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Development of multi stage multiple sampling plans for variable quality characteristics

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Abstract
The main advantages of variable sampling plans are the existence of a small sample size for effective decisions and receiving good information on the quality characteristics. But small samples sometimes may lead to wrong decisions during the inspection of the lot and can increase the risk of the producer or consumer. The same scenario may happen in the double or triple sampling also due to small sample sizes. The quality control professionals suggest multiple sampling inspections when the size of the lot is very large. Hence an attempt has been made to design and develop new multi stage multiple sampling plans for variable quality characteristics with unknown standard deviation. These plans are useful for quality characteristic that are measured in a continuous scale for sentencing the large lot. The efficiency measures of the sampling plan are derived and the designing methodology is presented. Tables are constructed for the selection of parameters of multiple sampling plans through standard quality levels.

Keywords: Multi stage multiple sampling, OC function (Operating Characteristic), AQL, LQL, unknown sigma, consumer's risk, producer's risk

1. Introduction

A variable sampling plan requires fewer samples than an attribute plan since more information is available during measurements. There are two methods available for a variable sampling plan, the first method is named the k-Method and the second method is named the M-Method. To describe a variable multiple sampling plan, the number of samples and variable criterion must be determined. The variable sampling systems are meant to be used for sampling a stream of lots from a supplier. The MIL-STD-414 and subsequent Sampling plan procedure for multiple sampling and its designing procedure fall short of an easy algorithm, designing the sampling plans through various quality levels and selection of the plans. Hence an attempt has been made to design and develop new multi stage multiple sampling plans for variable quality characteristics with known and unknown standard deviation. In the multi-stage multiple sampling plans for variables of unknown sigma, if the first stage does not lead to acceptance of the lot then the second stage of sampling continues, even in the second stage if the lot is not accepted then the third stage of variable sampling is continued and the procedure continues until m^{th} stage. At m^{th} stage final decision is taken about acceptance or rejection of the lot. The designing procedure is given and tables are constructed for easy selection of the plan.

Grant Ireson and George Resnikoff (1952) [6] have studied the variables inspection plan based on the normal distribution for one sided and two-sided specification limit for unknown standard deviation. Schilling (1967) [13] has designed the operating characteristics of mixed variables and attributes sampling plans. Steven Sidik (1971) [16] has designed the acceptance sampling plan for variables based on the normal distribution of unknown standard deviation. Helmut Schneider (1989) [7] has analysed the variable sampling plan based on failure censored samples based on Weibull and log-normal distribution. Schilling and Neubauer [2008] have reported the variable sampling plans for different process parameters. Devaarul and Rebecca Jebaseeli Edna [2011] [2] have designed the operating characteristic function and related measures act of mixed sampling product control for costly and destructive items. Sankle and Singh [2012] [12] have studied Single Sampling Plans for variables under the measurement error. Rebecca Jebaseeli Edna *et al*. [2013] [11], have developed Multi-dimensional variable sampling plans.

Senthil Kumar *et al.* [2014] [14] have developed multiple deferred state variable sampling plan by minimizing the sum of risks.
Muhammad Aslam *et al.* [2015] [8] have studied skip-lot sampling plans for variable quality characteristics. Shih-Wen Liu and Chien-Wei Wu [2015] [15] have developed new variable sampling plan based on several quality characteristic. Geetha [2016] [4] has mentioned Inverse Gaussian distribution used in the single sampling plans by variables. Mujahid Sayyed [2016] [9] studied about the correlated data for variable sampling plan with known standard deviation.
Chien-Wei-Wu *et al.* [2017] [1] have defined the lifetime performance index based on the variable sampling plan. Ramya and Devaarul [2019] [10] have studied the variable sampling plan for non-normal based on log-logistic failure distribution. Geetha and Pavithra [2019] [5] have studied single sampling plan by variables using range that is specified by two points on the operating characteristic curve.

