International Journal of Statistics and Applied Mathematics 2023; 8(3): 36-43

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2023; 8(3): 36-43

https://www.mathsjournal.com
Received: 08-03-2023
Accepted: 15-04-2023

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A point base appraisal of fuzzy edge detection techniques in computer vision

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DOI: https://doi.org/10.22271/maths.2023.v8.i3a.976

Abstract
Edge detection techniques play a vital role in performing computer vision tasks, especially in image processing. Nowadays the accuracy and power of the computer, it has received much attention of the research communities. Many researchers established the various fuzzy edge detection techniques, but still it is a challenging task of the researcher to detect the true edges completely from an image/blurred image. This paper discussed the recently entrenched edge detectors with their characteristics. Also, the accuracy of these edge detectors has been carried out in the context of the detection in a given image and the speed with the help of MATLAB software.

Keywords: Edge detection, Sobel, Harris, FAST & image processing

1. Introduction

An image is an array, or a matrix, of square pixels (picture elements) arranged in columns and rows [1]. The feature means characteristics of an object that is feature is a significant piece of information extracted from an image which provides a more detailed understanding of the image. Feature extraction has referred that dimensionality reduction of that object. It plays an important role in image processing. In image processing features can be classified into three types that are low, middle and high. Low level features are colour, texture and middle level feature is the shape and a high-level feature is a semantic gap of objects [2].

An edge is a sharp change in intensity of an image [3]. Edge detection process detects outlines of an object and boundaries between objects and the background in the image. An edge-detection filter can also be used to improve the appearance of blurred or anti-aliased video streams. The basic edge-detection operator is a matrix area gradient operation that determines the level of variance between different pixels. The edge-detection operator is calculated by forming a matrix centered on a pixel chosen as the center of the matrix area.

The fuzzy relative pixel value algorithm has been developed with the knowledge of vision analysis with low or no illumination, thus making this method optimized for applications requiring such methods [4]. The method helps us to detect edges in an image in all cases due to the subjection of pixel values to an algorithm involving a host of fuzzy conditions for edges associated with an image. The purpose of this paper is to present a new methodology for image edge detection which is undoubtedly one of the most important operations related to low level computer vision, in particular within the area of feature extraction with the plethora of techniques.

2. Types of fuzzy matrices

Definition 2.1: Fuzzy Matrix

A fuzzy matrix could be a matrix that has its components from [0, 1]. Let A is the fuzzy matrix and it general form can be written as,

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdot & \cdot & \cdot & A_{1n} \\ A_{21} & A_{22} & \cdot & \cdot & \cdot & A_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{m1} & A_{m2} & \cdot & \cdot & \cdot & A_{mn} \end{bmatrix}$$

Definition 2.2: Fuzzy Super row matrix
Let fuzzy super matrix is

$A_s = [A_1, A_2, \ldots, A_n]$

Where each A_i (i=1,2,…,t) is a fuzzy row vector (or) fuzzy super row matrix.

Definition 2.3: Fuzzy Super column matrix
Let $A_s = [A_1/A_2/ \ldots/A_m]$ (M>1) where each $A_j$is a fuzzy row vercor, j=1,2,…,N. we call $A_S$as the fuzzy super row matrix.

$$A_S = \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ A_N \end{bmatrix}$$

Definition 2.4: Super fuzzy matrix
Let us consider a fuzzy matrix,

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

Where A_{ij} (i=j=1,2,3) be fuzzy sub-matrices and A_{11} and A_{21} are equal. Also the columns in fuzzy sub-matrices of the $A_{12}= A_{22}$ & $A_{13} = A_{23}$ are equal. This is obvious from the instant index of the fuzzy sub-matrices. The remaining rows in fuzzy sub-matrices A_{11}, A_{12}, A_{13}, A_{21}, A_{22} and A_{23} are equal.

Definition 2.5: Super fuzzy matrix

$$\text{Let, } A = \begin{bmatrix} A_{11} & A_{12} \ldots & A_{1n} \\ A_{21} & A_{22 \ldots} & A_{2n} \\ & & \vdots \\ A_{m1} & A_{m2 \ldots} & A_{mn} \end{bmatrix}$$

Where Aij's are fuzzy sub-matrices; i = 1, 2, …, m and j =1, 2, …, n.

