Modelling long memory in volatility for weekly jute prices in the Malda district, West Bengal

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Abstract
In this study, the presence of long memory in the volatility process of weekly jute prices in the Samsi and Gajol markets of the Malda district (West Bengal) for the period of January 2009 to December 2022 has been investigated. For this objective, the ARCH-LM test and Hurst rescaled range (R/S) analysis are used to determine the ARCH effect and long memory in the volatility process for the series, and the results indicate the presence of the ARCH effect and long memory in conditional variance. Accordingly, the GARCH and FIGARCH models have been applied for modelling and forecasting the volatility of the jute prices. The wavelet method has been used to estimate the fractional difference parameter in the FIGARCH model. The AR (1)-GARCH (1, 1) and AR (1)-FIGARCH (1,0,270,1) models for the Samsi market and the AR (1)-GARCH (1,1) and AR (1)-FIGARCH (2,0,284,1) models for the Gajol market are found suitable at the training stage based on their minimum AIC and BIC. The forecasting performance of these models was evaluated in the validation period with the help of Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Mean Absolute Percentage Error (MAPE) criteria, and the residuals were examined to ensure that the fitted models were adequate. Finally, the AR (1)-FIGARCH (1,0,270,1) and AR (1)-FIGARCH (2,0,284,1) are found to be the best optimal models for forecasting the jute prices in the Samsi and Gajol markets, respectively.

Keywords: Jute prices, volatility, forecasting, GARCH model, FIGARCH model

1. Introduction
Jute (Corchorus capsularis L.) known as the “Golden Fiber” is one of the major commercial cash crops grown in India. It is the cheapest and strongest of the natural fibers and is considered the fiber of the future. After cotton, it is the most important fiber crop grown in India. India is the world's largest producer of raw jute, accounting for more than half of global jute production. Among the states, West Bengal ranks first in area and production of jute in the country with a total area of 0.52 million hectares (78.24%) and a total production of 7.61 million bales (79.68%) with a 2643 kg/hectare productivity during the years 2020–21. (Directorate of Economics & Statistics, DA&FW). In recent years modelling and forecasting of volatility concerning the long memory is an emerging area of scientific research. Volatility refers to changes in economic variables across time. The focus of this study is on changes in agricultural prices throughout time. Large price movements that do not reflect market fundamentals are problematic as they can lead to wrong decisions. Implied volatility reflects the price movements expected by market participants and is measured as the percentage of futures price deviation from the underlying expected value of the selected commodity. In contrast, the GARCH (Bollerslev, 1986) \(^\text{[2]}\) class of processes has a short memory feature and imposes substantially quicker exponential decay rates for the lagged squared residuals. (Baillie et al., 1996) \(^\text{[1]}\). The FIGARCH model, for instance, allows only a slow hyperbolic rate of decay for lagged squared or absolute innovations in the conditional variance function. (Tayefi and Ramanathan, 2012) \(^\text{[14]}\). In particular, the fractionally integrated GARCH model (FIGARCH) can be used for time series that display long memory in conditional variances. Several studies have been conducted in the country for modelling time series in the presence of volatility: Shireesha et al. (2016) \(^\text{[13]}\) investigated the price volatility of turmeric in the country’s major markets as well as future prices using ARCH-GARCH analysis, and they observed persistent volatility in market price series and futures prices. Khatkar et al. (2013) \(^\text{[15]}\),
Paul et al. (2015) [10], and Devi Bhavani et al. (2015) [3] conducted similar studies on commodities markets. Paul et al. (2016) [11] investigated the presence of long-memory features in the (log and squared log) return series and volatility of the daily spot price of gram in the Delhi market using GARCH, FIGARCH and several extension of GARCH. The study revealed that the FIGARCH model has better predictiveness accuracy compared to all other models. Similar works have been done by Paul et al. (2015) [12] and Lama et al. (2020) [8].