2. Formulation of multi stage multiple sampling plans for variables with unknown sigma

Let, N = Lot size and U = Upper Specification
n_1= First stage sample size
n_2= Second stage sample size
n_3= Third stage sample size
n_j= j[th] stage sample size
k_1= Variable factor such that lot is accepted, if $\bar{x}_1 + k_1s_1 \leq U$
k_2= Variable factor such that lot is accepted, if $\bar{x}_2 + k_2s_2 \leq U$
k_3 = Variable factor such that lot is accepted, if $\bar{x}_3 + k_3s_3 \leq U$
k_j= j[th] stage Variable factor such that lot is accepted, if $\bar{x}_j + k_js_j \leq U$
$\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots \bar{x}_j$= sample means where, $\bar{x}_j = \frac{\sum x_j}{n}$
$s_1, s_2, s_3 \ldots\ldots\ldots s_j$ = sample standard deviations, where $s^2 = \frac{1}{n-1}\sum_{i=1}^{n_j}(x_j - \bar{x}_j)^2$
β_1= 1-α (1-Producer's risk) and β_2 = β (Consumer's risk)
β_1 = P [Acceptance of the lot /AQL] and β_2 = P [Acceptance of the lot/LQL]
β_1^1 = Probability of acceptance in the 1[st] stage at AQL
$\beta_1^2, \beta_1^3, \ldots \ldots \ldots \ldots \beta_1^j$ = Probability of acceptance in the j[th] stage at AQL
β_2^1 = Probability of acceptance in the 1[st] stage at LQL
$\beta_2^2, \beta_2^3, \ldots \ldots \ldots \ldots \beta_2^j$ = Probability of acceptance in the j[th] stage at LQL
p_1= Acceptable Quality Level
p_2 = Limiting Quality Level

Algorithm of multi stage multiple sampling plans for variable quality characteristics with Unknown Sigma.

Stage: I

Let U be the upper specification limit

Step 1: Draw a random sample of size n_1.
Step 2: Determine the sample mean $\bar{x}_1$and sample standard deviation s_1.
Step 3: If $\bar{x}_1 + k_1s_1 \leq U$, accept the lot.
Step 4: If $\bar{x}_1 + k_1s_1 > U$, go to second stage sampling.

Stage: II

Step 5: Draw a random sample of size n_2.
Step 6: Determine the sample mean $\bar{x}_2$ and sample standard deviation s_2.
Step 7: If $\bar{x}_2 + k_2s_2 \leq U$, accept the lot.
Step 8: If $\bar{x}_2 + k_2s_2 > U$, go to stage three sampling.

Stage: III

Step 9: Draw a random sample of size n_3
Step 10: Determine the sample mean $\bar{x}_3$ and sample standard deviation s_3.
Step 11: If $\bar{x}_3 + k_3s_3 \leq U$, accept the lot, otherwise go to j[th] Stage

Stage: J

Step 12: Draw a random sample of size n_j
Step 13: Determine the sample mean $\bar{x}_j$ and sample standard deviation s_j
Step 14: If $\bar{x}_j + k_js_j \leq U$, accept the lot, otherwise reject the lot.

Operating Characteristic and associated measures of MSMSP

Theorem 1

Probability of acceptance

The probability of acceptance of multi stage multiple sampling plans for variables is as follows:

$P_a(p) = P[\bar{x}_1 + k_1s_1 \leq U]$
$+ P[\bar{x}_2 + k_2s_2 \leq U \text{ and } U < \bar{x}_1 + k_1s_1]$
$+ P[\bar{x}_3 + k_3s_3 \leq U \text{ and } U < \bar{x}_2 + k_2s_2 \text{and } U < \bar{x}_1 + k_1s_1] \ldots.$
$+ P[\bar{x}_j + k_js_j \leq U \text{ and } U < \bar{x}_{j-1} + k_{j-1}s_{j-1} \ldots\ldots U < \bar{x}_2 + k_2s_2 \text{ and } U < \bar{x}_1 + k_1s_1].$
$P_a(p) = P[\bar{x}_1 + k_1s_1 \leq U] + P[\bar{x}_2 + k_2s_2 \leq U] * P[U < \bar{x}_1 + k_1s_1] + P[\bar{x}_3 + k_3s_3 \leq U] * P[U < \bar{x}_2 + k_2s_2] * P[U < \bar{x}_1 + k_1s_1] \ldots. + P[\bar{x}_j + k_js_j \leq U] * P[U < \bar{x}_{j-1} + k_{j-1}s_{j-1}] \ldots * P[U < \bar{x}_1 + k_1s_1]$

