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To obtain IBFS of transportation problem through index of dispersion

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Abstract
The transportation problem aims to minimize transportation costs using linear programming. We have developed a new method for obtaining the initial basic feasible solution by calculating the Index of Dispersion. We then compare the obtained result with the VAM, LCM, and MODI methods.

Keywords: Transportation problem, initial basic feasible solution, index of dispersion

1. Introduction
The transportation problem, a well-known optimization challenge in operations research and logistics, focuses on the efficient movement of goods from sources to destinations with the objective of minimizing costs or maximizing profits. The original formulation of the basic transportation problem dates back to 1941 when Hitchcock introduced it [4]. Subsequently, efficient solution methods were developed, notably by Dantzig in 1951 [3], followed by Charnes *et al.* in 1953 [2]. In order to address the transportation problem, it can be mathematically expressed as a linear programming model. Various solution techniques, including the North-West Corner Method, Least Cost Method, Vogel's Approximation Method, and Modified Distribution Method, employ iterative allocation rules to optimize the assignment of goods and achieve an optimal solution.

The Index of Dispersion refers to a statistical measure that quantifies the variability in a dataset relative to its average. It is calculated by dividing the variance of the dataset by its mean. The variance measures the dispersion or spread of the data points, while the mean represents the average value. By computing the Index of Dispersion, we can gain insights into the degree of variability present in the dataset relative to its central tendency. In the context of the transportation problem, the Index of Dispersion can be utilized as a criterion to assess and compare different solutions or approaches, potentially aiding in the selection of the most suitable method for achieving an initial basic feasible solution.

2. Algorithm for Index of Dispersion
The algorithm for our proposed approach to determine the initial basic feasible solution for the transportation problem is presented as follows:

Step 1: Verify if the Transportation Problem (TP) is balanced by checking if the total supply is equal to the total demand. If the TP is not balanced, balance it by adding a dummy row or column to adjust the supply and demand values.

Step 2: Determine the variance $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$ and the mean $\bar{x}_i = \frac{\sum_{i=1}^{n} x_i}{n}$ for each row. Then obtain the respective Index of dispersion is, D = $\frac{\sigma^2}{\bar{x}_i}$ and perform the multiplication of the obtained value with its corresponding supply value (D × Supply = D_S).

Step 3: Determine the variance $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$ and the mean $\bar{x}_i = \frac{\sum_{i=1}^{n} x_i}{n}$ for each column. Then obtain the respective Index of dispersion is, D = $\frac{\sigma^2}{\bar{x}_i}$ and perform the multiplication of the obtained value with its corresponding demand value (D × Demand = D_D).

Step 4: Determine the row or column that exhibits the highest D_S or D_D among all the rows and columns, resolving ties arbitrarily. Find the cell within the chosen row or column that has the lowest cost and allocate as many units as feasible to that cell.

Step 5: Subtract the number of units assigned to the cell from the row supply and column demand, and mark the row supply or column demand as satisfied. Create a new tableau based on this adjustment. If both a row and a column are satisfied at the same time, mark only one of them as satisfied, and assign a zero demand (or supply) to the remaining column (or row). Additionally, when calculating subsequent Index of Dispersion, do not include any column or row with zero demand or supply.

Step 6: Recompute [D_S or D_D] for each row and columns in the reduced transportation tableau, as outlined in steps 2 and 3. Proceed to steps 4 and 5 accordingly. Repeat this iterative process until all the demands and supplies are fulfilled.

Step 7: Lastly, compute the total transportation cost of the Transportation Table (TT). This calculation involves summing the product of the cost and the corresponding assigned value for each cell in the TT.

Numerical Example with Illustration

	W1	W2	W3	W4	Supply
F1	5	4	2	6	20
F2	8	3	5	7	30
F3	5	9	4	6	50
Demand	10	40	20	30	

	W1	W2	W3	W4	Supply	(D)	D_S
F1	5	4	2	6	20	0.69	13.73
F2	8	[30]3	5	7	30/0	0.86	25.65
F3	5	9	4	6	50	0.78	38.89
Demand	10	40/10	20	30			
(D)	0.5	1.94	0.64	0.05			
D_D	5	77.5	12.73	1.58			

D_D = 77.5(Maximum) which is present in 2nd column. The least cost in this column is 3.
We allocate the minimum supply/demand to the corresponding cost in the 2nd column, 30 units. Row 2 is identified as satisfied and subsequently crossed out.

	W1	W2	W3	W4	Supply	(D)	D_S
F1	5	[10]4	2	6	20/10	0.69	13.73
F3	5	9	4	6	50	0.78	38.89
Demand	10	10/0	20	30			
(D)	0	1.92	0.67	0			
D_D	0	76.92	13.33	0			

D_D = 76.92(Maximum) which is present in 2nd column. The least cost in this column is 4.
We allocate the minimum supply/demand to the corresponding cost in the 2nd column, 10 units. Column 2 is identified as satisfied and subsequently crossed out.