2. Materials and Methods

2.1 Data Description

In order to carry out our analysis, historical weekly jute price data for Samsi and Gajol markets of Malda district has been taken from the Agricultural Marketing Information Network (https://agmarknet.gov.in) portal for the periods of January, 2009 to December, 2022 (672 weeks) and January, 2010 to December, 2022 (624 weeks), respectively. In the present study, statistical analyses have been carried out using the powerful software “RStudio” (https://www.rstudio.com).

2.2 Methodology

2.2.1 ARCH-LM Test

A methodology to test for the ARCH effect in univariate time series models using the Lagrange multiplier test was proposed by Engle (1982) [4]. Let \( \varepsilon_t \) be the residual series. Using LM test the squared series \( \{ \varepsilon_t^2 \} \) is examined for presence of conditional heteroscedasticity, vis-à-vis the ARCH effects. The test is equivalent to usual F-statistic for testing null hypothesis \( H_0: \alpha_i = 0, i = 1,2, ..., q \); i.e., absence of ARCH effect against alternative hypothesis \( H_1: \) at least one of the estimated \( \alpha_i \) coefficients must be significant i.e. presence of ARCH effect in the linear regression:

\[
\varepsilon_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + a_2 \varepsilon_{t-2}^2 + ... + a_q \varepsilon_{t-q}^2 + \varepsilon_t; \quad t = q + 1, ..., T
\]

Where, \( \varepsilon_t \) denotes error term, \( q \) is pre-specified positive integer and \( T \) is sample size. ARCH-LM statistic has an asymptotic distribution similar to that of a chi-squared random variable with \( q \) degrees of freedom.

2.2.2 Long Memory process

The concept of long memory, or long-range dependence, is an important one in time series analysis. A long memory feature occurs when the autocovariances for a stationary time series tend to zero like a power function but more slowly than an exponential decay. In this study, the Hurst exponent method (Hurst, 1951) [5] has been used to test the long memory. Hurst’s rescaled range (R/S) statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. The least square method is applied to these values and a regression is run, obtaining an estimate of the slope of the regression line. This estimate is a measure of the Hurst exponent, which is an indicator of market persistence. The value of Hurst exponent \( (H) \) varies from 0 to 1. If \( 0.5 < H < 1 \), it implies the presence of persistent long memory in the time series. For estimating the long memory parameter \( (d) \) of FIGARCH model, the algorithm based on wavelet (Jensen, 1999) [6] is followed.

2.2.3 GARCH Model

The ARCH \( (q) \) model for the real-valued discrete-time stochastic process \( \{ \varepsilon_t \} \) is defined by specifying the conditional distribution of \( \varepsilon_t \) given the information \( \Psi_{t-1} \) available up to time \( t - 1 \), whenever

\[
\varepsilon_t | \Psi_{t-1} \sim N(0, h_t) \quad \text{and} \quad \varepsilon_t = \xi_t \sqrt{h_t}
\]

\[
h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}
\]

Where is \( \{ \xi_t \} \) a white noise process i.e., \( \xi_t \sim \text{IID} (0,1) \). Where \( \omega > 0, \alpha_i \geq 0 \) for all \( i \) and \( \sum_{i=1}^{q} \alpha_i < 1 \) are required to be satisfied to ensure non-negativity and finite unconditional variance of stationary \( \{ \varepsilon_t \} \) series. Thus, by definition, the \( \{ \varepsilon_t \} \) process is serially uncorrelated with mean zero, but the conditional variance of the process, \( h_t \), is changing over time. Bollerslev (1986) [2] and Taylor (1986) [13] proposed the Generalized ARCH (GARCH) model independently of each other, in which conditional variance is also a linear function of its own lags and has the following form.

2.2.4 FIGARCH Model

Baijli et al. (1996) [11] proposed the FIGARCH \( (p, d, q) \) model as one way of modelling long memory in volatility. They develop the FIGARCH \( (p, d, q) \) model by allowing the differencing parameter in the IGARCH \( (p, q) \) model to take non-integer values as follows

\[
\phi(L)(1 - L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)] \eta_t
\]

Where \( d \) is a fraction \( 0 < d < 1 \), \( \eta_t = \varepsilon_t^2 - h_t \) and all the roots of \( \phi(L) = \frac{1}{(1 - \alpha(L) - \beta(L))(1 - L)^{-1}} \) of order \( m - 1 \) and \( 1 - \beta(L) \) lie outside the unit circle. An overall check of model adequacy is provided by the Ljung-Box Q statistic (1978) [9] which tells whether residuals follow a white noise process.

