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Modelling the volatility of the Ghana stock market: A comparative study

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Abstract

The Ghana stock market is considered attractive to both local and international investors, as it is a developing market with potential for growth. The volatility of stock returns is one of the crucial features of Ghana's stock market that should be carefully taken into account by any investor or policymaker. As a result, the GARCH, TGARCH, and EGARCH models were used in this study to analyze the volatility of the Ghanaian stock market. The models were assessed using Akaike Information Criterion (AIC), RMSE and MAPE. The TGARCH (1,1) with generalized error distribution was the model that suited the data the best based on the AIC, RMSE, and MAPE values.

Keywords: Ghana Stock Exchange (GSE), volatility, GARCH models, log returns

1. Introduction

Wealth creation is the aim of every investor as the value of financial assets changes over time. The stock market is a desirable place to invest. A stock market may be defined as the regulatory environment that permits the trading of shares of several businesses or organizations (Gregoriou, 2009) ^[1]. Long regarded as significant economic development stimulators are stock exchanges. They give people and organizations that want to invest their savings or extra money by buying securities access to a regulated market for trading securities. Since the Ghana Stock Exchange is a developing market, both local and foreign investors are thought to be interested in taking advantage of the chance to make money on stock market. The volatility of stock returns is one of the crucial factors that must be carefully taken into account by any investor or policymaker (Dufitinema, 2021 Henriksen, 2011) ^[2, 3]. Volatility refers to a statistical calculation that measures the level of dispersion or variability in the returns of a particular security or market index (Brooks, 2008) ^[4]. In the last three decades, scholars and finance industry experts have placed considerable importance on modeling and forecasting asset return volatility. This heightened focus is attributed to the significance of volatility in various economic and financial contexts, including portfolio optimization, risk management, and as an indicator of the potential risk associated with future changes in asset returns.

According to Tsay (2010) ^[5] one unique characteristic of volatility is that it is difficult to directly see the conditional variance of returns on the underlying assets. Since this conditional variance may be estimated accurately, financial analysts are particularly eager to do so in order to optimize portfolio allocation. In order to evaluate the conditional volatility of financial assets, a variety of models have been created since the 1980s. One such example is the Auto Regressive Conditional Heteroskedastic (ARCH) model, which was developed by Engle (1982) ^[6]. Bollerslev (1986) ^[7] introduced the Generalized ARCH (GARCH) model which has been useful in modeling volatility of modern financial time series. There have been other extensions to the GARCH model which includes the exponential GARCH (EGARCH) by Nelson (1991) ^[8] and GJR-GARCH by Glosten *et al.* (1993) ^[9], which is similar to the Threshold GARCH (TGARCH) by Zakoin (1994) ^[10].

Since then, many empirical applications of modeling the volatility of financial time series have been made using various specifications of these models and their numerous expansions. These include the work of Zakoin (1994)^[10], Theodossiou and Lee, (1995)^[11], Karmakar (2007)^[12], Angabini and Wasiuzzaman (2011)^[13], Lim and Sek (2013)^[14], Ugurlu *et al.* (2014)^[15], Lu *et al.* (2016)^[16] and Ndei *et al.* (2019)^[17].

While many studies have investigated stock market volatility in other countries, relatively little is known about the dynamics of volatility in the Ghanaian market. This research seeks to fill this gap by analyzing the historical data of the Ghana stock market to uncover the patterns of volatility in the market. The proposed research aims to address the current gap in the literature by exploring the volatility of the Ghana stock market using the GARCH family of models. By using these models, this research can provide insights into the dynamics of volatility in the Ghana stock market and inform the development of risk management strategies for investors and policymakers.

