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Infinite double integral representation for the polynomial set *Rn*(x, y)

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Abstract
In the present paper, an attempt has been made to express an Infinite Double Integral Representation for the polynomial set *Rn*(*x*, *y*). Many Interesting new results may be obtained as particular cases on separating the parameter, which may be useful for scientists, engineers and new technology.
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Keywords: Hypergeometric function, Appell function, integral representation

1. Introduction
We defined the polynomial set $R_n(x, y)$ by means of the generating relation.

$$(\xi + \lambda_1 y^{-m_1} t^{m_1})^{\sigma} F_1 \left[\begin{array}{l} (a_p); (b_u); b^1; \\ \quad \lambda_1 x^e t^e, \lambda_2 y^{-m_2} t^{m_2} \\ (c_q); \end{array} \right] = \sum_{n=0}^{\infty} R_{n,e;m_1;m_2;(c_q);}^{\lambda;\lambda_1;\lambda_2;\xi;\sigma;(a_p);(b_u);b'} (x,y) t^n \quad .$$

Where λ; λ_1; λ_2; σ; ξ; are real and m_1, m_2, and *e* are positive integers.

The left hand side of (1.1) contains generalized Appell's function of two variables [1] by notation of Burchnall and Chaundy [2].

2. Notations
A.
(i). $n = 1, 2, 3, \ldots\ldots n-1, n$.
(ii). $(a_p) = a_1, a_2, a_3, \ldots\ldots a_p$.
(iii). $[(a_p)] = a_1, a_2, \ldots\ldots a_p$.
(iv). $\left[(a_p)\right]_n = \prod_{i=1}^{p} (a_i)_n = (a_1)_n (a_2)_n (a_3)_n \ldots\ldots (a_p)_n$.

B.
(i) $\Delta(a;b) = \frac{b}{a}, \frac{b+1}{a}, \frac{b+2}{a}, \ldots\ldots \frac{b+a-1}{a}$.
(ii) $\Delta_k[a(i);b] = \frac{b}{a}, \frac{b+1}{a}, \ldots\ldots \frac{b+a-2}{a}$
(iii) $\Delta_k\left[m;(a_p)\right] = \left(\frac{a_i+r-1}{m}\right)_k \begin{array}{l} r = 1,2 \ldots\ldots m \\ i = 1,2, \ldots\ldots p \end{array}$
(iv) $\Delta(a; b \pm c \pm d) = \Delta(a; b + c + d)\, \Delta(a; b + c - d)$,
$\Delta(a; b - c + d)$, $\Delta(a; b - c - d)$.

C.
(i). (i)$\Delta_k[a,b] = \prod_{r=1}^{a} \left(\frac{b+r-1}{a}\right)_k = \left(\frac{b}{a}\right)_k, \left(\frac{b+1}{a}\right)_k \ldots\ldots \left(\frac{b+a-2}{a}\right)_k$
(ii). (ii) $\Delta_k\left[m;(a_p)\right] = \prod_{i=1}^{p} \prod_{r=1}^{m} \left(\frac{a;+r-1}{m}\right)_k$.

D.

(i). $\Gamma[(a_p)] = \prod_{i=1}^{p} \Gamma(a_i)$.

(ii). $\Gamma\left[a + \frac{(m)}{m}\right] = \prod_{r=1}^{m} \Gamma\left(a + \frac{r}{m}\right)$.

(iii).$\Gamma[(a;b)] = \prod_{r=1}^{a} \Gamma\left(\frac{b+r-1}{a}\right)$

(iv).v$\Gamma\left[\Delta\left(m;(a_p)\right)\right] = \prod_{i=1}^{p}\prod_{r=1}^{m}\left(\frac{a_i+r-1}{m}\right)$.

E.

(i). $\Gamma_*(a \pm b) = \Gamma(a+b)\Gamma(a-b)$.

(ii). $\Gamma_{**}(a+1) = \Gamma(a+b)\Gamma(a-b)$.

3. Theorem: For $m_2 > 1$

$$R_n(x,y) = \frac{4\sigma_1^a\sigma_2^b}{\Gamma a\,\Gamma b}\int_0^\infty\int_0^\infty e^{2a-1}\,\xi^{2b-1}e^{-(\sigma_1 e^2+\sigma_2\xi^2)}$$

$$\times\ M\sum_{h=0}^{\left[\frac{n}{m_1}\right]} z(h)\times F\left[\begin{matrix}\Delta(m_2;-n+m_1h), \Delta\left(m_2-1,1-(c_q)-n+m_1h\right), b' \\ \frac{\sigma_1\sigma_2\lambda_2\{-(m_2-1)\}^{(m_2-1)(q-p)}(-m_2)^{(1-u)^{m_2}}e^2\xi^2}{4(\lambda x^e y)^{m_2}(1-\xi)^2} \\ \Delta\left(m_2-1;1-(a_p)-n+m_1h\right), \Delta(m_2;1-(b_u)-n+m_1h), \\ a, b;\end{matrix}\right]d\xi \quad (3.1)$$

Proof : $I = M\sum_{h=0}^{\left[\frac{n}{m_1}\right]}\sum_{k=0}^{\left[\frac{n}{m_2}\right]}\frac{z(h)\Delta_k[m_2;-n+m_1h]}{\Delta_k[m_2-1;1-(a_p)-n+m_1h]}$

$\times\frac{\Delta_k[m_2-1;1-(c_q)-n+m_1h](b')_k e^{2k}\xi^{2k}\sigma_1^k\sigma_2^k\lambda_2^k}{\Delta_k[m_2;1-(b_u)-n+m_1h](a)_k(b)_k}$