2.2.5 Information criteria and accuracy measures

In this paper, we used two widely applied criterion Akaike information criterion (AIC) and the Bayesian information criterion (BIC) to select the best model among a set of candidate models.

Akaike’s Information criteria (AIC) = \( n \ln(SSE) - n \ln(n) + 2(k + 1) \)

Bayesian Information criteria (BIC) = \( n \ln(SSE) - n \ln(n) + (k + 1) \ln(n) \)

Where, \( n \) is sample size and \( k \) is number of predictor terms so \((k + 1)\) is total number of parameters in the model being
evaluated. The model with the lowest AIC and BIC values are treated as the best model. Furthermore, the RMSE, MAE and MAPE are used as an accuracy measure to evaluate the performance of the models.

Root Mean Square Error (RMSE) = $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$

Mean Absolute Error (MAE) = $\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|

Mean Absolute Percentage Error (MAPE) = $100\frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|

Where, $y_i$ and $\hat{y}_i$ are the actual value and predicted value of response variable.

### 3. Results and Discussions

#### 3.1 Primary statistical analysis

The weekly series of jute prices for the Samsi and Gajol markets of West Bengal are shown in Figure 1(a-b), which depict a gradually up-and-down pattern. The descriptive statistics to summarize information from the weekly jute price data are listed in Table 1. As Table 1 shows, the series of jute prices in the Samsi market has a mean of 3657.36, a standard deviation of 1381.23, and a 37.77% coefficient of variation, while the Gajol market has a mean of 3873.07, a standard deviation of 1437.66, and a 37.12% coefficient of variation, suggesting presence of volatility. In addition, skewness and kurtosis statistics show that the price series is not normally distributed.

![Fig 1](image_url)

**Fig 1:** Weekly jute price series for the Samsi and Gajol markets, including all structural breaks and confidence intervals.

<table>
<thead>
<tr>
<th>Markets</th>
<th>No. of observations</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>CV (%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samsi Market</td>
<td>672</td>
<td>1650</td>
<td>7836</td>
<td>3657.36</td>
<td>3200</td>
<td>1381.23</td>
<td>37.77</td>
<td>0.95</td>
<td>3.25</td>
</tr>
<tr>
<td>Gajol Market</td>
<td>624</td>
<td>1760</td>
<td>8545.56</td>
<td>3873.07</td>
<td>3455.59</td>
<td>1437.66</td>
<td>37.12</td>
<td>0.97</td>
<td>3.26</td>
</tr>
</tbody>
</table>

To begin with the implementation of GARCH and FIGARCH models, the data series are divided into two sets: the training set and the testing set. In the first step, the models are fitted to the training data set consisting of 538 observations taken from the Samsi market and 499 observations from the Gajol market. Thereafter, the said models are validated using the test data sets consisting of the last 134 and 125 observations for the Samsi and Gajol markets, respectively.

#### 3.2 Test for volatility

The first step in implementing the GARCH and FIGARCH models is to determine whether or not the time series is volatile. In order to test for volatility, the AR(p) model is first fitted to the jute price series data of the two markets. AR(1) models are judged to be acceptable for the data under consideration based on the partial autocorrelation function and AIC, BIC, MAE, RMSE, and MAPE values. In the next step, the residuals of the fitted AR (1) models are obtained and graphically depicted in Figure 2(a-b), which explain the phenomenon of volatility clustering. In other words, a large fluctuation is likely to follow a previous large fluctuation, whereas a small fluctuation is likely to follow a previous small fluctuation.
Again, the presence of the ARCH effect was investigated for both the residuals and squared residuals series. The results of the ARCH-LM test are shown in Table 2, where p-values for all series are less than 5%, indicating that the null hypothesis of constant variance is rejected. Rejecting $H_0$ implies that the ARCH effect exists in the residuals and squared residuals series.