Definition 2.6: Modified regular Inverse of Matrix
Modified regular Inverse of Matrix A modified regular inverse of a non-singular matrix is giving the specific answer of a positive set of equations. This modified regular inverse exists for any (possibly square) matrix by any means with complex elements. It is used right here, for solving linear matrix equations, and among other programs for locating an expression for the main idempotent elements of a matrix.

Definition 2.7: Square Fuzzy Supermatrix
Let A is fuzzy matrix and then A is called a super fuzzy square matrix, if A can be partitioned randomly among the columns satisfies the below relationship, i_1 and i_{1+1}, i_2 and i_2 and i_{2+1}, …,i_r and $i_{r+1.}$
Similarly among the rows i_1 and i_{1+1}, i_2 and i_2 and i_{2+1}, …,i_r and i_{r+1} (r+1< n).

Definition 2.8: Square Fuzzy Symmetric Super matrix
Let A be a fuzzy super matrix or fuzzy square super matrix. Then A is called as a symmetric fuzzy super square matrix if is such that A_{ii} are square fuzzy matrices and each of these fuzzy square matrices are symmetric square matrices and diagonal sub-matrices A_{ii} are symmetric square fuzzy matrices.
i.e., $A^u = A_{ii}$
The non-diagonal fuzzy submatrices are symmetric about the diagonal.
i.e., $A^u = A_{ji}$ for $1 \leq i, j \leq n$.

Definition 2.9: Fully Fuzzy Matrix
A Matrix system such as,

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \hat{*} \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nn} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \dots & d_{nn} \end{pmatrix}$$

Where a_{ik}; $1 \leq i; k \leq n$, are arbitrary triangular fuzzy numbers, the elements d_{ij} in the right-hand matrix are fuzzy numbers and the unknown elements x_{kj} are non-negative ones, is called a FFMEs.

Definition 2.10: General fuzzy super matrix
Let us consider the matrix,

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

Where A_{11}, A_{12}, A_{13}, A_{21}, A_{22} and A_{23} be fuzzy submatrices where number of columns in the fuzzy submatrices A_{11} and A_{21}areequal. Similarly, the columns in fuzzy submatrices of A_{12} and A_{22} are equal and columns of fuzzy matrices A_{13} and A_{23} are equal. This is evident from the second index of the fuzzy submatrices. One can also see, the number of row in fuzzy submatrices A_{11}, A_{12} and A_{13} are equal.
The fuzzy submatrices of A_{ij}, (i=2, j=1, 2, 3) are equal. Thus, a general super fuzzy matrix can written as,

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}$$

where A_{ij}'s are fuzzy submatrices; i = 1, 2, …, m and j =1, 2, …, n.

Definition 2.11: Inverse of k regular super fuzzy matrix
Let A be matrix is belongs to super fuzzy matrix then is said to be a right inverse of k regular and it can be written as,
$A^kXA=A^k$
It's some positive integer k. So X is called the inverse of K regular super fuzzy matrix A.
$A_\mu=\{1^k\}=\{X/A^kXA=A^k\}$.

Definition 2.12: K-G-Inverse of Fuzzy matrix
Let A matrix A϶$(IF)_n$, is said be right (or) left k-regular if there exists a matrix X ϵ $(IF)_n$ such that $A^kXA = A^k$, for some positive integer k. X is called a right k-g-inverse of A.

Definition 2.13: G-inverse of fuzzy matrix
The a k-g inverse of A when each element of the set $A\{1^k\}$. Let A is k-regular & g-regular for all integers $q \geq k$ (k=1).

Definition 2.14: K-Regular Fuzzy Matrix
Let A be a Inverse of Fuzzy Matrix (IFM) and its right (or left) k-regular IFM is k regular in fuzzy.

Definition 2.15: Fuzzy symmetric Equations (FSE)
The fuzzy symmetric matrix equation of the form

$$A\tilde{X} = \tilde{B}$$

A is a m×m non-singular matrix and $\tilde{B}$is an m×n non-zero spreads. The fuzzy matrix is renewed to a fuzzy system of linear equations based on the Kronecker Methods.

