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Several features of a complete graphs and regularity graphs in the rings

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Abstract
Suppose that $\mathrm{Reg}(\psi(\mathrm{R}))$ and $\Omega(\psi(R))$ have been the sub-graphs of complete graph $\mathrm{C}(\psi(R))$ generated as the sets of all regularity components and nil in R. We find out when every one of the graphs $\mathrm{C}(\psi(R))$, $\mathrm{Reg}(\psi(\mathrm{R}))$ and $\Omega(\psi(R))$ is linked and homogenous. Whenever $\mathrm{Reg}(\psi(\mathrm{R}))$ and $\Omega(\psi(R))$ are both Eulerian and regularity.

Keywords: Graphs of eulerian, regularity graphs, complete graphs

1. Introduction

The concept of graph theory was introduced via (Harary, F. 1972) [11]. Also, in 1981 Vince introduced the concept of locally homogeneous graphs in the groups (Vince, A. 1981). R shall be employed during this whole study to signify ring having unity $1 \neq 0$. Have $\Omega(R)$ denote the sets of all R is nil and R entire graphs are a simple graph having vertices subset R in which two different vertices ζ *and* $\eta$are contiguous if $\zeta + \eta \in \Omega(R)$. Anderson and Badawi presented these graphs, designated as $\mathrm{C}(\psi(R))$ in (Anderson, D. F. and Badawi, A. 2008) [1], where the researchers provided a detailed discussion of the situation whenever $\Omega(R)$ is indeed an ideal. They did, however, calculate several graphic symmetries including the radius and circumference of $\mathrm{C}(\psi(R))$. In 2009 (Akbari, S. *et al*. 2009) [5] demonstrated that only if R is a ring, a connecting complete graph is Hamilton. After that (Maimani, H. R. *et al*. 2012) [12] explored the species $\mathrm{C}(\psi(R))$. Pucanovi and Petrov, (Pucanovi, Z. and Petrovi, Z. 2011; Anderson, D. F., & Badawi, A. 2012) [15, 2] estimated the circumference of $\mathrm{C}(\psi(R))$. In 2012 (Shekarriza, M. H. *et al*. 2012) provides the properties of Eulerian $\mathrm{C}(\psi(R))$. The concept of $\mathrm{C}(\psi(R))$ predominance number is calculated separately in (Chelvam, T. T., & Asir, T. 2013; Chelvam, T. T., & Selvakumar, K. 2014) [8-9]. Ramin discusses the vertices connectivity of $\mathrm{C}(\psi(R))$wherein R is a ring (Ramin, A. 2013) [16] and (Asir, T., & Chelvam, T. T. 2013) [8] investigates the complements of $\mathrm{C}(\psi(R))$. for a finite ring R (Sander, T., & Nazzal, K. 2014) [17] [17] considers minimal nil k-flows for $\mathrm{C}(\psi(R))$. Akbari and Heydari (Akbari, S., & Heydari, F. 2013) [6] investigates several characteristics of a regular graphs $\mathrm{Reg}(\psi(\mathrm{R}))$. Erić and Pucanović (Erić, A. L., & Pucanović, Z. S. 2013) [10] investigates the graphic of $\mathrm{C}(\psi(R))$. Additionally, (Anderson, D. F., & Badawi, A. 2013) [3] defines R's generalized complete graphs. The readers can consult (Müller, H. *et al*. 2014; Nazzal, K. 2016; Singh, P. and Bhat, V. K. 2020; Anderson, D. F. *et al*. 2012; Tamizh Chelvam, T. 2022) [13, 14, 19, 2, 20] for just a review on the complete graphs of a ring.
The result that follows fully describes the graphs $\mathrm{C}(\psi(R))$. whenever $\mathrm{C}(R)$ is an ideal of R.

1.1. Remark (Anderson, D. F., & Badawi, A. 2008) [1]: Assume R is a ring, and $\Omega(R)$ is an ideal of R. Suppose that $|\Omega(R)| = \omega$, $|R/\Omega(R)| = \varphi$.

a) $\Omega(\psi(R))$is the complete graph K_{ω}.
b) If $2 \in \Omega(R)$, then $\mathrm{Reg}(\psi(R))$ is indeed the unions of $\varphi - 1 \cap K_{\acute{\omega}} s$.
c) $R/\Omega(R) \cong \mathbb{Z}_2$ or $R/\Omega(R) \cong \mathbb{Z}_3$ iff $\mathrm{Reg}(\psi(R))$ is linked.
d) If $2 \in \mathrm{Reg}(R)$, then $\mathrm{Reg}(\psi(R))$ is indeed the unions of $(\varphi - 1)/2 \cap K_{\omega\acute{\omega}} s$.

