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New mappings in fuzzy topological spaces

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Abstract
In this paper, we introduce the concept of fuzzy Sgw-continuous maps and fuzzy Sgw-irresolute maps in fts. We prove that the composition of two fuzzy Sgw-continuous maps need not be fuzzy Sgw-continuous and study some of their properties.

Keywords: Fuzzy Sgw-continuous maps, fuzzy Sgw-irresolute maps

1. Introduction

The concept of a fuzzy subset was introduced and studied by L.A. Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, L.A.Zadeh [1] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L. Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces. Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function

$$\mu_A : X \rightarrow [0,1] \text{ of A, defined by}$$
$$\mu_A(x) = 1, \quad \text{if } x \in A,$$
$$= 0, \quad \text{if } x \notin A.$$

Thus an element $x \in X$ isin A if $\mu_A(x) = 1$ and is not in A if $\mu_A(x) = 0$. In general if X is a set and A is a subset of X then A has the following representation. $A = \{ (x, \mu_A(x)): x \in X\}$, here $\mu_A(x)$ may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A. Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1; he called such a class as fuzzy subset.

A fuzzy subset A in X is characterized as a membership function μ_A: X [0,1], which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy Sgw-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy Sgw-closed but not conversely. Also we introduce fuzzy Sgw-open sets in fuzzy topological spaces and study some of their properties.

Fuzzy logic means approximate reasoning, information granulation, computing with words and so on. Fuzzy logic provides an inference structure that enables the human reasoning capabilities to be applied to artificial knowledge-based systems. It provides a means for converting linguistic strategy into actions and thus offers a high- level computation.Fuzzy logic provides mathematical strength to the emulation of certain perceptual and linguistic attributes associated with human cognition, whereas the science of neural networks provides a new computing tool with learning and adaptation capabilities. Several researchers have worked on topology using fuzzy sets and developed the theory of fuzzy topological spaces.

The notion of fuzzy subsets naturally plays a very significant role in the study of fuzzy topology introduced by C. L. Chang, N. Levine introduced the concepts of generalized closed sets in general topology in the year 1970. G. Balasubramanian and P. Sundaram introduced and studied generalized closed fuzzy sets in fuzzy topology. K.K. Azad introduced semi-closed fuzzy sets in the year 1981. H. Maki, T. Fukutake, M. Kojima and H. Harada introduced semi-generalized closed fuzzy sets (briefly fsg - closed) in fuzzy topological space in the year 1998.
T. Kong, R. Kopperman and P. Meyer shown some of the properties of generalized closed set have been found to be useful in computer science and digital topology. Caw, Ganster and Reilly and has shown that generalization of closed set is also useful to characterize certain classes of topological spaces and there variations.

1.1 Preliminaries

1.1.1 Definition [1]: A fuzzy subset A in a set X is a function A: X → [0, 1]. A fuzzy subset in X is empty iff its membership function is identically 0 on X and is denoted by 0 or μ_ϕ. The set X can be considered as a fuzzy subset of X whose membership function is identically 1 on X and is denoted by $\mu_x$or I_X. In fact, every subset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

1.1.2 Definition [1]: If A and B are any two fuzzy subsets of a set X, then A is said to be included in B or A is contained in B iff A(x) ≤ B(x) for all x in X. Equivalently, A ≤ B iff A(x) ≤ B(x) for all x in X.

1.1.3 Definition [1]: Two fuzzy subsets A and B are said to be equal if A(x) = B(x) for every x in X.
Equivalently A = B if A(x) = B(x) for every x in X.

1.1.4 Definition [1]: The complement of a fuzzy subset A in a set X, denoted by A′ or 1 − A, is the fuzzy subset of X defined by A′(x) = 1 − A(x) for all x in X. Note that (A′)′ = A.

1.1.5 Definition [1]: The union of two fuzzy subsets A and B in X, denoted by A ∨ B, is a fuzzy subset in X defined by (A ∨ B)(x) = Max{$\mu_A(x)$, $\mu_B(X)$} for all x in X.

1.1.6 Definition [1]: The intersection of two fuzzy subsets A and B in X, denoted by A ∧ B, is a fuzzy subset in X defined by (A ∧ B)(x) = Min{A(x), B(x)} for all x in X.