Definition 2.16: S-Inverse of Super Fuzzy matrix
The given matrix is the S-inverse of super fuzzy matrix.

$$A^s = \begin{vmatrix} 1-a_{11} & 1-a_{12} & \cdot & \cdot & \cdot & 1-a_{1n} \\ 1 \quad a_{21} & 1 \quad a_{22} & \cdot & \cdot & \cdot & 1 \quad a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1-a_{m1} & 1-a_{m2} & \cdot & \cdot & \cdot & 1-a_{mn} \end{vmatrix}$$

Main Properties (i). (As)S=A & (ii). A+AS=U, where U is the unit matrix.

3. Edge detection

Edges can be modelled according to their intensity profiles and can be of following types:

1. **Step edge:** The image intensity abruptly changes from one value to one side of the discontinuity to a different value on the opposite side.
2. **Ramp edge:** A step edge where the intensity change is not instantaneous, but occurs over a finite distance.
3. **Ridge edge:** The image intensity abruptly changes value, but then returns to the starting value within some short distance.
4. **Roof edge:** A ridge edge where the intensity change is not instantaneous, but occurs over a finite distance, usually generated by the intersection of surfaces.

3.1 Harris operator

Harris proposed a method to solve the problems of noisy response due to a binary window function, namely Harris detector by applying the Gaussian noisy filter. Harris operator detector [5, 6] is based on the local auto-correlation function of a signal which measures the local changes of the signal with patches shifted by a small amount in different directions. Given a shift (x, y) and a point the auto-correlation function is defined as,

$$C(x,y) = \Sigma w[\, I(xi,yi) - I(xi+\Delta x,\, yi+ \Delta y)]2$$

Where the image function I(.) is approximated by a Taylor expansion truncated to the first order terms. C(x, y), the auto-correlation matrix which captures the intensity structure of the local neighborhood. Finally, find the operator points as local maxima of the operator response by characterizing operator by Eigenvalues of C(x, y).

3.2 Wang-Brady operator

In general, the operator is based on the operators, measurement of total curvature, i.e., the second order tangential derivative. Conventionally directional derivatives are obtained from linear combinations of first and second derivatives with respect to the x and y components. These methods don't provide the reliable result and also more computationally expensive. Wang and Brady [7] proposed method to solve this problem and get the improved accuracy of operator localization by adopting the linear interpolation scheme. The empirical parameters, constant measure of image surface curvature S and threshold T are determined depending on the context of the image.

3.3 SUSAN operator

Smith and Brady [9] entrenched a new approach to low level image processing, in particular, edge and operator detection and structure preserving noise reduction. These resulting methods are accurate noise resistant, namely Smallest Univalues Segment Assimilating Nucleus (SUSAN). The SUSAN principle is formulated in the following equation, where $n(x_0)$ is the USAN size at x0, on the simplification,

$$\frac{dI}{dx}(x_0 + a(x_0)) - \frac{dI}{dx}(x_0 + b(x_0)) = 0$$

The SUSAN detectors are based on the minimizing of the local image region, and the noise reduction method uses the region in the smoothing the neighborhood.

3.4 Trajkovic-Hedley operator

Trajkovic and Hedley [10] developed an operator based on the property of operators the change of image intensity should be high in all directions. The comer response function (CRF) is computed as a minimum change of intensity over all possible directions. A multigrid approach is employed to reduce the computational complexity and to improve the quality of the detected operators. To overcome the problem of lines at certain orientations being detected as comers an interpixel approximation was used. The operator detection operator is as follows;

$$r1(\alpha)=A\cos 2\,\alpha + B1\sin 2\alpha + C \,\&\, r2(\alpha)=A\cos 2\,\alpha + B2\sin 2\alpha + C, \text{ Where } A= (r_A\text{-}r_B/2) \,\&\, A= (r_{A+}r_B/2)$$

3.5 FAST operator

The several feature detectors, many of them are really good. But when looking for a real-time application point of view, they are not fast enough. Like, SLAM (Simultaneous Localization and Mapping) mobile robot which have limited computational resources. As a solution to this, FAST (Features from Accelerated Segment Test) algorithm was proposed by Edward Rosten and Tom Drummond [11-13] which is summarized as follows. In an image, identify an interest point, then considering a circle of sixteen pixels around the pixel. Find the number of contiguous pixels to the interest point with respect to threshold value T. This version does not perform well when the number of contiguous pixels less than 12. To improve the speed and efficiency the author has introduced the machine learning approach. Detection of multiple interest points adjacent to one another is can be dealt with by applying non maximal suppression after detecting the interest points.

