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# Approximating the solution of a nonlinear delay integral equation by an efficient iterative algorithm in hyperbolic spaces

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**Abstract**
This paper uses an efficient iterative method to approximate the solution of a nonlinear delay integral equation in hyperbolic spaces. A numerical example is given to support our main results. Our result improves several existing iterative methods in the literature.

**Keywords:** Hyperbolic space, strong convergence and nonlinear integral equation

## 1. Introduction

Fixed point theory is concerned with some properties which ensure that a self-map $M$ defined on a set $B$ admits at least one fixed point. By fixed point of $M$, we mean a point $w \in B$ which solves an operator equation $w = Mw$, known as fixed point equation. Now, let $F(M) = \{w \in B : w = Mw\}$ stand for the set of all fixed points of $M$. The theory of fixed point plays significant role in finding the solutions of problems which arise in different branches of mathematical analysis. For some years now, the advancement of fixed point theory in metric spaces has captured considerable interests from many authors as a result of its applications in many fields such variational inequality, approximation theory and optimization theory.

Banach Contraction principle still remains one of the fundamental theorems in analysis. It states that if $(B,d)$ is a complete metric space and $M : B \to B$ fulfills

$$d(Mf, Mh) \leq ed(f,h) \tag{1}$$

For all $g,h \in B$ with $e \in [0,1)$, then there exists a unique fixed point of $M$. Mappings satisfying (1.1) are known as contraction mappings.

In this work, we will consider the following $A^*$ iterative method introduced in [6]:

$$\begin{cases} g_1 \in W, \\ j_m = K((1-\zeta_m)g_m + \zeta_m K g_m), \\ i_m = K((1-\vartheta_m)j_m + \vartheta_m K j_m), & m \in N, \\ h_m = K i_m, \\ g_{m+1} = K h_m, \end{cases} \tag{2}$$

where $\{\zeta_m\}$ and $\{\vartheta_m\}$ are sequences in $(0,1)$. The authors showed that $A^*$ iterative algorithm has a better rate of convergence than most leading iterative algorithms for almost contraction mappings and Suzuki generalized non-expansive mappings.

In recent years, several methods have been developed to solve nonlinear integral equations, see for example [3-6, 9, 7, 8].
Delay integral equations play significant role in mathematical science and engineering. A large classof initial and boundary valued problems can be transformed into Volterra integral equations. These equations are applied in mathematical physics models such as electric circuits, conformal mapping, scattering in quantum mechanics, diffraction problems, electromagnetic scattering problems, propagation of elastical and acoustical waves (see [10] and the references therein).

Let the interval $I=[c,d](c<d)$ be fixed. We will consider the space $C(I)$ of all continuous functions endowed with the metric

$$d(g,h)=\sup_{z\in I}\frac{|g(z)-h(z)|}{\Gamma(z)}$$

Where $\Gamma : I \to (0,\infty)$ is a non-decreasing continuous function and $g,h\in C(I)$ It is known that $(C(I),d)$ is a complete metric space and hence a hyperbolic space [1].
In this article, we consider the following delay nonlinear Volterra integral equation:

$$g(z)=f(z)+\varphi\left(\int_c^z p(z,\eta,g(\eta),g(\overline{w}(\eta)))d\eta\right), \tag{3}$$

Where $z,\eta\in I,\varphi : C(I)\to C(I)$ is a bounded function, $f:I\to C$ and $p:I\times I\times C\times C\to C$ arecontinuous functions and $\overline{w}:I\to I$ is a continuous function with $\overline{w}(z)\le z$, for all $z\in I$. a
We assume that the following conditions hold:
(C1) Let $f:I\to C$ and $\overline{w}:I\to I$ be continuous functions such that $\overline{w}(z)\le z,$ for all $z\in I.$
(C2) If the functions $\delta,\gamma : I\times I\times\to[0,\infty)$ are continuous and satisfy

$$\int_c^z \delta(z,\eta)\Gamma(\eta)dz\le M_1\Gamma(z)$$

And

$$\int_c^z \gamma(z,\eta)\Gamma(\eta)dz\le M_2\Gamma(z)$$

$M_1,M_2\in[0,1)$. Then the continuous function $p:I\times I\times C\times C\to C$ satisfies

$$|p(z,\eta,g(\overline{w}(\eta))-p(z,\eta,h(\eta),h(\overline{w}(\eta))|$$

$$\delta(z,\eta)|g(\eta)-h(\eta)|+\gamma(z,\eta)|g(\overline{w}(\eta))-h(\overline{w}(\eta))|,$$

(C3) The function $\varphi : C(I)\to C$ is bounded such that if there exists $\lambda>0,$ then we have

$d(\varphi(p),\varphi(p_2))\le\lambda d(p_1,p_2)$. (C4) $\lambda(M_1+M_2<1)$.