$\times\frac{\{-m_2-1\}^{(m_2-1)(q-p)k}(-m_2)^{(1-u)m_2}}{(\lambda x^e y)^{m_2k}\,k!}$

$\times\int_0^\infty\int_0^\infty e^{2a+2k-1}\,\xi^{2b+2k-1}e^{-(\sigma_1e^2+\sigma_2\xi^2)}de\,d\xi$

$= M\sum_{h=0}^{\left[\frac{n}{m_1}\right]}\sum_{k=0}^{\left[\frac{n}{m_2}\right]}\frac{z(h)\Delta_k[m_2;-n+m_1h],\Delta_k[m_2-1;1-(c_q)-n+m_1h](b')_k}{\Delta_k[m_2-1;1-(a_p)-n+m_1h],\Delta_k[m_2;1-(b_u)-n+m_1h]}$

$\times\frac{\{-m_2-1\}^{(m_2-1)(q-p)k}(-m_2)^{(1-u)m_2}e^{2k}\xi^{2k}\sigma_1^k\sigma_2^k\lambda_2^k\Gamma(a+k)\Gamma(b+k)}{(\lambda x^e y)^{m_2k}(a)_k(b)_k4^k\sigma_1^{a+k}\sigma_2^{b+k}k!}$ (3.2)

Hence, $I = \frac{\Gamma(a)\Gamma(b)}{4\,\sigma_1^a\,\sigma_2^b}R_n(x,y)$

This complete the theorem.
On using [4].

$$\int_0^\infty\int_0^\infty e^{2a-1}\,\xi^{2b-1}e^{-(\sigma_1e^2+\sigma_2\xi^2)}de\,d\xi = \frac{\Gamma(a)\Gamma(b)}{4\,\sigma_1^a\,\sigma_2^b}$$

Particular Cases of (3.1)

i) On putting $p = 0 = q = u = h = \lambda_1$; $\lambda = 1 = \lambda_2 = \xi = \sigma = e = m_2$; $b' = x$, we get

$$\phi_n(x) = \frac{4^k\,\sigma_1^a\,\sigma_2^b\,x^n}{\Gamma(a)\,\Gamma(b)}\int_0^\infty\int_0^\infty e^{2a-1}\,\xi^{2b-1}e^{-(\sigma_1e^2+\sigma_2\xi^2)}$$

$$\times F\left[\begin{matrix}-n, x; \\ \frac{\sigma_1\sigma_2e^2\xi^2}{x} \\ a, b;\end{matrix}\right]de\,d\xi$$

where $\phi_n(x)$ are Sylvester Polynomials.

ii) On taking $p = 0 = q = h = \lambda_1$; $u = 1 = y = x = e = \lambda = m_2$; $\lambda_2 = \frac{1}{c}$ $b' = -x$, $b_1 = (\beta_1 + x)$, we get

$$m_n(\alpha,\beta,c) = \frac{4\,\sigma_1^a\,\sigma_2^b\,(\beta_1+x)_n}{\Gamma(a)\,\Gamma(b)\,n!}\int_0^\infty\int_0^\infty e^{2a-1}\,\xi^{2b-1}e^{-(\sigma_1e^2+\sigma_2\xi^2)}$$

$$\times F\begin{bmatrix} -n,-x; & \\ & \frac{\sigma_1\sigma_2 e^2\xi^2}{c} \\ 1-\beta,-x-n,a,b; & \end{bmatrix} ded\xi$$

where $m_n(x;\beta_1,c)$ are the Meixner Polynomials.

iii) On taking $p = 0 = q = h = \lambda_1$; $u = 1 = e = \lambda = \lambda_1 = \lambda_2$; $x = x = m_2 = 1$, $b_1 = b$, $y = \frac{1}{y}$, $b' = a$, we arrive at

$$g_n^{(a,b)}(x,y) = \frac{4\,\sigma_1^a\,\sigma_2^b\,(b)_n x^n}{n!\,\Gamma(a)\,\Gamma(b)}\int_0^\infty\int_0^\infty e^{2a-1}\,\xi^{2b-1}e^{-(\sigma_1 e^2+\sigma_2\xi^2)}$$

$$\times F\begin{bmatrix} -n,a; & \\ & \frac{y\,\sigma_1\sigma_2 e^2\xi^2}{x} \\ 1-b-n,a,b; & \end{bmatrix} ded\xi$$

where $g_n^{(a,b)}(x,y)$ are the Lagrange Polynomials [3].

iv) On setting $p = 0 = q = h = b' = \lambda_1$; $u = 1 = e = x = y = \lambda_2 = \xi = m_2$; $b_1 = -x$, we have

$$c_n(x,a) = \frac{4\,\sigma_1^a\,\sigma_2^b\,(-x)_n(-a)^{-n}x!}{n!}\int_0^\infty\int_0^\infty e^{2a-1}\,\xi^{2b-1}e^{-(\sigma_1 e^2+\sigma_2\xi^2)}$$

$$\times F\begin{bmatrix} -n; & \\ & a\,\sigma_1\sigma_2\xi^2 e^2 \\ x-n+1,a,b; & \end{bmatrix} ded\xi$$

Where $c_n(x;a)$ are the Charlier Polynomials.

2. Reference

1. Appell P, Kampe de Feriet J. Functions Hypergeometricques et Hyperspheriques, Paris; c1926.
2. Burchnall JL, Chaundy TW. Expansions of Appell's double Hypergeometric function II Quart. Journal of Mathematics. Oxford Ser. 1941;12:112-128.
3. Erdèlyi A, *et al*. Higher transcendental functions, Vol. I, II and III. Bateman Manuscript Project, California Institute of Technology; c1953.
4. Rainville ED. Special functions. Mac Millan Co. New York; c1960.