2. Materials and Methods

2.1 Standard Garch Model

The GARCH model by Bollerslev (1986)^[7] was developed to overcome the weakness in the ARCH model (Dralle, 2011)^[18] and has the following form

$$x_t = \varepsilon_t \sigma_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i x_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (1)$$

here ε_t is iid with mean 0 and variance 1 and x_t represent a discrete-time stochastic process. One condition that is enough to warrant the positivity of the conditional variance is: $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1, 2, \dots, p$, $\beta_j \geq 0$, $j = 1, 2, \dots, q$. If and only if $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j < 1$ then GARCH (p, q) is weakly stationary (Lamma *et al.*, 2015)^[19].

2.2 Exponential GARCH Model

The EGARCH model is another model which is asymmetric model and takes into account the impact of price changes' leverage effects on the conditional variance. (Mohammed *et al.*, 2020)^[20]. It was first introduced by (Nelson, 1991)^[8].

The EGARCH (p, q) model is given in equation (2).

$$x_t = \varepsilon_t \sigma_t$$

$$\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^p \gamma_i \left(\frac{|x_{t-i}|}{\sigma_{t-i}} + E \left(\frac{|x_{t-i}|}{\sigma_{t-i}} \right) \right) + \sum_{j=1}^q \beta_j \ln(\sigma_{t-j}^2) \quad (2)$$

see (Brooks, 2008 and Dash and Dash, 2016)^[4, 21].

2.3 Threshold GARCH (TGARCH) Model

Another commonly used model for addressing leverage effects in volatility analysis is the TGARCH model (Zakoian, 1994)^[10]. A TGARCH (p, q) is expressed in equation

$$\sigma_t^2 = \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{i=1}^q (\alpha_i + \gamma_i N_{t-i}) x_{t-i}^2 + \alpha_0 \quad (3)$$

N_{t-i} is an index for negative x_{t-i} , that is

$$N_{t-i} = \begin{cases} 1 & \text{for } x_{t-i} < 0, \\ 0 & \text{for } x_{t-i} \geq 0 \end{cases}$$

And α_0 , α_i , γ_i and β_j are nonnegative parameters that satisfies the conditions similar to those of the GARCH model. In this model, the threshold is set at zero in order to distinguish between the effects of past shocks. It is possible to model the conditional volatility if an ARCH effect is present in a given data series, hence it is always critical to check the data for ARCH effect before conditional volatility can be considered.

2.4 Testing for ARCH Effect

There are two commonly employed tests for examining the presence of an ARCH effect. We first consider the Ljung-Box statistics $B(m)$ which are applied to x_t^2 . The null hypothesis for this test is that the initial m lags of the autocorrelation function of the x_t^2 are 0 (Tsay, 2010)^[5]. Equation (4) expresses the Ljung-Box.

$$B(m) = N(N+2) \sum_{k=1}^m \frac{\hat{\rho}_k}{T-k} \quad (4)$$

here N is the size of the sample, the number of lags is m and $\hat{\rho}_k$ is the estimation of k^{th} autocorrelation of the squared residuals. Equation (5) gives the expression for $\hat{\rho}_k$.

$$\hat{\rho}_k = \frac{\sum_{t=k+1}^N (x_t^2 - \hat{\mu})(x_{t-k}^2 - \hat{\mu})}{\sum_{t=1}^N (x_t^2 - \hat{\mu})^2} \tag{5}$$

$\hat{\mu}$ is the mean of the sample.

Assuming the null hypothesis is true, $B(m)$ follows an asymptotic chi-squared distribution with m degrees of freedom (Box *et al.*, 2016)^[22]. The null hypothesis is rejected if $B(m) > \chi_m^2(\alpha)$ (Tsay, 2010)^[5].