In [2], Castro and Guerra showed that under the assumptions (C1)–(C4), the problem (1.3) possess a unique solution in C(1).
Our aim in this section is to approximate the unique solution of the delay nonlinear Volterra integralequation via our efficient iterative algorithm (1.2). Our main result in this section is given in the

**Following theorem**

**Theorem 1.1.** *Suppose all the conditions (C1)–(C4) are satisfied. Then the problem (1.3) has a uniquesolution* $l\in C(I)$ *and the sequence* $\{g_m\}$ *defined by (1.2) converges to* $l$.

**Proof**. Suppose $\{g_m\}$ is a sequence defined by (1.2). Define an operator $K:C(I)\to C(I)$ by

$$Kg(z) = f(z) + \varphi\left(\int_c^z p(z,\eta,g(\eta),g(\overline{w}(\eta)))d\eta\right), \tag{4}$$

Where $z,\eta \in I$ and $g \in C(I)$. We now show that $g_m \to l$ as $m \to \infty$.

From (1.2), setting $u_m = ((1-\zeta_m)g_m \oplus \zeta_m Kg_m, l)$ and $s_m = (1-\vartheta_m)j_m \oplus \vartheta_m Kj_m$, we have

$$u_m = d((1-\zeta_m)g_m \oplus \zeta_m Kg_m, l)$$

$$\leq (1-\zeta_m)d(g_m,l) + \zeta_m d(Kg_m,l)$$

$$= (1-\zeta_m)d(g_m,l) + \zeta_m d(Kg_m,Kl)$$

$$= (1-\zeta_m)d(g_m,l) + \zeta_m \sup_{z\in I}\frac{|Kg_m(z) - Kl(z)|}{\Gamma(z)}$$

$$= (1-\zeta_m)d(g_m,l) + \zeta_m$$

$$\times \sup_{z\in I}\frac{\left|\varphi\left(\int_c^z p(z,\eta,g_m(\eta),g_m(\overline{w}(\eta)))d\eta\right) - \varphi\left(\int_c^z p(z,\eta,l(\eta),l(\overline{w}(\eta)))d\eta\right)\right|}{\Gamma(z)}$$

$$\leq (1-\zeta_m)d(g_m,l) + \zeta_m \times$$

$$\lambda \sup_{z\in I}\frac{\left|\int_c^z p(z,\eta,g_m(\eta),g_m(\overline{w}(\eta)))d\eta - \int_c^z p(z,\eta,l(\eta),l(\overline{w}(\eta)))d\eta\right|}{\Gamma(z)} \tag{1.5}$$

$$\leq (1-\zeta_m)d(g_m,l) + \zeta_m \times$$

$$\lambda \sup_{z\in I}\frac{\left|\int_c^z p(z,\eta,g_m(\eta),g_m(\overline{w}(\eta)))d\eta - p(z,\eta,l(\eta),l(\overline{w}(\eta)))\right|d\eta}{\Gamma(z)}$$

$$= (1-\zeta_m)d(g_m,l) + \zeta_m \times$$

$$\lambda \sup_{z\in I}\frac{\left|\int_c^z \delta(z,\eta)\Gamma(\eta)\frac{|g_m(\eta)-l(\eta)|}{\Gamma(z)}d\eta + \int_c^z \gamma(z,\eta,\Gamma(\overline{w}(\eta)))\frac{|g_m(\overline{w}(\eta)) - l(\overline{w}(\eta))|}{\Gamma(\overline{w}(\eta))}\right|d\eta}{\Gamma(z)}$$

$$= (1-\zeta_m)d(g_m,l) + \zeta_m \times$$

$$\lambda\left[\sup_{z\in I}\frac{|g_m(\eta)-l(\eta)|}{\Gamma(z)}\sup_{z\in I}\frac{\int_c^z \delta(z,\eta)\Gamma(z)d\eta}{\Gamma(z)}\right.$$

$$\left.+\sup_{z\in I}\frac{|g_m(\overline{w}(\eta)) - l(\overline{w}(\eta))|}{\Gamma(\overline{w}(\eta))}\sup_{z\in I}\frac{\left|\int_c^z \gamma(z,\eta,\Gamma(\overline{w}(\eta))\right|}{\Gamma(\overline{w}(z))}d\eta\right]$$