Table 2: ARCH-LM test for residuals and squared residuals

<table>
<thead>
<tr>
<th>Data series</th>
<th>Chi-squared</th>
<th>Df</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samsi residuals</td>
<td>37.134</td>
<td>12</td>
<td>0.002125</td>
</tr>
<tr>
<td>Samsi squared residuals</td>
<td>476</td>
<td>12</td>
<td>0.00</td>
</tr>
<tr>
<td>Gajol residuals</td>
<td>112.46</td>
<td>12</td>
<td>&lt; 2.2e-16</td>
</tr>
<tr>
<td>Gajol Squared residuals</td>
<td>46.447</td>
<td>12</td>
<td>5.81e-06</td>
</tr>
</tbody>
</table>

3.3 Test for long memory and estimation

The presence of long memory in both the residuals and squared residuals series is confirmed by investigating the autocorrelation function (ACF) plot. The results, shown in Figures 3-4 indicate that the autocorrelation functions of the residuals are small and the ACF plots have no particular form. Most of the autocorrelation values stay inside the 95% confidence intervals. This is suggestive of their short memory property. However, the autocorrelation functions of the squared residuals, on the other hand, are greater and remain significant over many lags. More crucially, they decay slowly, indicating that the time series are highly autocorrelated up to a considerable lag.

Fig 2: Residuals of fitted AR (1) models a. in the Samsi market b. in the Gajol market

Fig 3: ACF and PACF plots of squared residuals for weekly jute prices in the Samsi market

Fig 4: ACF and PACF plots of squared residuals for weekly jute prices in the Gajol market
Accordingly, the presence of long memory in conditional variance is tested as discussed in Section 2.2.2, and it is found that the R/S Hurst values ($H$) of squared residuals for Samsi and Gajol markets are 0.595 and 0.588, respectively (Table 3) (higher than 0.5), which firmly conclude the existence of the long memory characteristic in volatility. Models using the long memory property are very sensitive to the estimation of the long-memory parameter $d$ (i.e., the fractional differencing parameter). Hence, it has been estimated by using the wavelet-based ordinary least squares estimator and presented in Table 3.

Table 3: Long memory tests for squared residuals series

<table>
<thead>
<tr>
<th>Data series</th>
<th>R/S Hurst Method ($H$)</th>
<th>Long memory parameter ($d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squared residuals (Samsi)</td>
<td>0.595 ($H &gt; 0.5$)</td>
<td>0.270</td>
</tr>
<tr>
<td>Squared residuals (Gajol)</td>
<td>0.588 ($H &gt; 0.5$)</td>
<td>0.284</td>
</tr>
</tbody>
</table>

3.4 Model Identification

Once the presence of volatility and long memory in the volatility are confirmed in the data, we proceed with our analysis to further estimate the parameters of both of the conditional mean and conditional variance equations. For this purpose, we employed the GARCH and FIGARCH models. To establish GARCH ($p, q$) and FIGARCH ($p, d, q$) specifications with different combinations of $p$ and $q$, where $p$ and $q$ are either 1 or 2, which are listed in Table 4.

Table 4: AIC and BIC values of the AR (1)-GARCH($p,q$) and AR(1)-FIGARCH($p,d,q$) models

3.5 Parameter estimation

The estimated parameters of the selected models at the end of the training period are reported in Table 5. A perusal of Table 5 indicates that all the parameters are statistically significant. For the volatility component, the long memory parameter $d$ (i.e., d-Figarch) is also significant at the 5% significance level, indicating the presence of long-range memory phenomenon for volatilities. The price volatility was captured through AR(1)-GARCH analysis, i.e., the sum of $\alpha$ and $\beta$ coefficients ($\alpha + \beta$) in GARCH models close to one indicates the persistence of volatility in the markets. These results confirmed that there is persistent volatility in jute prices in the Gajol market (0.99), whereas no persistence volatility was found in the Samsi market (0.510) (Table 5). The conditional variance of the best fitted GARCH and FIGARCH models, depicted in Figures 5-6, indicates that conditional variance is very much time dependent.