3.6 Robert operator

The calculation of the gradient magnitude of an image is obtained by the partial derivatives Gx and Gy at every pixel location. The simplest way to implement the first order partial derivative is by using the Roberts cross gradient operator [14].

Gx= f (i, j) - f (i+1, j+1) Gy= f (i+1, j) – f (i,j+1) (1)

The above partial derivatives can be implemented by approximating them to two 2x2 masks.

3.7 Prewitt operator

The Prewitt edge detector [15] is a much better operator than Roberts's operator. This operator having a 3 x 3 masks deals better with the effect of noise. An approach using the masks of size 3 x 3 is given below, the arrangement of pixels about the pixels [i,j].

3.8 LOG Operator

The Laplacian of Guassion [16] is the first and also most common blob detector. It is the two partial derivative approximations for the Laplacian for a 3 x 3 region are given as,

Gx = 4(a8) - (a1+ a3+ a5+ a7) & Gy = 8(a8) - (a0+a1+ a2+ a3+ a4+ a5+ a6+ a7)

3.9 Zero-crossing operator

Zero crossing operator [16] was established by Haralick in 1984. The Edge detection is done by convolving an image with the Laplacian at a given scale and marks the points where the result have zero value, which is called the zero-crossings. These points should be checked to ensure that the gradient magnitude is large.

$$f(x) = \frac{Ir-Il}{2}\left(erf\left(\frac{x}{\sqrt{2}\sigma}\right)+1\right)+Il$$

Where *Ir* right side of the edge and *Il* left side of the edge.

3.10 Canny operator

Canny technique [17] is a very important method to find edges by isolating noise from the image before finding the edges of the image, without affecting the features of the edges in the image and then applying the tendency to find the edges and the critical value for threshold. The algorithmic steps for canny edge detection technique are followed:

1. Convolve image f (r, c) with a Gaussian function to get smooth image.

f(r,c).f(r,c)=f(r,c)*G(r,c,6)

2. Apply first difference gradient operator to compute edge strength, then the edge magnitude and direction are obtained as before.
3. Apply non-maximal or critical suppression of the gradient magnitude.
4. Apply threshold of the non-maximal suppression image.

3.11 Sobel Operator

The Sobel edge detector [18] is very much similar to the Prewitt edge detector. The difference between the both is that the weight of the central coefficient is 2 in the Sobel operator. The partial derivatives of the Sobel operator are calculated as

$G_x = (a_6 + 2_a 5 + a_4) - (a_0 + 2_a 1 + a_2)$ & $G_y = (a_2 + 2_a 3 + a_4) - (a_0 + 2_a 7 + a_6)$

4. Experimental result

All algorithm was implemented in MATLAB software. The performance of the various method of edge detection is tested with images. The various edge detector is used for the extraction of edge points and gaps for 1 pixel wide are filled for the detection of edge features. The processing time for detection of edges in images is summarized in Table 1. The edge detected in images are shown in figures 1.

Table 1: Time taken for various edge detection operators

Image	Processing Time (in Seconds)									
	b	c	d	e	f	g	h	i	j	k	l
Building	0.412	0.424	0.415	0.393	0.376	0.642	0.301	0.299	0.215	0.314	0.371
Cell image	0.735	0.576	0.510	0.399	0.381	0.511	0.312	0.316	0.211	0.332	0.296

(b) Wang Brady (c) Trajkovic Hedley (d) Robert (e) Prewitt (f) Harris (g) Susan
(h) LOG (i) Zero Crossing (j) Canny (k) Sobel (l) FAST

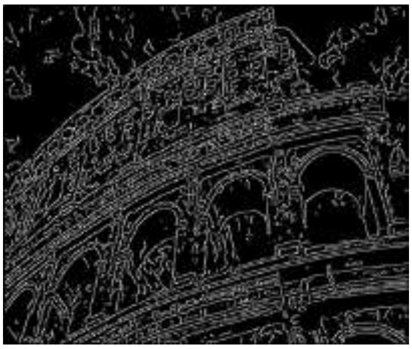
It is observed from the table 1, the performance of the various edge detection methods. It is noted that edge detection timings are considerably lower in FAST procedure, the rest of them most probability same.