The other test is the Lagrange multiplier test. It is equivalent to the F statistics for testing $\alpha_i = 0$ for $i = 1, 2, \dots, m$ in the regression.

$$x_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \dots + \alpha_m x_{t-m}^2 + e_t \tag{6}$$

$t = m + 1, \dots, N$. e_t is the error term, m is a specified integer, and N is the size of the sample (Lee, 1991)^[23]. The null hypothesis is then.

$$H_0: \alpha_1 = \dots = \alpha_m = 0$$

The statistic for this test written as equation (7).

$$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(T - 2m - 1)} \tag{7}$$

The null hypothesis is rejected if $F > \chi_m^2(\alpha)$ (Tsay, 2010)^[5]. Here,

$$SSR_0 = \sum_{t=m+1}^T (x_t^2 - \bar{u})^2 \tag{8}$$

And

$$SSR_1 = \sum_{t=m+1}^T (e_t)^2 \tag{9}$$

Where \bar{u} is the mean of x_t^2 and \hat{e}_t is the least squares residual from the regression equation in (6).

2.5 Model Evaluation

2.5.1 The Root Mean Square Error (RMSE)

RMSE is used for evaluating the predicting performance of the models. It is the most favoured measure among practitioners and academics (Lim and Sek 2013)^[14]. It is given by equation.

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^n (\sigma_t - \hat{\sigma}_t)^2} \tag{10}$$

where σ_t is the realized volatility while $\hat{\sigma}_t$ is the estimated conditional volatility. The true volatility is hidden and thus unobserved. Due to this, the realized volatility is used (Audrino and Bühlmann, 2009 and Lunde and Hansen, 2005)^[24, 25].

2.5.2 Mean Absolute Percentage Error (MAPE)

MAPE is a statistical measure used to evaluate the accuracy of a forecasting method. It quantifies the accuracy as a ratio, which is typically calculated using equation.

$$MAPE = \frac{1}{n} \sum_{t=1}^n \left| \frac{\sigma_t - \hat{\sigma}_t}{\sigma_t} \right| \times 100 \tag{11}$$

2.6 Data Exploration

This study utilized data from the database of the Ghana Stock Exchange, specifically the Ghana Stock Exchange Composite Index (GSE-CI). The dataset comprises 2403 observations covering the period from 3rd January, 2012 to 10th October 2021. The Ghana Stock Exchange Composite Index is based on the volume weighted average of the closing price of all the listed stocks. It includes all ordinary shares that are listed on its platform in the GSE-CI, except companies listed on other markets. The GSE-CI is a market capitalization weighted index, meaning that the weight assigned to each constituent is determined by its market capitalization. The index's base date is December 31, 2010, and the base index value is set at 1000. Figure 1 displays a chart of the GSE-CI daily stock index, which illustrates that there are phases of significant and minor price fluctuations (volatility clustering) in the series. The log return series is made up of 2402 observations, as one observation is omitted during the calculation of the log return. Figure 2 shows the plot of the daily log returns for the series. The plot of the log returns of the series depicts evidence of volatility clustering. Figure 3 and Figure 4 show the ACF and PACF plot of the daily log return series.