We discuss whether every of those graphs $C(\psi(R))$, $Reg(\psi(R))$, and $\Omega(\psi(R))$ is locally linked in this work. Likewise, we studied regularity of the graphs $Reg(\psi(R))$ and $\Omega(\psi(R))$. Finally, the Eulerian graphs $Reg(\psi(R))$ and $\Omega(\psi(R))$ are investigated.

2. Some Properties of $C(\psi(R))$, $Reg(\psi(R))$, and $\Omega(\psi(R))$

First, we will start with the following definition

2.1 Definition

Consider G to be a graph having vertices and edges collections $M(G)$ and $N(G)$, correspondingly. Take $m \in M(G)$ be the open neighbor of m, which is given as $\Lambda(m) = \{\varsigma \in M(G): \varsigma m \in \Lambda(G)\}$.If for all $m \in M(G)$ $\Lambda(m)$ the graph G is considered to be local linked. Therefore, when G is a union of whole graph, G is local linked; otherwise, if G has such vertices and edges part besides the $K_{1,1}$ G isn't really local linked.

2.2 Proposition

Suppose $\Omega(R)$ be an ideal of R.

1. $\Omega(\psi(R))$ is a locally linked graph.
2. R is an integral domain iff $Reg(\psi(R))$ and $C(\psi(R))$ are locally connected graphs.

2.3 Proposition

Suppose R be the sum of two rings R_1 and R_2. If neither R_1 nor R_2 is an integral domain, so $\Omega(\psi(R))$ is locally linked.

Proof. There is no pathway linking (1,0) and (0,1) in $\Lambda((0,0))$ when R is a combination of two integral domain. As a result, $\Omega(\psi(R))$ is not locally linked. Suppose that neither R_1 nor R_2 are integral domains. Because (0,0) in $\Lambda((\varsigma,\sigma))$ is a pathway combining (ζ,η) and (λ,ξ) in $\Lambda((\varsigma,\sigma))$, we have $(\zeta,\eta)-(0,0)-(\lambda,\xi)$. As a result, $\Lambda((\varsigma,\sigma))$ is locally linked for all (ς,σ) in $\Omega(R)-0$. Therefore, the connectedness of a graphs created by $\Lambda((0,0))$ must still be investigated. If (ζ,η) and (λ,ξ) are two non-neighboring vertex in $\Lambda((0,0))$, then $\zeta \in \Omega(R_1)\setminus\{0\}$ means that $(\zeta,\eta)-(-\zeta,-\xi)-(\lambda,\xi)$ *is* a pathway in $\Lambda((0,0))$ and η is a pathway in $\Omega(R)-0$. Then $\eta \in \Omega(R_1)\setminus\{0\}$ denotes a pathway in $\Lambda((0,0))$ as $(\zeta,\eta)-(-\lambda,\eta)-(\lambda,\xi)$.

2.4 Proposition

Suppose R be a ring product and. If neither R_1 nor R_2 is an integral domain. Consequently, $Reg(\psi(R))$ is locally linked.

Proof. Assuming that $(\varsigma,\sigma) \in Reg(R)$ and $(\zeta,\eta),(\lambda,\xi)$ are two non-contiguous vertexes in $\Lambda((\varsigma,\sigma))$. Thus, $\zeta \in \Lambda(\varsigma)$ offers the pathway $(\zeta,\eta)-(\varsigma,-\sigma)-(-\varsigma,-\xi)-(\lambda,\xi)$ in $\Lambda((\zeta,\eta))$ and $\eta \in \Lambda(\sigma)$ offers the pathway $(\zeta,\eta)-(-\varsigma,\sigma)-(-\lambda,-\sigma)-(\lambda,\xi)$ in $\Lambda((\zeta,\eta))$.Now, since R be a ring product it becomes obvious that if $|Reg(R_1)| = |Reg(R_2)| = 2$, so neither R_1 nor R_2 is an integral domain (According to the Proposition. 2.3) and $Reg(\psi(R))$ is a full graph. Thus, locally linked.