The above data Simulated in TLS MATLAB toolbox. It is randomly generated data based on the sample size. It is also to fix the threshold values are 2 in the toolbox in the image.

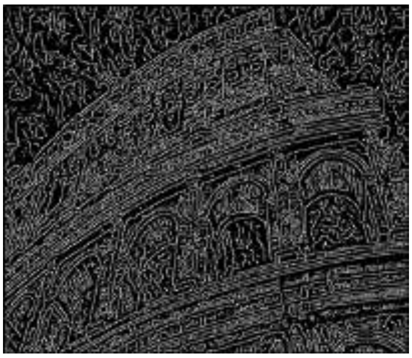

(a)

(b)

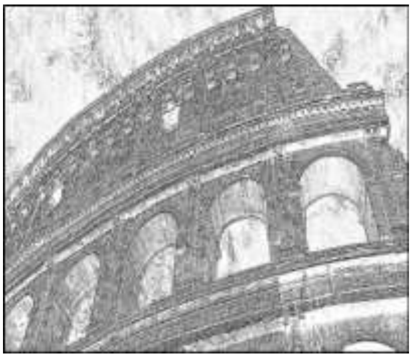
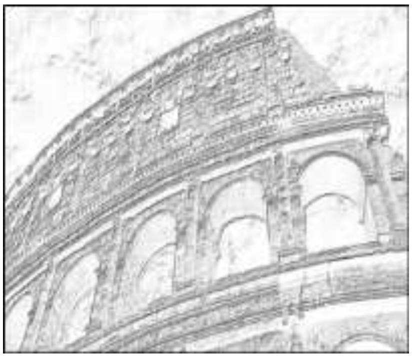
(c)


(d)

(e)

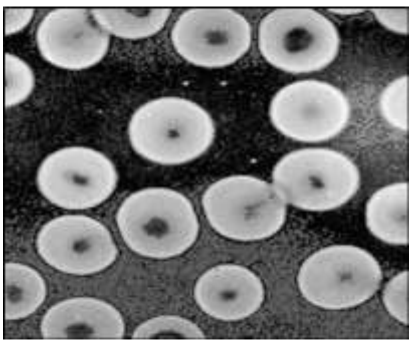
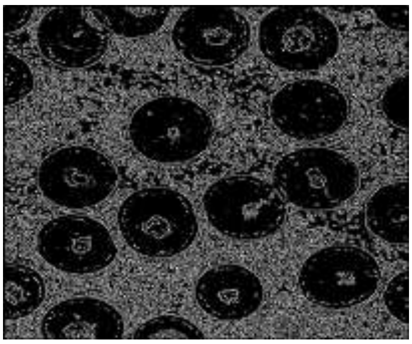
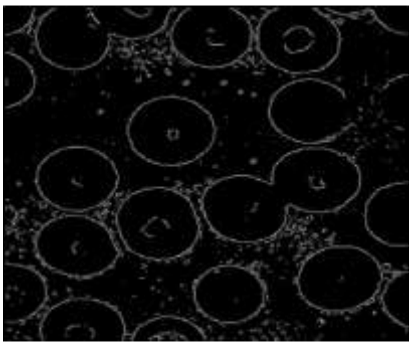
(f)


(g)

(h)

(i)


(j)

(k)

(l)


(a)

(b)

(c)