Table 5: Parameter estimates

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As shown in Table 6, comparing the validation results of all the models indicates that all are likely to perform well in the forecasting phase, and it is observed that the AR(1)-FIGARCH (1, 0.270, 1) and AR (1)-FIGARCH (2, 0.284, 1) models produce the lowest MAE, RMSE, and MAPE for Samsi and Gajol markets, respectively. It can be concluded that the FIGARCH models are the most accurate compared to the GARCH model in the presence of long memory in volatility, where predictions indicate that there are narrow differences between the actual and predicted values of jute prices (Figures 7-8). Finally, two models i.e. AR (1)-FIGARCH (1, 0.270, 1) and AR (1)-FIGARCH (2, 0.284, 1) are found to forecast accurately the weekly jute prices in the Samsi and Gajol markets of West Bengal, respectively.

3.6 Validation and Diagnostic checking
The model validation process is concerned with examining residuals obtained from fitted models to see if they contain any systematic pattern that could still be removed to improve the chosen models. This has been done through the Ljung-Box diagnostic test, and it is found that the all p-values of the Ljung-Box test are more than 5% (Table 6), which means that the model residual meets the assumption of white noise residuals. The evaluation of forecasting performance has been done for the test set as an out of-sample period of the last 134 and 125 observations for the Samsi and Gajol markets, respectively. Table 6 represents the results of the models based on the three different accuracy performance measures: MAE, RMSE, and MAPE.

<table>
<thead>
<tr>
<th>Method</th>
<th>MAE</th>
<th>RMSE</th>
<th>MAPE</th>
<th>Ljung-Box test</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)-GARCH (1, 1)</td>
<td>104.488</td>
<td>199.579</td>
<td>1.830</td>
<td>0.538 [0.463]</td>
</tr>
<tr>
<td>AR (1)-FIGARCH (1, 0.270, 1)</td>
<td>104.011</td>
<td>199.711</td>
<td>1.824</td>
<td>0.253 [0.615]</td>
</tr>
<tr>
<td>AR(1)-FIGARCH (2, 0.284, 1)</td>
<td>152.220</td>
<td>250.633</td>
<td>2.428</td>
<td>2.575 [0.109]</td>
</tr>
<tr>
<td>AR (1)-FIGARCH (2, 0.284, 1)</td>
<td>147.110</td>
<td>251.128</td>
<td>2.354</td>
<td>0.351 [0.553]</td>
</tr>
</tbody>
</table>

p-values of the Ljung & Box statistics are reported between square brackets.

4. Conclusion and Future scope
The aim of this paper is to introduce an appropriate model for modelling and forecasting the volatility of weekly jute prices in the Samsi and Gajol markets of the Malda district (West Bengal). For this purpose, the ARCH-LM test and Hurst rescaled range (R/S) analysis are employed to identify the ARCH effect and long memory in volatility of the series. The presence of the ARCH effect and long memory in conditional variance found for jute prices indicates that it would be better to develop and employ the GARCH and FIGARCH models. We considered wavelet methods for estimating the fractional...
difference parameter. The AR-GARCH and AR-FIGARCH models are fitted to the jute price series, and the tentative models selected at the training stage are the AR (1)-GARCH (1,1) and AR (1)-FIGARCH (1,0.270,1) models for the Samsi market and the AR (1)-GARCH (1,1) and AR (1)-FIGARCH (2,0.284,1) models for the Gajol market, based on the minimum AIC and BIC values. Based on forecasting performances, a comparative study has been made between the tentatively selected models, and finally, the AR (1)-FIGARCH (1,0.270,1) and AR (1)-FIGARCH (2,0.284,1) are found to be the best optimal models to forecast the jute prices for the Samsi and Gajol markets, respectively. The model demonstrated good performance in terms of explained variability and predicting power. The study has revealed that the FIGARCH model is efficient for capturing and forecasting volatility phenomenon, especially for data with the long memory property in conditional variance.

5. References