2.5 Proposition

If $R = \prod_{j=2}^{n+1} R_i, n \geq 2$, so $Reg(\psi(R))$ is locally linked.

Proof. Let $\varsigma = (\varsigma_i) \in Reg(R)$ and $\varphi = (\varphi_j)$ and $\phi = (\phi_j)$ represent two non-neighboring vertexes in $\Lambda(\varsigma)$. Because $\varphi \in \Lambda(\varsigma), \varsigma_i + \varphi_i \in \Omega(R_j)$, for some $j = 2$. If take $\varrho = (\varrho_j)$ so $\varrho_1 = \varphi_1, \varrho_2 = -\varphi_2, \varrho_3 = -\phi_3$, $\varrho_4 = -\phi_4$ and $\varrho_i = 2\ \forall j + 1 \geq 5$, therefore $\varphi - \varrho - \phi - 1$ is a pathway in $\Lambda(\varsigma)$.

2.6 Definition

Consider G to be a graph having vertices and edges collections $M(G)$ and $N(G)$, correspondingly. Then

1. $C(\psi(\mathbb{Z}_k))$ is not locally linked iff $k = \upsilon^{p+1}$, with υ is a prime and $p \geq 3$ or $k = \upsilon_1\upsilon_2\upsilon_3$, and $\upsilon_1, \upsilon_2$and υ_3 are separate primes.
2. $\Omega(\psi(\mathbb{Z}_k))$ is not locally linked iff $k = \upsilon_1\upsilon_2\upsilon_3$ where $\upsilon_1, \upsilon_2$and υ_3 are distinct primes.
3. $Reg(\psi(\mathbb{Z}_k))$ is not locally linked iff $k = \upsilon^{p+1}$, where υ is prime and $p \geq 3$.

2.7 Proposition

Assume R is the product of two rings R_1 and R_2 are two rings for which $|Reg(R_1)| = k_1$ and $|Reg(R_2)| = k_2$. Set $(\varphi_1,\varphi_2) \in Reg(R)$ and $\deg_1(\varphi_1) = \varepsilon_1$ and $\deg_2(\varphi_2) = \varepsilon_2$ with $\deg_j(\varphi_j)$ is indeed the grade of φ_j in $Reg\left(\psi(R_j)\right)$.The vertices grade (φ_1,φ_2) in $Reg\left(\psi(R_j)\right)$ is therefore provided as,

$$\deg\big((\varphi_1,\varphi_2)\big) = \begin{cases} k_2\varepsilon_1 + k_1\varepsilon_2 - \varepsilon_1\varepsilon_2 - 1, & \text{if } 2 \in Reg(R); \\ k_1\varepsilon_2 + k_2\varepsilon_1 + (k_1+k_2) - (\varepsilon_1+\varepsilon_2) - \varepsilon_1\varepsilon_2 - 3, & \text{if } 2 \in Z(R_1)\text{and } Z(R_2); \\ k_1\varepsilon_2 + k_2\varepsilon_1 - \varepsilon_2 + k_2 - \varepsilon_1\varepsilon_2 - 2, & \text{if } 2 \in Z(R_1) \text{ and } 2 \in Reg(R_2). \end{cases}$$

Proof. Suppose that $2 \in Reg(R)$ so $\Lambda\big((\varphi_1,\varphi_2)\big) = \{(\varsigma,\sigma) \in Reg(R): \varsigma \in \Lambda(\varphi_1)$ or $\in$ $\Lambda(\varphi_2)\}$. So, $\left|\Lambda\big((\varphi_1,\varphi_2)\big)\right| = k_2\varepsilon_1 + k_1\varepsilon_2 - \varepsilon_1\varepsilon_2 - 1$. Now, If $2 \in Z(R_1)$ and $Z(R_2)$, thus $\Lambda\big((\varphi_1,\varphi_2)\big) = \{(\varsigma,\sigma) \in Reg(R) \setminus \{(\varphi_1,\varphi_2)\}: \varsigma \in \Lambda(\varphi_1) \cup \{\varphi_1\}$ or $\sigma \in \Lambda(\varphi_2) \cup \{\varphi_2\}\}$. Hence, $\left|\Lambda\big((\varphi_1,\varphi_2)\big)\right| = (\varepsilon_2+1)k_1 + (\varepsilon_1+1)k_2 - (\varepsilon_1+1)(\varepsilon_2+1) - 3$. If $2 \in Z(R_1)$ and $2 \in Reg(R_2)$, then $\Lambda\big((\varphi_1,\varphi_2)\big) = \{(\varsigma,p) \in Reg(R) \setminus \{(\varphi_1,\varphi_2)\}: \varsigma \in \Lambda(\varphi_1) \cup \{\varphi_1\}$ or $\sigma \in \Lambda(\varphi_2)\}$. Therefore, $\left|\Lambda\big((\varphi_1,\varphi_2)\big)\right| = (\varepsilon_1+1)k_2 + k_1\varepsilon_2 - (\varepsilon_1+1)\varepsilon_2 - 2$.