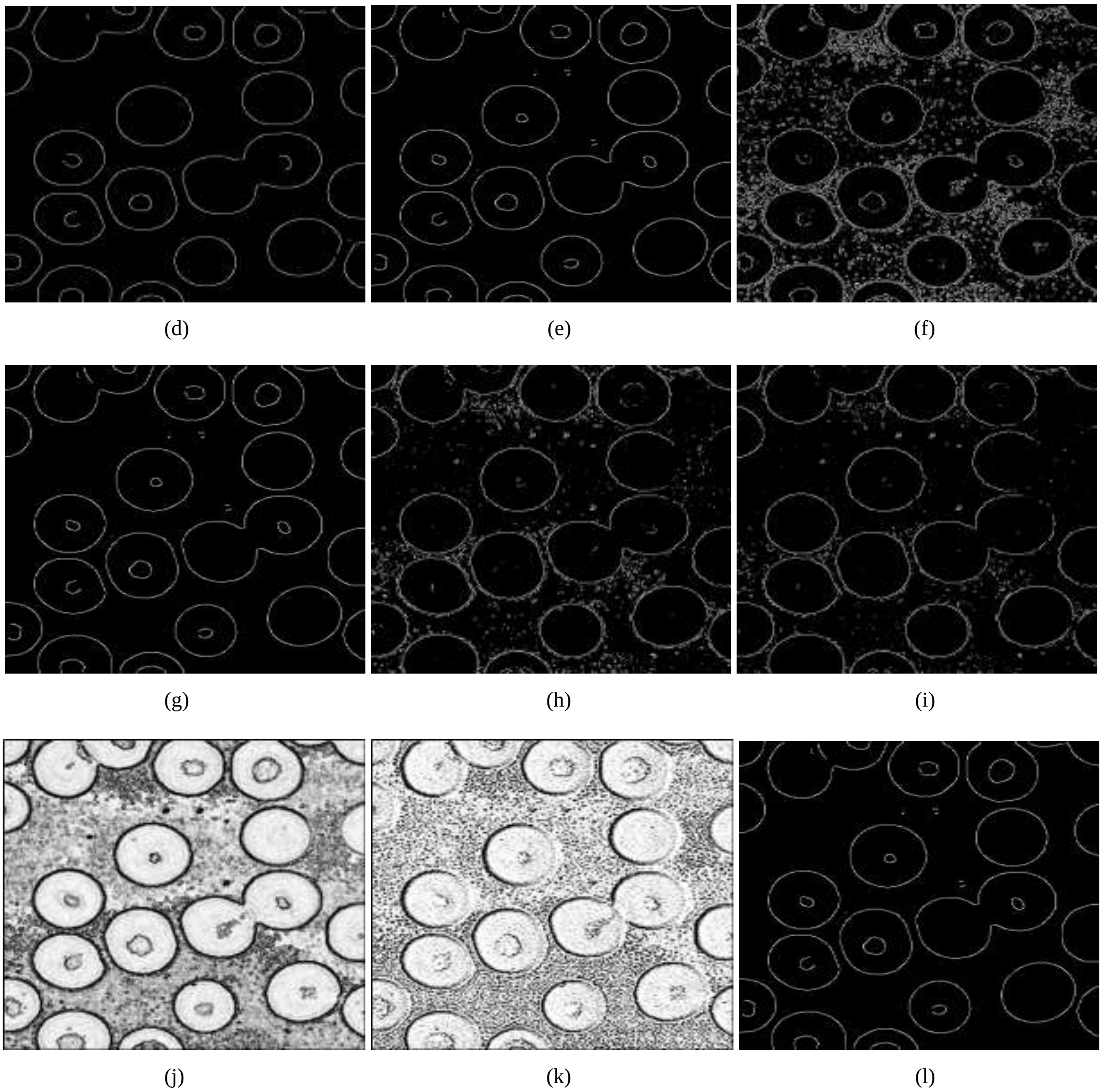


Fig 1: Edge Extracted for (a) original Image, (b) Wang Brady, (c) Trajkovic Hedley, (d) Robert, (e) Prewitt, (f) Harris, (g) Susan, (h) LOG, (i) Zero Crossing, (j) Canny, (k) Sobel, (l) FAST

4. Conclusion

The field of computer vision is undergoing tremendous development in recent years. Computer vision concerns with developing systems that can interpret the content of natural scenes. In this paper, a comparative study of various edge detection techniques for image processing. The relative performance of various edge detection techniques is carried out with an image by using MATLAB software. It is observed from the results Harris, LoG and Canny edge detectors produce almost the same edge map. The FAST result is superior when compared to all for a selected image since different edge detections work better under different conditions. Even though, so many edge detection techniques are available in the literature, since it is a challenging task for the research communities to detect the exact image without noise from the original image.

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