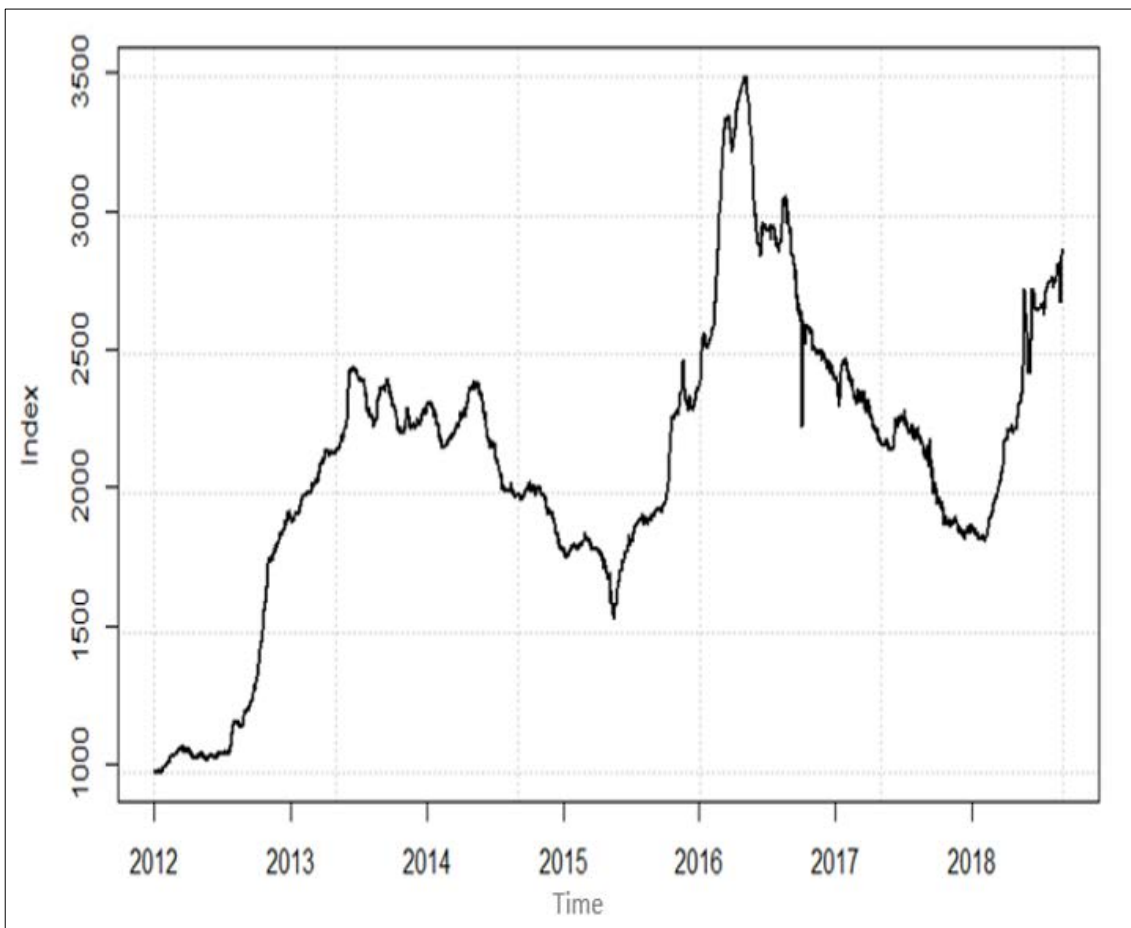


Fig 1: Plot of GSE-CI

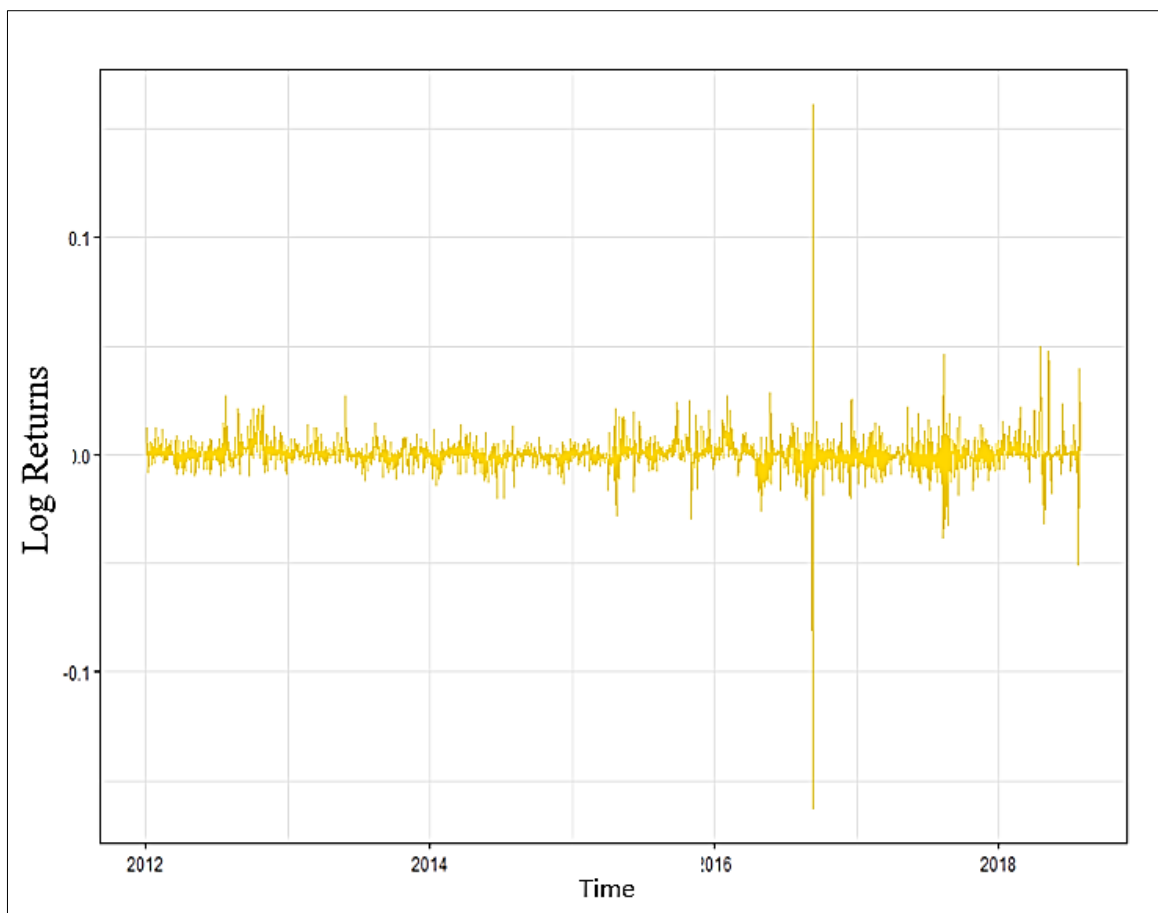


Fig 2: Plot of the daily log returns of GSE-CI

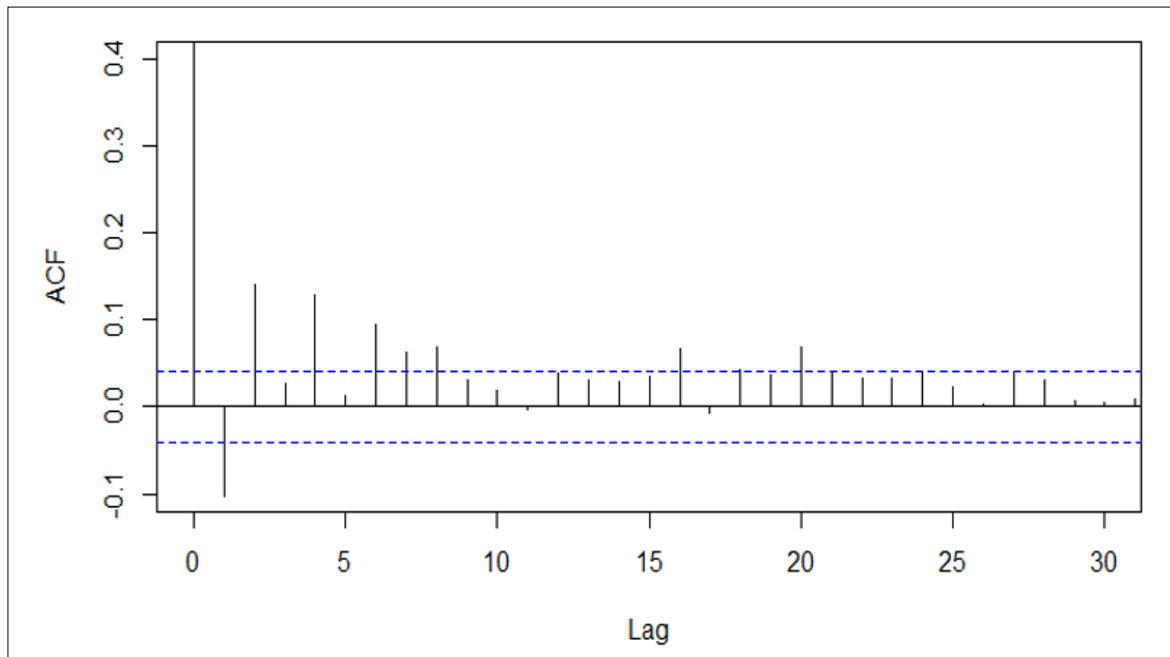


Fig 3: Plot of ACF of the daily log returns of GSE-CI

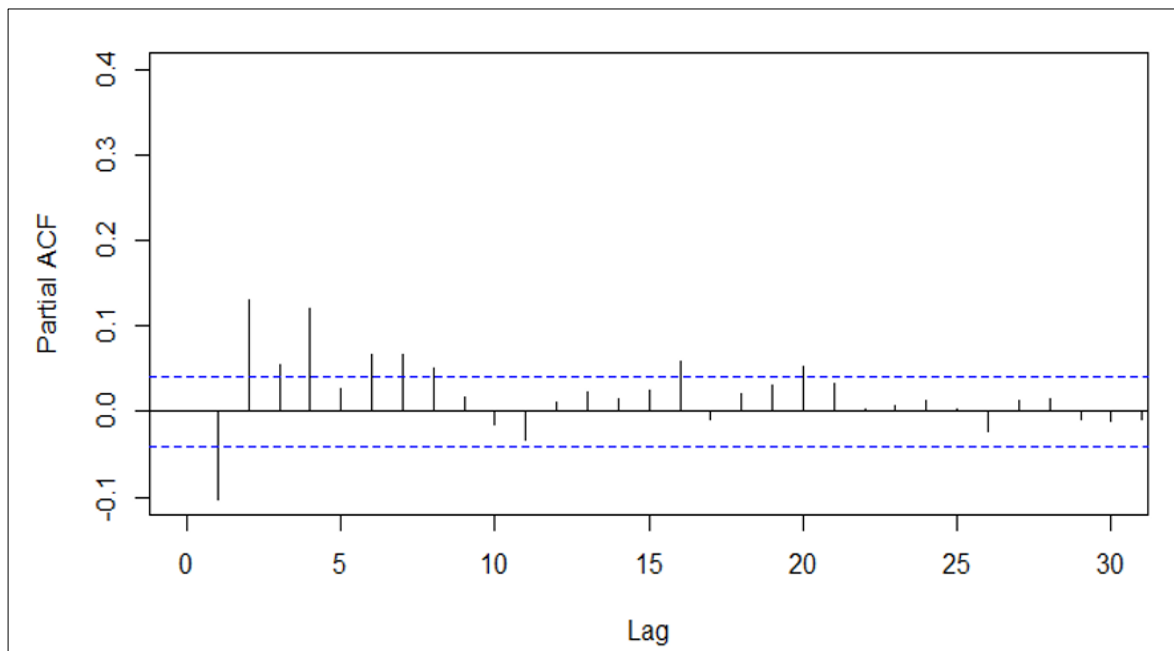


Fig 4: Plot of PACF of the daily log returns of GSE-CI

The summary statistics for the daily log return series are given in Table 1. The summary statistics for the log return series indicate a high kurtosis, which implies that the series is not distributed normally. This is supported by the normality tests presented in Table 2, as well as a visual examination of the return's histogram and density plot depicted in Figure 3. The p-value for the Jarque-Bera, Kolmogorov-Smirnov and the Anderson-Darling tests are all less than 0.05 significant level which depict nonnormality of the data. Positive skewness also depicts evidence of asymmetry. Thus, the skewness coefficient depicts that the distribution of the series has fat right tails which means that small positive movements of the index are not likely to be followed by equally small negative movements.