1.1.7 Definition [1]: A fuzzy set on X is 'Crisp' if it takes only the values 0 and 1 on X.

1.1.8 Definition [2]: Let X be a set and be a family of fuzzy subsets of (X,τ) is called a fuzzy topology on X iff τ satisfies the following conditions.

1. μ_ϕ; μ_X∈τ: That is 0 and 1 ∈τ
2. If G_i∈ τ for i ∈ I then $\vee G_i$∈τ
   i ∈ I
3. If G,H∈τ then G ∧ H ∈τ

The pair (X, τ) is called a fuzzy topological space (abbreviated as fts). The members of τare called fuzzy open sets and a fuzzy set Ain X is said to be closed iff 1 − A is an fuzzy open set in X.

1.1.9 Remark [2]: Every topological space is a fuzzy topological space but not conversely.

1.1.10 Definition [2]: Let X be a fts and A be a fuzzy subset in X. Then ∧ {B: B is a closed fuzzy set in X and B ≥ A} is called the closure of A and is denoted by A or cl(A).

1.1.11 Definition [2]: Let A and B be two fuzzy sets in a fuzzy topological space (X,τ)and let A ≥ B. Then B is called an interior fuzzy set of A if there exists G ∈τ such that A ≥ G ≥ B, the least upper bound of all interior fuzzy sets of A is called the interior of A and is denoted by A^0.

1.1.12 Definition [3]: A fuzzy set A in a fts X is said to be fuzzy semi open if and only if there exists a fuzzy open set V in X such that V ≤A ≤cl(V).

1.1.13 Definition [3]: A fuzzy set A in a fts X is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set V in X such that int (V)≤ A ≤V. It is seen that a fuzzy set A is fuzzy semiopen if and only if 1-A is a fuzzy semi-closed.

1.1.14 Theorem [3]: The following are equivalent:

(a) μ is a fuzzy semiclosed set,
(b) μ^C is a fuzzy semiopen set,
(c) int(cl(μ)) ≤ μ.
(d) int(cl(μ)) ≥ μ^c

1.1.15 Theorem [3]: Any union of fuzzy semi open sets is a fuzzy semi open set and (b) any intersection of fuzzy semi closed sets is a fuzzy semi closed.

1.1.16 Remark [3]

(a) Every fuzzy open set is a fuzzy semi open but not conversely.
(b) Every fuzzy closed set is a fuzzy semi-closed set but not conversely.
(c) The closure of a fuzzy open set is fuzzy semi open set
(d) The interior of a fuzzy closed set is fuzzy semi-closed set

1.1.17 Definition [3]: A fuzzy set μof a fts X is called a fuzzy regular open set of X if int(cl(μ).= μ.

1.1.18 Definition [3]: A fuzzy set μof fts X is called a fuzzy regular closed set of X if cl(int(μ)) = μ.

1.1.19 Theorem [3]: A fuzzy set μof a fts X is a fuzzy regular open if and only if μ^C fuzzy regular closed set.

1.1.20 Remark [3]

- Every fuzzy regular open set is a fuzzy open set but not conversely.
- Every fuzzy regular closed set is a fuzzy closed set but not conversely.

1.1.21 Theorem [3]

- The closure of a fuzzy open set is a fuzzy regular closed.
- The interior of a fuzzy closed set is a fuzzy regular open set.

1.1.22 Definition [4]: A fuzzy set α in fts X is called fuzzy rwclosedif cl(α)≤ μwhenever α ≤ μ and μ is regular semi-open in X.

1.1.23 Definition [5]: A fuzzy set α in fts X is called fuzzy Sgw closed if p-cl(α)$\leq \mu$ whenever $\alpha \leq \mu$and μ is rgα −open set in X.

1.1.24 Defintion 5]: A fuzzy set α of a fts X is fuzzy Sgw-open set, if it's complement α^c is a fuzzy Sgw-closed in fts X.

1.1.25 Definition [2]: Let X and Y be fts. A map f: X Y is said to be a fuzzy continuous mapping if $f^{-1}(\mu)$ is fuzzy open in X for each fuzzy open set μ in Y.

1.1.26 Definition [6]: Let X and Y be fts. A map f: X Y is said to be a fuzzy almost continuous mapping if $f^{-1}(\mu)$ is fuzzy open in X for each fuzzy regular open set μ in Y.

1.1.27 Definition [6]: Let X and Y be fts. A map f: X Y is said to be a fuzzy irresolute if $f^{-1}(\mu)$ is fuzzy semi-open set in X for each fuzzy semi-open set μ in Y.