$P_a(p) = P(i) + P(ii) + P(iii) + \ldots\ldots\ldots + P(j)$

Proof

Case 1: In the first stage, if $\bar{x}_1 + k_1s_1 \leq U$ the lot is accepted otherwise, case 2 is implemented
Case 2: From the first stage, if $\bar{x}_1 + k_1s_1 > U$ and in the second stage of $\bar{x}_2 + k_2s_2 \leq U$ the lot is accepted, otherwise case 3 is implemented.
Case 3: From the second stage, if $\bar{x}_1 + k_1s_1 > U$ and $\bar{x}_2 + k_2s_2 > U$ and in the third stage if $\bar{x}_3 + k_3s_3 \leq U$ the lot is accepted, otherwise the next case is implemented

Case J: From the j-1[th] stage, if $\bar{x}_1 + k_1s_1 > U$ and $\bar{x}_2 + k_2s_2 > U \ldots$and $\bar{x}_{j-1} + k_{j-1}s_{j-1} > U$ and in the j[th] stage of $\bar{x}_j + k_js_j \leq U$ the lot is accepted, and if $\bar{x}_j + k_js_j > U$ the lot is rejected.
Here Cases 1 to j are mutually exclusive cases, hence by the law of addition,
We get,

$P_a(p) = P(i) + P(ii) + P(iii) + \ldots\ldots\ldots P(j)$

Hence the Probability of Acceptance of the Lot is

$P_a(p) = P[\bar{x}_1 + k_1s_1 \leq U]$
$+ P[\bar{x}_2 + k_2s_2 \leq U \text{ and } U < \bar{x}_1 + k_1s_1]$
$+ P[\bar{x}_3 + k_3s_3 \leq U \text{ and } U < \bar{x}_2 + k_2s_2 \text{ and } U < \bar{x}_1 + k_1s_1] \ldots.$
$+ P[\bar{x}_j + k_js_j \leq U \text{ and } U < \bar{x}_{j-1} + k_{j-1}s_{j-1} \ldots\ldots U < \bar{x}_2 + k_2s_2 \text{ and } U < \bar{x}_1 + k_1s_1].$

Theorem 2

Average Outgoing Quality [AOQ]

The average outgoing quality of multi stage multiple sampling plans for variables is as follows.

AOQ = p.P_a(p)

AOQ = p.[$\frac{N-n}{N}$]P_a(p)

Proof

AOQ = p.P_a($\frac{N-n}{N}$) + 0(1-P_a)($\frac{N-n}{N}$)

= p.P_a($\frac{N-n}{N}$)

AOQ = p.P_a(1-$\frac{n}{N}$)

Sample size is very small in the proportion of the lot $\frac{n}{N} \sim 0$
So we get,

AOQ = p.P_a(p)

Then, AOQ = p.P_a(p) = P [$\bar{x}_1 + k_1s_1 \leq U$]
+ P [$\bar{x}_2 + k_2s_2 \leq$ U and U < $\bar{x}_1 + k_1s_1$]
+ P [$\bar{x}_3 + k_3s_3 \leq$ U and U< $\bar{x}_2 + k_2s_2$ and U < $\bar{x}_1 + k_1s_1$] ….
+ P [$\bar{x}_j + k_js_j \leq$ U and U< $\bar{x}_{j-1} + k_{j-1}s_{j-1}$ …… U < $\bar{x}_2 + k_2s_2$ and U < $\bar{x}_1 + k_1s_1$].