$$\leq (1-\zeta_m)d(g_m,l) + \zeta_m\lambda[d(g_m,l).M_1 + d(g_m,l).M_2]$$

$$= (1-\zeta_m)d(g_m,l) + \zeta_m\lambda(M_1 + .M_2)d(g_m,l). \tag{1.6}$$

Since $0 < \zeta_m < 1$ and by assumption (C4) we have $\lambda(M_1 + M_2) < 1$. So, (1.6) yields

$$d(u_m,l) \leq d(g_m,l). \tag{1.7}$$

Also,

$$d(S_m,l)=d(1-\vartheta_m)j_m\oplus\vartheta_m(Kj_m,l)$$

$$\le(1-\vartheta_m)d(j_m,l)+\vartheta_m d(Kj_m,l)$$

$$=(1-\vartheta_m)d(j_m,l)+\vartheta_m d(Kj_m,Kl)$$

$$=(1-\vartheta_m)d(j_m,l)+\vartheta_m\sup_{z\in I}\frac{|Kj_m(z)-Kl(z)|}{\Gamma(z)}$$

$$=(1-\vartheta_m)d(j_m,l)+\vartheta_m\times$$

$$\sup_{z\in I}\frac{\left|\varphi\left(\int_c^z p(z,\eta,j_m(\eta),j_m(\overline{w}(\eta)))d\eta\right)-\varphi\left(\int_c^z p(z,\eta,l(\eta),l(\overline{w}(\eta)))d\eta\right)\right|}{\Gamma(z)}$$

$$\le(1-\vartheta_m)d(j_m,l)+\vartheta_m\times$$

$$\lambda\sup_{z\in I}\frac{\left|\left(\int_c^z p(z,\eta,j_m(\eta),j_m(\overline{w}(\eta)))d\eta\right)-\left(\int_c^z p(z,\eta,l(\eta),l(\overline{w}(\eta)))d\eta\right)\right|}{\Gamma(z)}$$

$$\le(1-\vartheta_m)d(j_m,l)+\vartheta_m\times$$

$$\lambda\sup_{z\in I}\frac{\int_c^z|p(z,\eta,j_m(\eta),j_m(\overline{w}(\eta)))-p(z,\eta,l(\eta),l(\overline{w}(\eta)))|d\eta}{\Gamma(z)}$$

$$\le(1-\vartheta_m)d(j_m,l)+\vartheta_m\times$$

$$\lambda\sup_{z\in I}\frac{\int_c^z\left(\delta(z,\eta)|j_m(\eta)-l(\eta)|+\gamma(z,\eta)|j_m(\overline{w}(\eta))-l(\overline{w}(\eta))|\right)d\eta}{\Gamma(z)}$$

$$=(1-\vartheta_m)d(j_m,l)+\vartheta_m\times$$

$$\lambda\sup_{z\in I}\frac{\int_c^z\delta(z,\eta)\Gamma(z)(\eta)\frac{|j_m(\eta)-l(\eta)|}{\Gamma(\eta)}d\eta+\int_c^z\gamma(z,\eta)\Gamma(\overline{w}(\eta))\frac{|j_m(\overline{w}(\eta))-l(\overline{w}(\eta))|}{\Gamma(\overline{w}(\eta))}d\eta}{\Gamma(z)}$$

$$\le(1-\vartheta_m)d(j_m,l)+\vartheta_m\times$$

$$\lambda\left[\sup_{\eta\in I}\frac{|j_m(\eta)-l(\eta)|}{\Gamma(\eta)}\sup_{z\in I}\frac{\int_c^z\delta(z,\eta)\Gamma(\eta)d\eta}{\Gamma(z)}\right.$$

$$\left.+\sup_{\eta\in I}\frac{|j_m(\overline{w}(\eta))-l(\overline{w}(\eta))|}{\Gamma(\overline{w}(\eta))}\sup_{z\in I}\frac{\int_c^z\gamma(z,\eta)\Gamma(\overline{w}(\eta))}{\Gamma(\overline{w}(z))}d\eta\right]$$

$$\le(1-\vartheta_m)d(j_m,l)+\vartheta_m\lambda\left[d(j_m,l)\cdot M_1+d(j_m,l)\cdot M_2\right]$$

$$=(1-\vartheta_m)d(j_m,l)+\vartheta_m\lambda(M_1+M_2)d(j_m,l). \tag{8}$$

Since $0<\vartheta_m<1$ and by assumption (C4) we have $\lambda(M_1+M_2)<1$. so (1.8) yields