2.8 Remark Assume R is a ring. Consequently

1. If $|R|$ is an even number, so $|\Omega(R)|$ and $|Reg(R)|$are indeed odd if R is a field with even order,
2. When $|R|$ is an odd number, then $|Reg(R)|$ is an even number, while$|\Omega(R)|$ is an odd number.

2.9 Definition Assume R is a ring, so

1. if R is a ring then $\Omega(\psi(R))$ is a regular graph,
2. R is a field iff $\Omega(\psi(R))$ is a regular graph of even grade.

2.10 Remark

Suppose R be a product of two rings R_1 and R_2 $(\varphi_1,\varphi_2) \in Reg(R)$. The grade of vertices (φ_1,φ_2) in $Reg(\psi(R))$ then is even iff $|Reg(R_1)|$: $|Reg(R_2)|$ are even and deg $\deg_1(\varphi_1), \deg_2(\varphi_2)$ are odd.

2.11 Proposition

Suppose R be a ring. Consequently $Reg(\psi(R))$ is a regular graph of odd grade iff R is a field.

Proof. Assume that $R = \prod_{j=2}^{n+1} R_i, n \geq 3$, and R_j is a finite ring $\forall\, i$. By using Definition 2.9. and Remark 2.10. we obtain $Reg(\psi(R))$ is a regular graph of odd grade. Next consider the situation if R is a product of fields of odd orders then $R \cong$

$H \times L$, with H is the product including all fields $R_j'h$ and $R_j'l$ is the product of all rings which are not field of odd orders. Therefore $\text{Reg}(\psi(R))$ is a regular graph of odd order. Lastly, if $|R| = 2^{k+1}p^k$, with $p > 1$, we can construct $R \cong H \times L$, where $|H| = 2^{k+1}$, and $|L| = p^k$. $\text{Reg}(\psi(R))$ is thus a regular graph of odd order. Therefore, R is a field.

3. Some Properties of Eulerian Graphs

First, we will start with the following definition:

3.1 Definition

If a graph seems to have a complete path that contains each of its edges, it is classified as Eulerian. Alternatively, a linked graph G is Eulerian iff every vertices in $\text{M}(G)$ has an even grade.

$\text{Reg}(\psi(R))$ is obviously Eulerian iff $R \cong \mathbb{Z}_2$, whereas $\Omega(\psi(R))$ is Eulerian iff $|R|$ is even, but $\text{C}(\psi(R))$ not Eulerian if R is a finite ring.

3.2 Proposition

If R is a finite ring, thus $\text{C}(\psi(R))$ is Eulerian graph iff R is a product of odd-ordered fields.

Proof: Allow R to be the straight product of two rings. Thus $\text{Reg}(\psi(R))$ is linked, because for every two non-adjacent vertex (ς, σ) and (ζ, η) in $\text{Reg}(\psi(R))$, $(\varsigma, \sigma) - (-\varsigma, -\eta) - (\zeta, \eta)$ is a pathway. As a result, for every finite ring R, $\text{Reg}(\psi(R))$ is linked.

3.3 Proposition

If R is a finite ring, thus $\Omega(\psi(R))$ is Eulerian graph iff R is a product of even-ordered fields.

Proof: obvious.

3.4 Proposition

Assume R is a finite ring. Then $\text{Reg}(\psi(R))$ is Eulerian graph iff $R \cong \mathbb{Z}_2$ or R is a product of odd order fields.

Proof: By applying Proposition. 2.11, we get the result.

3.5 Proposition

Assume R is a ring. If $|R|$ is odd, then $\Omega(\psi(R))$ is Eulerian graph.

Proof. If R is a ring, so $\Omega(\psi(R))$ is obviously Eulerian graph iff R is a field or $|R|$ is even. Assume $R = \prod_{j=1}^{k+1} R_{j+1}$, with R_{j+1} being a finite ring for every i. Thus, there are two possibilities.