Theorem 3
Average Sample Number (ASN)
The Average Sample Number of multi stage multiple sampling plans for variables is given

ASN = n_1P_I+ (n_1+ n_2) P_{II} + (n_1 + n_2 + n_3) P_{III} ++ (n_1 + n_2 + n_3 + ……n_j) P_J

Where,
P_{II} = 1-P_I
P_{III} = 1-P_{II}
……….
P_J = 1-P_{j-1}

P_I, P_{II}, P_{III},…P_J are probability of a decision [acceptance or rejection of the lot] on the basis of the samples.

Theorem 4
Average Total Inspection
The average total inspection of multi stage multiple sampling plan for variables is as follows:

ATI = n_1P(i) + (n_1+n_2)(P_a(p) - P(i)) + (n_1+n_2+n_3) (P_a(p) - P(ii)) + (n_1+n_2+n_3+……….+n_j)(P_a(p) - P(j-1) + N(1-P_a(p))

Designing of multi stage multiple sampling plan inspection by variables for unknown sigma
In the case of multi stage multiple sampling plan by variables under assumptions of normal distributions for a quality characteristic. The relationship between the fraction defective p and the probability of acceptance P_a(p) is given by

$$Z_p = k_j + \frac{1}{\sqrt{n_j}} Z_{1-p_{a(p)}} \sqrt{1+\frac{k_j^2}{2}}$$

This relationship is used to determine the parameters for various values of the fractional defective. The acceptance probabilities are decreasing starting from 1 to 0. This means that Pa(p) is decreasing function of p. In order to determine the procedure value n_j and k_j, we require the points (p_1, 1-α_j) and (p_2, β_j) on the operating characteristic function. In the multi stage multiple sampling plans the criterion are $P_a(p_1) \geq (1-\alpha) = \beta_1$ and $P_a(p_2) \leq \beta = \beta_2$.

Designing multi stage multiple variable sampling plan through AQL
Given probability of acceptance of the lot is $P_a(p_1) \geq (1-\alpha) = \beta_1$, Let β_1= 0.95

Step 1: Let the 1st stage of probability of acceptance at AQL be

β_1^1= 0.65

Step 2: Let the second stage of probability of acceptance at AQL be

$$\beta_1^2 = \frac{\beta_1 - \beta_1^1}{1-\beta_1^1}$$

Step 3: Let the third stage of probability of acceptance be

$$\beta_1^3 = \frac{\beta_1 - \beta_1^1 - (\beta_1^1 - \beta_1^2)}{1-\beta_1^1 - (\beta_1^1 - \beta_1^2)}$$
………………………..

Step j: Let the jth stage of probability of acceptance be

$$\beta_1^j = \frac{\beta_1 - \beta_1^1 - (\beta_1^1 - \beta_1^2) \dots \dots \dots \dots ((\beta_1^{j-2} - \beta_1^{j-1})}{1-\beta_1^1 - \cdots \dots \dots (\beta_1^{j-2} - \beta_1^{j-1})}$$

The jth stage sampling parameters for n_j and k_j for the known AQL are determined as follows:

$$Z_{p_1} = k_j + \frac{1}{\sqrt{n_j}} Z_{1-p_{a(p_j)}} \sqrt{1+\frac{k_j^2}{2}}$$

$$Z_{p_1} = k_j + \frac{1}{\sqrt{n_j}} \beta_1^j \sqrt{1+\frac{k_j^2}{2}}$$

$$(Z_{p_1} - k_j) = \frac{1}{\sqrt{n_j}} \beta_1^j \sqrt{1+\frac{k_j^2}{2}}$$

$$(Z_{p_1} - k_j)\sqrt{n_j} = \beta_1^j \sqrt{1+\frac{k_j^2}{2}}$$

$$\sqrt{n_j} = \frac{\beta_1^j \sqrt{1+\frac{k_j^2}{2}}}{(Z_{p_1} - k_j)}$$

$$n_j = \left[\frac{\beta_1^j \sqrt{1+\frac{k_j^2}{2}}}{(Z_{p_1} - k_j)}\right]^2$$

Based on the above equations, the parameters are determined and given in table (1).