$$d(s_m, l) \le d(j_m, l). \tag{9}$$

From (1.2) and (1.7), we have

$$d(j_m, l) = d(Ku_m, l)$$

$$= d(Ku_m, Kl)$$

$$= \sup_{z\in I} \frac{|Ku_m(z) - Kl(z)|}{\Gamma(z)}$$

$$= \sup_{z\in I} \frac{\left|\varphi\left(\int_c^z p(z,\eta,u_m(\eta),u_m(\overline{w}(\eta)))d\eta\right) - \varphi\left(\int_c^z p(z,\eta,l(\eta),l(\overline{w}(\eta)))d\eta\right)\right|}{\Gamma(z)}$$

$$\le \lambda \sup_{z\in I} \frac{\left|\left(\int_c^z p(z,\eta,u_m(\eta),u_m(\overline{w}(\eta)))d\eta\right) - \left(\int_c^z p(z,\eta,l(\eta),l(\overline{w}(\eta)))d\eta\right)\right|}{\Gamma(z)}$$

$$\le \lambda \sup_{z\in I} \frac{\int_c^z |p(z,\eta,u_m(\eta),u_m(\overline{w}(\eta))) - p(z,\eta,l(\eta),l(\overline{w}(\eta)))|d\eta}{\Gamma(z)}$$

$$\le \lambda \sup_{z\in I} \frac{\int_c^z \left(\delta(z,\eta)|u_m(\eta) - l(\eta)| + \gamma(z,\eta)|u_m(\overline{w}(\eta)) - l(\overline{w}(\eta))|\right)d\eta}{\Gamma(z)}$$

$$= \lambda \sup_{z\in I} \frac{\int_c^z \delta(z,\eta)\Gamma(\eta)\frac{|u_m(\eta) - l(\eta)|}{\Gamma(\eta)}d\eta + \int_c^z \gamma(z,\eta)\Gamma(\overline{w}(\eta))\frac{|u_m(\overline{w}(\eta)) - l(\overline{w}(\eta))|}{\Gamma(\overline{w}(\eta))}d\eta}{\Gamma(z)}$$

$$\le \lambda\left[\sup_{\eta\in I}\frac{|u_m(\eta) - l(\eta)|}{\Gamma(\eta)}\sup_{z\in I}\frac{\int_c^z \delta(z,\eta)\Gamma(z)d\eta}{\Gamma(z)}\right.$$

$$\left. + \sup_{\eta\in I}\frac{|u_m(\overline{w}(\eta)) - l(\overline{w}(\eta))|}{\Gamma(\overline{w}(\eta))}\sup_{z\in I}\frac{\int_c^z \gamma(z,\eta)\overline{)\ }(\overline{w}(\eta))}{\Gamma(\overline{w}(z))}d\eta\right]$$

$$\le \lambda[d(u_m, l)\cdot M_1 + d(u_m, l)\cdot M_2]$$

$$= \lambda(M_1 + M_2)d(u_m, l).$$

$$\le d(u_m, l) \le d(g_m, l) \tag{10}$$

Using (1.2), (1.9) and (1.10), we have

$$d(i_m, l) = d(Ks_m, l)$$

$$= d(Ks_m, Kl)$$

$$= \sup_{z\in I} \frac{|Ks_m(z) - Kl(z)|}{\Gamma(z)}$$

$$= \sup_{z\in I} \frac{\left|\varphi\left(\int_c^z p(z,\eta,s_m(\eta),s_m(\overline{w}(\eta)))d\eta\right) - \varphi\left(\int_c^z p(z,\eta,l(\eta),l(\overline{w}(\eta)))d\eta\right)\right|}{\Gamma(z)}$$

$$\leq \lambda \sup_{z\in I} \frac{\left|\left(\int_c^z p(z,\eta,s_m(\eta),s_m(\overline{w}(\eta)))d\eta\right) - \left(\int_c^z p(z,\eta,l(\eta),l(\overline{w}(\eta)))d\eta\right)\right|}{\Gamma(z)}$$

$$\leq \lambda \sup_{z\in I} \frac{\int_c^z |p(z,\eta,s_m(\eta),s_m(\overline{w}(\eta))) - p(z,\eta,l(\eta),l(\overline{w}(\eta)))| d\eta}{\Gamma(z)}$$

$$\leq \lambda \sup_{z\in I} \frac{\int_c^z \left(\delta(z,\eta)|s_m(\eta) - l(\eta)| + \gamma(z,\eta)|s_m(\overline{w}(\eta)) - l(\overline{w}(\eta))|\right)d\eta}{\Gamma(z)}$$