1.1.28 Definition [2]: Let X and Y be fts. A map f: X Y is said to be a fuzzy semi-continuous if
$f^{-1}(\mu)$ is fuzzy semi-open set in X for each fuzzy open set μ in Y.

1.1.29 Definition [4]: Let X and Y be fts. A map f: X Y is said to be a fuzzy rw-continuous if
$f^{-1}(\mu)$ is fuzzy rw-open set in X for each fuzzy open set μ in Y.

1.1.30 Definition [7]: Let X and Y be fts. A map f: X Y is said to be a fuzzy completely semi-continuous if $f^{-1}(\mu)$ is fuzzy regular semi-open set in X for each fuzzy open set μ in Y.

2. Fuzzy Sgw-continuous maps and fuzzy Sgw-irresolute maps in fuzzy topological spaces

2.1 Definition: Let X and Y be fts. A map f: X Y is said to be fuzzy Sgw-continuous map if the inverse image of every fuzzy open set in Y is fuzzy Sgw-open in X.

2.2 Theorem: If a map f: (X,T) (Y,T) is fuzzy continuous, then f is fuzzy Sgw-continuous map.

Proof: Let μ be a fuzzy open set in afts Y. Since f is fuzzy continuous map $f^{-1}(\mu)$ is a fuzzy open set in fts x,as every open set is fuzzy Sgw-open, we have $f^{-1}(\mu)$ is fuzzy a Sgw-open set in fts X. Therefore f is fuzzy continuous map.
The converse of the above theorem need not be true in general as seen from the following example

2.3 Example: Let X=Y= {a, b, c,d} and the functions $\alpha, \beta, \gamma, \delta$: X [0, 1] be defined as
α (x) = 1 if x = a
0 otherwise
β(x) = 1 if x = b
0 therwise
γ(x) = 1 if x = a,b
otherwise
δ(x) = 1 if x = a,b,c
otherwise.

Consider T= {1,0,$\alpha,\beta,\gamma,\delta$},$\sigma$ = {1,0,$\alpha,\beta,\gamma,\delta$}.Now (X,T) and (Y, σ) are the fts.
Define a map f: (X,T) (Y,T) by f(a)=c,f(b)=a,f(c)=b,f(d)=d, then f is fuzzy Sgw-continuous map but not fuzzy continuous map as the inverse image of the fuzzy set γ in (Y, σ) is
μ: X [0 1] defined as
μ (x) = 1 if x = b,c
0 otherwise.

This is not a fuzzy open set in (X,T).

2.4 Theorem: A map f:(X,T) (Y, σ) is fuzzy Sgw-continuous map iff the inverse image of every fuzzy closed set in a fts Y is a fuzzy Sgwclosed set in fts X.

Proof: Let δ be a fuzzy closed set in a fts Y then δ^c is fuzzy open in fts Y. Since f is fuzzy Sgw-continuousmap,$f^{-1}(\delta^c)$ is Sgw-open in ftsX but $f^{-1}(\delta^c) = 1- f^{-1}(\delta^c)$ and so $f^{-1}(\mu)$ is a fuzzy Sgw-closed set in fts X. Conversely, Assume that the inverse image of every fuzzy closed set in Y is fuzzy Sgw closed in fts X. Let μ be a fuzzy open set in fts Y then μ^c is fuzzy closed in Y;
by hypothesis $f^{-1}(\mu^c)= 1-f^{-1}(\mu)$ is fuzzy Sgwclosed in Xand so $f^{-1}(\mu)$ is a fuzzy Sgw-open set in fts X. Thus f is fuzzy Sgw-continuous map.

2.5 Theorem: If a function f: (X,T) (Y, σ) is fuzzy almost continuous map, then it is fuzzy Sgw-continuous map.

Proof: Let a function f:(X,T) (Y, σ) be a fuzzy almost continuous map and μ befuzzy open set in fts Y. Then $f^{-1}(\mu)$ is a fuzzy regular open set in fts X. Now, $f^{-1}(\mu)$ is fuzzy Sgw-open in X, as every fuzzy regular open set is fuzzy Sgw-open.Therefore f is fuzzy Sgw-continuous map.
The converse of the above theorem need not be true in general as seen from the following example.