Table 1: The table gives the values of n_1, n_2 and n_3 for the known variable factors k_1, k_2 and k_3 of multi stage multiple sampling plan for variables indexed through AQL.

Multi Stage Multiple Sampling through AQL						
p_1	k_1	n_1	k_2	n_2	k_3	n_3
0.001	3.1400	356	3.2684	474	3.2853	608
0.002	2.9391	213	3.0686	372	3.0785	516
0.003	2.8181	149	2.9454	323	2.9540	456
0.004	2.7291	118	2.8669	260	2.8701	389
0.005	2.6620	91	2.8058	218	2.8063	336
0.006	2.6056	75	2.7814	154	2.7685	266
0.007	2.5791	43	2.7584	121	2.7616	188
0.008	2.5812	22	2.7744	82	2.7817	127

Note

Table (1) is developed for the given probability of acceptance (1-α)= β_1= 0.95, the first stage of probability of acceptance is $\beta_1^1 = 0.65$, the second stage of probability of acceptance is $\beta_1^2 = \frac{\beta_1-\beta_1^1}{1-\beta_1^1}$ = 0.94 and the third stage of probability of acceptance is $\beta_1^3 = \frac{\beta_1-\beta_1^1-(\beta_1^1-\beta_1^2)}{1-\beta_1^1-(\beta_1^1-\beta_1^2)}$ = 0.97.

Illustration 1

In a production process, let the incoming lot quality be p_1= 0.001 (AQL) and the given probability of acceptance is (1-α) = β_1= 0.95. Obtain the multi stage multiple sampling plan by variables indexed through AQL.

Solution

It's given that value of p_1 = 0.001 and the probability of acceptance is β_1= 0.95 (β_1= 1-α). Let the first stage of assuming probability of acceptance be $\beta_1^1 = 0.65$, the second stage of probability of acceptance is derived from this formula i.e., $\beta_1^2 = \frac{\beta_1-\beta_1^1}{1-\beta_1^1}$ = 0.94 and the third stage of probability of acceptance is derived from this formula i.e., $\beta_1^3 = \frac{\beta_1-\beta_1^1-(\beta_1^1-\beta_1^2)}{1-\beta_1^1-(\beta_1^1-\beta_1^2)}$ = 0.97.Then from the table (1), one can get the parameters of multi stage multiple sampling plans for variables as [n_1=356, n_2=474, n_3=608 and k_1=3.1400, k_2=3.2684, k_3=3.2853].

Designing multi stage multiple sampling plans by variables through AQL and LQL

Stage: I

To determine the first stage sampling parameters n_1 and k_1. It is known that for given p_1 and p_2. Then consider

$P_a(p_1) \geq 1-\alpha_1=\beta_1^1$

$P_a(p_2) \leq \beta_2 = \beta_2^1$

Assume that X ~ N (μ, σ^2), Where σ^2 is unknown

Let p = P [X > U] 1

$p = P\left[\frac{X-\mu}{\sigma} > \frac{U-\mu}{\sigma}\right]$

p = P [Z > Z_p], Where [$Z_p = \frac{U-\mu}{\sigma}$]

$P_a(p) = P[\bar{x}_1 + k_1 s_1 \leq U]$ 2

It is known that for given p_1 and p_2

$P_a(p_1) \geq 1-\alpha_1= \beta_1$

$P_a(p_2) \leq \beta_2 = \beta_2$

From 1, substitute p = p_1

$$Z_{p_1} = k_1 + \frac{1}{\sqrt{n_1}} Z_{1-Pa(p1)}\sqrt{1+\frac{k_1^2}{2}}$$

$$Z_{p_1} = k_1 + \frac{1}{\sqrt{n_1}} Z_{\alpha 1}\sqrt{1+\frac{k_1^2}{2}} \qquad 3$$

p = p_2

$$Z_{p_2} = k_1 + \frac{1}{\sqrt{n_1}} Z_{1-Pa(p2)}\sqrt{1+\frac{k_1^2}{2}}$$

$$Z_{p_2} = k_1 + \frac{1}{\sqrt{n_1}} Z_{-\beta_1}\sqrt{1+\frac{k_1^2}{2}}$$

$$Z_{p_2} = k_1 - \frac{1}{\sqrt{n_1}} Z_{\beta_1}\sqrt{1+\frac{k_1^2}{2}} \qquad 4$$

