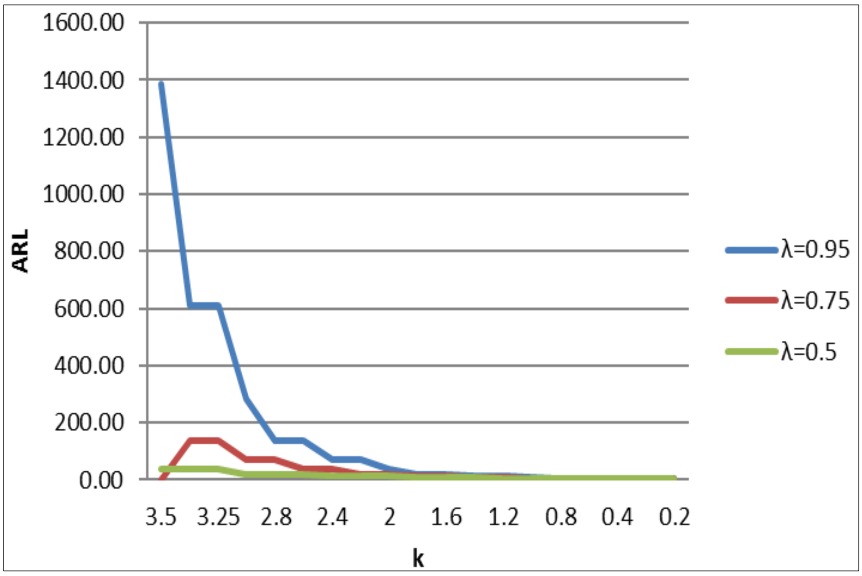

Fig 4: ARL curve for 0-2 TPEWMA

6. Conclusion

In this article, a new control chart is developed for unstabilized process. The control charts namely, 0 TPEWMA, 0-1 TPEWMA and 0-2 TPEWMA are designed for various variable factors. The control limits for various values of λ and k are obtained. It is found that when λ and k values decreases then out of control alarm is raised. The Average Run Length (ARL) for the newly developed charts is derived. In the case of large values of λ and k, the ARL becomes smaller for 0 TPEWMA, 0-1 TPEWMA, 0-2 TPEWMA than PEWMA. In the case of small values λ and k, the ARL for three developed charts and PEWMA chart are the same. The ARL curve becomes steeper when λ and k value gets increased and the ARL curve becomes straight line when λ and k values are decreased. Hence one can conclude that TPEWMA can be implemented in industries to detect small shifts.

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