2.6 Example: consider the fts (X,T) and (Y, σ) as defined in example 2.3. define a map
f:(X,T) (Y, σ) by f(a)= c,f(b)=a,f(c)=b,f(d)=d. Then f is fuzzy Sgw-continuous map but it is not almost continuous map.

2.7 Example: fuzzy semi-continuous maps and fuzzy Sgw-continuous maps are independent as seen from the following examples.

2.8 Example: Let X= Y={a, b, c,d} and the functions $\alpha, \beta, \gamma, \delta$: X [0, 1] be defined as

α (x) = 1 if x = a
otherwise
β(x) = 1 if x = b
otherwise
γ(x) = 1 if x = a,b
otherwise
δ(x) = 1 if x = a,b,c
otherwise.

Consider T={1,0,$\alpha,\beta,\gamma,\delta$}, σ = {1,0,α}.Let map f: X Y defined by f(a)=b,f(b)=b,f(c)=a,f(d)=c, then f is fuzzy Sgw-continuous map but it is not fuzzy semi continuous map as the inverse image of fuzzy set α^c in (Y, σ) is

μ (x) = 1 if x = a,b,d
otherwise.
This is not a fuzzy semi closed set in fts X.

2.9 Example
Let X= Y={a, b, c} and the functions α, β,: X [0, 1] be defined as

α (x) = 1 if x = a

otherwise

β(x) = 1 if x = b,c

otherwise

Consider T= {1,0,α,β}, σ = {1,0,α}.Let map f: X Y defined by f(a)=b,f(b)=a,f(c)=c.

Then f is fuzzy semi-continuous map but f is not fuzzy Sgw continuous map as the inverse image of fuzzy set α^c in (Y, σ) is

μ (x) = 1 if x = a,c

otherwise.

This is not fuzzy Sgw-closed set in fts X.

3. Remark: fuzzy rw-continuous maps and fuzzy Sgw-continuous maps are independent as seen from the following examples.

3.1 Example: Let X= {a, b, c,d},Y={a,b,c} and the functions α, β, γ, δ: X [0, 1] be defined as

α (x) = 1 if x = a
0 otherwise
β(x) = 1 if x = b
otherwise
γ(x) = 1 if x = a,b
otherwise.
δ(x) = 1 if x = a,b,c
otherwise

Consider T= {1,0,α,β, γ, δ},σ= {1,0,α}.Let map f:X Y defined by f(a)=b,f(b)=a,f(c)=a,f(d)=c, then f is fuzzy Sgw-continuous map but it is not fuzzy rw continuous map as the inverse image of fuzzy set α^c in (Y, σ) is
μ (x) = 1 if x = a,d
otherwise.

This is not fuzzy rw-closed set in fts X.

3.2 Example: Let X= Y={a, b, c} and the functions α, β: X [0, 1] be defined as

α (x) = 1 if x = a
otherwise

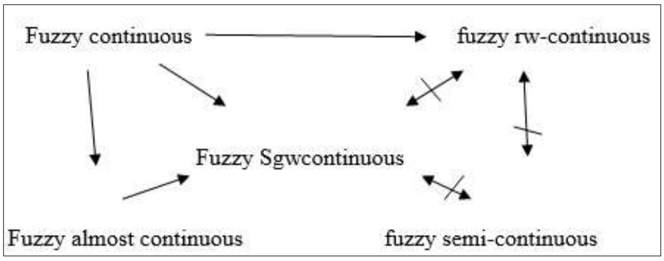
β(x) = 1 if x = b,c
otherwise

Consider T= {1,0,α,β}, σ = {1,0,α}.Let map f: X Y defined by f(a)=b,f(b)=a,f(c)=c.

Then f is fuzzy rw-continuous map but f is not fuzzy Sgw continuous map as the inverse image of fuzzy set α^c in (Y, σ) is
μ (x) = 1 if x = a,c
otherwise.

This is not fuzzy Sgw-closed set in fts X.

3.3 Remark: From the above discussion and known results we have the following implication



3.4 Theorem: If a function f: (X,T) (Y, σ) is fuzzy Sgw-continuous map and fuzzy completely continuous mapthen it is fuzzy continuous map.