$$Z_{p_1} - Z_{p_2} = k_1 + \frac{1}{\sqrt{n_1}} Z_{\alpha 1}\sqrt{1+\frac{k_1^2}{2}} - k_1 + \frac{1}{\sqrt{n_1}} Z_{\beta_1}\sqrt{1+\frac{k_1^2}{2}}$$

$$Z_{p_1} - Z_{p_2} = \frac{1}{\sqrt{n_1}}\sqrt{1+\frac{k_1^2}{2}}\,(Z_{\alpha_1} + Z_{\beta_1})$$

$$\sqrt{n_1} = \frac{(Z_{\alpha_1} + Z_{\beta_1})}{Z_{p_1} - Z_{p_2}}\sqrt{1+\frac{k_1^2}{2}}$$

Here $Z_{\alpha_1} = 1 - \alpha_1 = \beta_1^1$

$Z_{\beta_1} = \beta_2 = \beta_2^1$

$$n_1 = \left(\frac{Z_{\beta_1^1} + Z_{\beta_2^1}}{Z_{p_1} - Z_{p_2}}\right)\left(1+\frac{k_1^2}{2}\right)$$

equation 3 * Z_{β_1}

$$Z_{p_1} Z_{\beta_1} = k_1 Z_{\beta_1} + \frac{1}{\sqrt{n_1}} Z_{\alpha_1} Z_{\beta_1}\sqrt{1+\frac{k_1^2}{2}} \qquad 5$$

equation 3 * Z_{α_1}

$$Z_{p_2} Z_{\alpha_1} = k_1 Z_{\alpha_1} - \frac{1}{\sqrt{n_1}} Z_{\alpha_1} Z_{\beta_1}\sqrt{1+\frac{k_1^2}{2}} \qquad 6$$

Add equation 5 and 6

$$Z_{p_1} Z_{\beta_1} + Z_{p_2} Z_{\alpha_1} = k_1 Z_{\beta_1} + \frac{1}{\sqrt{n_1}} Z_{\alpha_1} Z_{\beta_1}\sqrt{1+\frac{k_1^2}{2}} + k_1 Z_{\alpha_1} - \frac{1}{\sqrt{n_1}} Z_{\alpha_1} Z_{\beta_1}\sqrt{1+\frac{k_1^2}{2}}$$

$Z_{p_1}Z_{\beta_1} + Z_{p_2}Z_{\alpha_1} = k_1 Z_{\beta_1} + k_1 Z_{\alpha_1}$

$$k_1 = \frac{Z_{\beta_1^1}Z_{p2} + Z_{p1}Z_{\beta_2^1}}{Z_{\beta_1^1}+Z_{\beta_2^1}}$$

Similarly

$$n_2 = \left(\frac{Z_{\beta_1^2} + Z_{\beta_2^2}}{Z_{p_1}-Z_{p_2}}\right)^2 \left(1+\frac{k_2^2}{2}\right)$$

$$k_2 = \frac{Z_{\beta_1^2}Z_{p2} + Z_{p1}Z_{\beta_2^2}}{Z_{\beta_1^2}+Z_{\beta_2^2}}$$

$$n_j = \left(\frac{Z_{\beta_1^j} + Z_{\beta_2^j}}{Z_{p_1}-Z_{p_2}}\right)^2 \left(1+\frac{k_j^2}{2}\right)$$

$$k_j = \frac{Z_{\beta_1^j}Z_{p2} + Z_{p1}Z_{\beta_2^j}}{Z_{\beta_1^j}+Z_{\beta_2^j}}$$

Based on the above designing procedure table (2) is constructed.

Table 2: The table gives the values of n_1, n_2, n_3 and variable factors k_1, k_2, k_3 for the parameters of multi stage multiple sampling plan for variables of unknown sigma. [producer's risk (α) = 0.05 and consumer's risk (β) = 0.10].