	W1	W3	W4	Supply	(D)	D_S
F1	5	[10]2	6	10/0	1	10
F3	5	4	6	50	0.2	10
Demand	10	20/10	30			
(D)	0	0.67	0			
D_D	0	13.33	0			

D_D = 13.33(Maximum) which is present in 2nd column. The least cost in this column is 2.
We allocate the minimum supply/demand to the corresponding cost in the 2nd column,10 units. Row 1 is identified as satisfied and subsequently crossed out.

	W1	W3	W4	Supply	(D)	D_S
F3	5	[10]4	6	50/40	0.2	10
Demand	10	10/0	30			
(D)	0	0.67	0			
D_D	0	13.33	0			

D_D = 13.33(Maximum) which is present in 2nd column. The least cost in this column is 4.
We allocate the minimum supply/demand to the corresponding cost in the 2nd column,10 units. Column 2 is identified as satisfied and subsequently crossed out.

	W1	W4	Supply	(D)	D_S
F3	[10]5	6	40/30	0.09	3.64
Demand	10/0	30			
(D)	0	0			
D_D	0	0			

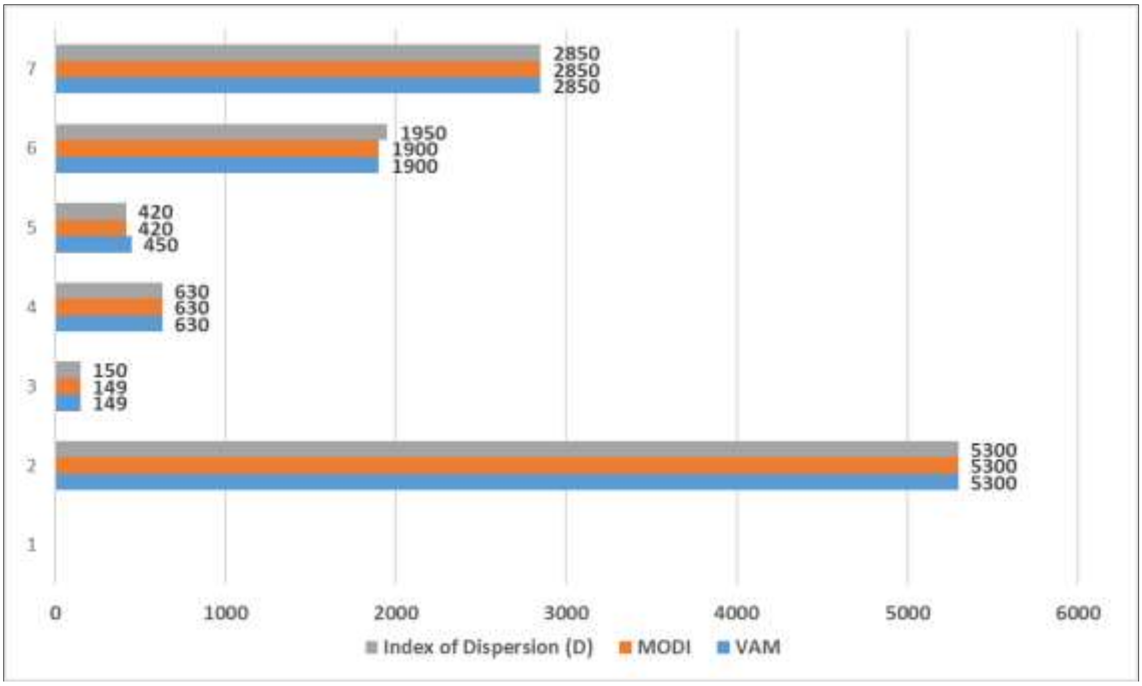
D_S = 3.64(Maximum). The least cost in this row is 5.
We allocate the minimum supply/demand to the corresponding least cost in this row,10 units. Column 1 is identified as satisfied and subsequently crossed out.

	W4	Supply
F3	[30]6	30/0
Demand	30/0	

In the end, we allocate 30 units to the leftover cost in TP.
Total cost $(30 \times 3) + (4 \times 10) + (10 \times 2) + (10 \times 4) + (10 \times 5) + (30 \times 6) = 420$
By observing that the total number of allocations is 6 in the transportation problem equals to m + n - 1, i.e., 3+4-1= 6 we can infer that the solution is non-degenerate.

3. Comparison

Example	Problem size	VAM	MODI	Index of Dispersion (D) Method
1	4×4	5300	5300	5300
2	3×4	149	149	150
3	4×4	630	630	630
4	3×4	450	420	420
5	3×3	1900	1900	1950
6	3×4	2850	2850	2850



4. Conclusion

In this research paper, we have presented a novel approach called the Index of Dispersion (D) to tackle the transportation problem. We have introduced a new approximation method to construct an efficient IBFS (Initial Basic Feasible Solution) algorithm. Through extensive testing, this method has demonstrated superior performance, yielding comparatively better results. In summary, our approach offers an improved Initial Basic Feasible Solution, ensuring minimal transportation costs in most of the problems. In most instances, the proposed method demonstrates a beneficial quality of generating an optimal or nearly optimal solution while effectively preventing degeneracy.

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