Proof: Let a function f: (X,T) (Y, σ) be a fuzzy Sgw-continuous map and fuzzy completely semi-continuous map. Let μ be a fuzzy closed set in fts Y. Then $f^{-1}(\mu)$ is both fuzzy closed set in ftsY.Then$f^{-1}(\mu)$ is both fuzzy rgα-open and fuzzy Sgw-closed set in fts X.If a fuzzy set α of fts X is both fuzzy rgα and fuzzy Sgw-closed then α is a fuzzy closed in fts X thus $f^{-1}(\mu)$ is a fuzzy closed set in fts X.Therefore f is fuzzy continuous map.

3.5 Theorem: If f:(X,T) (Y, σ) is fuzzy Sgw-continuous map and g:(Y, σ) (Z,ρ) is fuzzy continuous map, then their composition gof:(X, T) (Z,ρ) is fuzzy Sgw-continuous map.

Proof: Let μ be a fuzzy open set in fts Z. Since g is fuzzy continuous map, $g^{-1}(\mu)$ is fuzzy open set in fts Y. Since f is fuzzy Sgw-continuousmap,$f^{-1}(g^{-1}(\mu))$ is a fuzzy Sgw-open set in fts X.
But $(gof)^{-1}(\mu)= f^{-1}(g^{-1}(\mu))$ thus gof is fuzzy Sgw-continuous map.

3.6 Definition: Let X and Y be fts. A map f: X Y is said to be a fuzzy Sgw-irresoulute map if the inverse image of every fuzzy Sgw-open in Y is a fuzzy Sgw-open set in X.

3.7 Theorem: If a map f: X Y is fuzzy Sgw- irresolute map then it is fuzzy Sgw-continuous map.

Proof: Let β be a fuzzy open set in Y. since every fuzzy open Set is fuzzy Sgw-open, β is a
Fuzzy Sgw-open set in Y. Since f is fuzzy Sgw-irresolutemap,$f^{-1}(\beta)$ is fuzzy Sgw-open in x. Thus f is fuzzy Sgw-continuous map.

The converse of the above theorem need not be true in general as seen from the following example.

3.8 Example: Let X={a, b, c,d}, Y={a,b,c} the functions α, β, γ, δ: X [0, 1] be defined as
α (x) = 1 if x = a
0 otherwise
β(x) = 1 if x = b
0 therwise
γ(x) = 1 if x = a,b
0 otherwise
δ(x) = 1 if x = a,b,c
0 otherwise

Consider T= {1,0,α,β, γ, δ},σ = {1,0,μ}.Let map f: X Y defined by f(a)=b,f(b)=a,f(c)=a,f(d)=c, then f is fuzzy Sgw-continuous map but it is not fuzzy Sgw-irresolutemap. Since for the fuzzy Sgw-closed set μ: Y [0,1] defined by μ (x) = 1 if x = b

otherwise in Y

$f^{-1}(\mu) = \alpha$ is not fuzzy Sgw-closed in (X,T).

3.9 Theorem: Let X,Y,Z be fts. If f: X Y is fuzzy Sgw-irresolute map and g:Y Z is fuzzy Sgw-continuous map then their composition gof:X Z is fuzzy Sgw-continuos map.

Proof: Let α be any fuzzy open set in fts Z, Since g is fuzzy Sgw-continuous map,($g^{-1}(\alpha)$) is a fuzzy Sgw-irresolute map$f^{-1}((g^{-1}(\alpha))$ is a fuzzy Sgw-open set in fts X
But $(gof)^{-1}(\alpha) = f^{-1}(g^{-1}(\alpha))$.Thus gof is fuzzy Sgw-continuous map.

3.10 Theorem: Let X,Y,Z be fts and f: X Y and g:Y Z be fuzzy Sgw-irresolute maps then their composition maps then their composition gof: X Z is fuzzy Sgw-irresoulte map.

Proof: Let α be a fuzzy Sgw-open set in fts Z. since g is fuzzy Sgw-irresolute map,$g^{-1}(\alpha)$ is a fuzzy Sgw-open set in fts Y. since f is fuzzy Sgw-irresolute map, $f^{-1}(g^{-1}(\alpha))$ is a fuzzy Sgw-open set in fts X. But $(gof)^{-1}(\alpha) = f^{-1}(g^{-1}(\alpha))$. Thus gof is fuzzy Sgw-irresolute map.

4. Conclusion
In this paper, a new class of maps called Fuzzy Sgw-Continuous maps and Fuzzy Sgw-irresolute in fuzzy topological spaces are introduced and investigated. In future, the same process will be analyzed for Fuzzy Sgw-properties.

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