Multi stage multiple sampling plan through AQL and LQL							
p_1	p_2	k_1	n_1	k_2	n_2	k_3	n_3
0.001	0.01	2.94525	38	2.619442	78	2.674991	96
0.002	0.02	2.721691	29	2.41827	60	2.430017	71
0.003	0.03	2.58323	24	2.291932	50	2.276494	58
0.004	0.04	2.48099	21	2.197629	43	2.162085	50
0.005	0.05	2.399133	18	2.121424	39	2.069758	44
0.006	0.06	2.330438	17	2.056936	35	1.991725	40
0.007	0.07	2.270983	15	2.000692	32	1.923743	36
0.008	0.08	2.218389	14	1.950578	30	1.863234	33
0.009	0.09	2.171102	13	1.905211	28	1.80851	31
0.01	0.1	2.128049	12	1.863633	26	1.758405	29
0.011	0.11	2.088455	12	1.825151	24	1.712073	27
0.012	0.12	2.051742	11	1.78925	23	1.668885	25
0.013	0.13	2.017469	10	1.755531	22	1.628358	24
0.014	0.14	1.98529	10	1.723684	21	1.590112	22
0.015	0.15	1.954926	9	1.693458	20	1.553843	21
0.016	0.16	1.926153	9	1.664652	19	1.519305	20
0.017	0.17	1.898786	8	1.637098	18	1.486294	19
0.018	0.18	1.872671	8	1.610656	17	1.454639	18

Illustration 2

In a production process let p_1 = 0.001 (AQL) and p_2 = 0.01 (LQL) and the corresponding probability of acceptance be (1-α) = β_1=0.95 and β_2 = 0.10. Obtain the multi stage multiple sampling plan for variables of unknown sigma.

Solution: It's given that p_1= 0.001, p_2 = 0.01 and the probability of acceptance is β_1= 95% (0.95) and β_2 =10% (0.10). From Table 2, we get the parameters of multi stage multiple sampling plans for variables [n_1=38, n_2=78, n_3=96 and k_1=2.9453, k_2=2.61944, k_3= 2.67499].

Property 1: When j=1 MSMSP reduces to single sampling plans for variables

Proof: We know that the samples sizes for MSMSP is

$$n_j = \left(\frac{Z_{\beta_1^j} + Z_{\beta_2^j}}{Z_{p_1}-Z_{p_2}}\right)^2 \left(1+\frac{k_j^2}{2}\right)$$

Let j =1. Then

$$n = \left(\frac{Z_{\beta_1} + Z_{\beta_2}}{Z_{p_1}-Z_{p_2}}\right)^2 \left(1+\frac{k^2}{2}\right)$$

The above sample size is the sample size of single sampling plans by variables with unknown sigma. Hence the proof.

Property 2: When j=1, MSMSP reduces to single sampling plans for variables

Proof: We know that the variable factor for MSMSP is

$$k_j = \frac{Z_{\beta_1^j}Z_{p_2} + Z_{p_1}Z_{\beta_2^j}}{Z_{\beta_1^j}+Z_{\beta_2^j}}$$

Let j =1. Then

$$k = \frac{Z_{\beta_1}Z_{p_2} + Z_{p_1}Z_{\beta_2}}{Z_{\beta_1}+Z_{\beta_2}}$$

The above variable factor is the variable factor for single sampling plans by variables with unknown sigma. Hence the proof.

3. Conclusion

In this research paper, a new multi-stage multiple sampling plan is developed with a final decision based on the results of each stage of the sampling plan, which will lead to accurate and effective decisions on the lots. The Operating characteristic function, ASN, ATI and AOQ are derived and tables are constructed for the multi-stage multiple sampling plans. The new algorithm helps to operate in the quality control section especially when the decision is based on several stages. Hence in the manufacturing industries one can maintain the standard quality of the lot or process by implementing multi-stage multiple sampling plans. It is found that when the index j=1, MSMSP converges to ordinary Single Sampling Plans.

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