$$= \lambda \sup_{z\in I} \frac{\int_c^z \delta(z,\eta)\Gamma(\eta)\frac{|s_m(\eta) - l(\eta)|}{\Gamma(\eta)}d\eta + \int_c^z \gamma(z,\eta)\Gamma(\overline{w}(\eta))\frac{|s_m(\overline{w}(\eta)) - l(\overline{w}(\eta))|}{\Gamma(\overline{w}(\eta))}d\eta}{\Gamma(z)}$$

$$\leq \lambda\left[\sup_{\eta\in I}\frac{|s_m(\eta) - l(\eta)|}{\Gamma(\eta)} \sup_{z\in I}\frac{\int_c^z \delta(z,\eta)\Gamma(z)d\eta}{\Gamma(z)}\right.$$

$$\left. + \sup_{\eta\in I}\frac{|s_m(\overline{w}(\eta)) - l(\overline{w}(\eta))|}{\Gamma(\overline{w}(\eta))} \sup_{z\in I}\frac{\int_c^z \gamma(z,\eta)\Gamma(\overline{w}(\eta))}{\Gamma(\overline{w}(z))}d\eta\right]$$

$$\leq \lambda[d(s_m,l)\cdot M_1 + d(s_m,l)\cdot M_2]$$

$$= \lambda(M_1 + M_2)d(s_m,l) \leq d(s_m,l)$$

$$\leq d(j_m,l) \leq d(g_m,l). \tag{12}$$

Also, by following similar arguments to those given above, we have the following inequalities:

$$d(h_m,l) \leq d(i_m,l) \leq d(g_m,l) \tag{13}$$

And

$$d(g_{m+1},l) \leq d(h_m,l) \leq d(g_m,l) \tag{14}$$

If we put $d(g_m,l) = \psi_m$, then (1.14) takes the form

$$\psi_{m+1} \leq \psi_m, \qquad \forall m \in N$$

Hence, $\{\psi_m\}$ is a monotone decreasing sequence of real numbers. Furthermore, it is a bounded sequence, so we have

$$\lim_{n\to\infty}\psi_m = \inf\{\psi_m\} = 0$$

**Example 1.2.** Consider the integral equation

$$g(z)=\frac{z}{3}+\frac{2}{3z}+\int_1^z \frac{1}{z}(g(\eta)+g(\overline{w}(\eta))d\eta), \qquad z\in I=[1,5] \tag{15}$$

where $\overline{w}(z)=\frac{\eta}{3},$ for all $\eta\in I$. Let $\Gamma:[1,5]\to(0,\infty)$ be defined by $\Gamma(z)=z^3$. clearly, the functions $f:I\to C$ and $\overline{w}:I\to I$ are continuous with $\overline{w}(\eta)\le\eta$ for all $\eta\in I$. Furthermore, the function $p:I\times I\times C\times C\to C$ is continuous such that

$$\left|p(z,\eta,g(\eta),g(\overline{w}(\eta))-p(z,\eta,h(\eta),h(\overline{w}(\eta))\right|$$

$$\le\left|\frac{1}{z}(g(\eta)+g(\overline{w}(\eta))-\frac{1}{z}(h(\eta)+h(\overline{w}(\eta))\right|$$

$$\le\frac{1}{z}|g(\eta)-h(\eta|)+\frac{1}{z}|g(\overline{w}(\eta)-h(\overline{w}(\eta))| \tag{16}$$

From (1.16), we can see that the functions $\delta,\gamma:I\times I\times\to[0,\infty)$ are defined by $\delta(z,\eta)=\gamma(z,\eta)=\frac{1}{z}.$.

Next, we sort for $M_1$ and $M_2$ as follows:

$$\int_1^z\delta(z,\eta)\Gamma(\eta)d\eta=\int_1^z\gamma(z,\eta)\Gamma(\eta)d\eta$$

$$=\int_1^z\frac{1}{z}\eta^3d\eta$$

$$=z^3\left(\frac{1}{4}-\frac{1}{4z^4}\right)$$

$$\le\frac{1}{4}\Gamma(z),$$ for all $z\in I.$

Therefore, $M_1=M_2=\frac{1}{4}$4. Notice that $d(\varphi(p_1)),\varphi(p_2)\le 1\cdot d(p_1,p_2)$ and so $\lambda=1.$ Thus, $\lambda(M_1+M_2)=1.\left(\frac{1}{4}+\frac{1}{4}\right)=\frac{1}{2}<1.$ It easy to check that the function $g(z)=z, z\in[1,5]$ is the unique solution of the problem (1.15). Thus, all the assumptions in Theorem 1.1 are satisfied. Hence, $\{g_m\}$ converges to the unique solution of (1.15).

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