Situation 1: $|R|$ is an odd number. If $\Omega(\psi(R))$ is Eulerian graph, $\deg((0,0,\dots,0)) = |\Omega(R)| - 2$ is an odd number. Since R is a product of field of odd orders, according to Remark 2.8. So $\deg((1,1,0,\dots,0) = |\Omega(R)| - 2 - \prod_{j=3}^{k} |\text{Reg}(R_{j+1})|$ is strange, even contradictory.

Situation 2: Assume that $|R|$ is even. Then $|R_{j+1}|$ is odd for all $j+1$. Consider $\varrho = (\varrho_{j+1}) \in \Omega(R)$.Now, $L = \{l \in \{1,2,.,k+1\} : \varrho_l \in \Omega(R_{l+1})\}$ and $H = \{1,2,.,k+1\} \setminus (L+1)$ are defined. About any finite ring of even order P, the summation of every two components are a zero-divisor iff all elements are zero-divisors. Thus, whenever $\varsigma = (\varsigma_{j+1}) \in \Omega(R) \setminus \{\varrho_{j+1}\}$ including all $j+1 \in L$, and left, the vertices $\varsigma_{j+1} \in R_{j+1} \setminus -\varrho_{j+1} + \Omega(R_{j+1\_}))$ $\forall\, j+1 \in \Phi$ and $\varsigma_{j+1} \in \Omega(R_{j+1\_})$ for some $j+1 \in \Phi$ is non-adjacent to ϱ. Because $|-\varrho_{j+1} + \Omega(R_{j+1})| = |\Omega(R_{j+1})|$ $\forall\, j+1$ we have $\deg(\varrho_{j+1}) = (|\Omega(R_{j+1})| - 1) - \left(\prod_{(j+1)\in L} |\text{Reg}(R_{j+1})| \left(\prod_{(j+1)\in L} |\text{Reg}(R_{j+1})| - \prod_{(j+1)\in L} (|\text{Reg}(R_{j+1})| - |\Omega(R_{j+1})|)\right)\right.$. Because $|\Omega(R_{j+1})|$ is even and $|\text{Reg}(R_{j+1})|$ is odd $\forall\, j+1 \in \Phi$ we may conclude that $\deg(\varrho_{j+1})$ is odd. Furthermore, $\Omega(\psi(R))$ is a linked graph because there are 0 neighboring vertex to every other vertex in $\Omega(\psi(R))$. As a result, $\Omega(\psi(R))$ is Eulerian graph.

3.6 Corollary

Assume R is a finite ring. Then $\Omega(\psi(R))$ is Eulerian graph iff $R \cong \mathbb{Z}_2$ or R is a product of odd order fields.

Proof: By applying Proposition. 2.11, we get the result.

4. Conclusion

In this study we obtained the following results:

1. $C(\psi(\mathbb{Z}_k))$ is not locally linked iff $k = \upsilon^{p+1}$, with υ is a prime and $p \geq 3$ or $k = \upsilon_1\upsilon_2\upsilon_3$, and υ_1, υ_2 and υ_3 are separate primes.
2. $\Omega(\psi(\mathbb{Z}_k))$ is not locally linked iff $k = \upsilon_1\upsilon_2\upsilon_3$ where υ_1, υ_2 and υ_3 are distinct primes.
3. $\text{Reg}(\psi(\mathbb{Z}_k))$ is not locally linked iff $k = \upsilon^{p+1}$, where υ is prime and $p \geq 3$.
4. $\Omega(\psi(R))$ is a regular graph, if R is a ring.
5. R is a field iff $\Omega(\psi(R))$ is a regular graph of even grade.
6. R is a field iff $\text{Reg}(\psi(R))$ is a regular graph of odd grade.
7. $\text{Reg}(\psi(\mathbb{Z}_k))$ is Eulerian iff $k = 3$.
8. $\Omega(\psi(\mathbb{Z}_k))$ is Eulerian iff $k = 3$ or k is an even number.
9. $C(\psi(\mathbb{Z}_k))$ is never Eulerian.
10. $\text{Reg}(\psi(R))$ is Eulerian graph iff $R \cong \mathbb{Z}_2$.
11. $\Omega(\psi(R))$ is Eulerian graph iff $R \cong \mathbb{Z}_2$.

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