Table 1: Summary Statistics of daily log returns of GSE-CI

Statistic	GSE-CI
Mean	0.000450
Maximum	0.161049
Minimum	-0.162585
Standard Deviation	0.008015
Skewness	0.177693
Kurtosis	141.387

Table 2: Normality Test for GSE-CI

Test	Statistic	P-Value
Jarque-Bera	2003281	< 0.001
Kolmogorov-Smirnov	0.48636	< 0.001
Anderson-Darling	147.15	< 0.001

2.6.1 ARCH Effect Test

The ARCH-LM test and the Ljung-Box test was performed at the 5% significant level to determine the presence of ARCH effect in the series and the results presented in Table 3. The null hypothesis states that there is no ARCH effect in the rate of returns, whereas the alternative hypothesis posits the presence of such an effect (Forsberg and Bollerslev, 2002)^[26]. Both tests show that the daily log returns are not homoscedastic but rather heteroscedastic since all p-values is approximately zero which is far less than the 5% significant level. This implies that there is an ARCH effect in the series and hence conditional variance can be computed Atoi 2014^[27].

Table 3: ARCH Effect Test for GSE-CI

Test	Chi-Squared	P-Value
ARCH-LM	955.43	< 0.001
Ljung-Box	581.12	< 0.001

3. Results and Discussion

3.1 Model Selection

The estimate of GARCH-family models received support from the ARCH effect's existence. In order to choose the optimum estimating model, the estimate is implemented in three different error distributions. The data was separated into two subgroups for the purpose of cross-validation. The GARCH models for the log return series were constructed using the in-sample data, which comprises the first 70% of the subset. The performance of volatility forecasting was verified using the rest often known as the out-of-sample data set. The summaries of the Akaike Information Criteria (AIC) for the models taken into account for three distinct error distributions-Normal (Norm), Student t-distribution (STD), and Generalized Error Distribution (GED) are shown in Table 4. GARCH_{GED} (2,2), EGARCH_{GED} (1,1), and TGARCH_{GED} (1,1) had the least AIC values among the GARCH, EGARCH and TGARCH models considered respectively which indicate that they were the best fitting models in their category. However, the TGARCH_{GED} (1,1) was observed to be the best fitting model in all the considered models because it had the smallest AIC value. This result is contrary to the findings of Lunde and Hansen (2005)^[25] that nothing consistently beat the GARCH (1,1) model. TGARCH was developed to deal with asymmetry. Thus, it implies that the GSE-CI is characterized by big negative shocks than positive shocks. Shamiri and Isa (2009)^[28] argues that, the best-fitted models selected based on AIC criterion may not necessarily provide the most accurate forecast of volatility in terms of RMSE and MAPE. Consequently, the accuracy of the volatility forecast was evaluated using MAPE and RMSE measures for both in-sample and out-of-sample data to verify the credibility of the discoveries of Shamiri and Isa (2009)^[28] and determine the appropriate GARCH-type model for modelling and forecasting the volatility of the Ghana stock market. Table 5 shows the MAPE and RMSE evaluation of the best fitted models. Again, among the best fitted models according to the AIC, the TGARCH_{GED} (1,1) had the least MAPE and RMSE values for both in-sample and out-of-sample evaluation. This is contrary to the findings of Shamiri and Isa (2009)^[28] that the best fitted model does not necessarily provide the best forecast.

Table 4: Summary of AIC of GARCH-Type Models

Model	GARCH			EGARCH			TGARCH		
	Norm	STD	GED	Norm	STD	GED	Norm	STD	GED
(1, 0)	-7.679	-7.933	-6.679	NA	NA	NA	-7.768	-7.939	0.007
(0,1)	NA	-7.885	0.282	-7.586	-7.891	-7.957	70.479	NA	31.391
(1,1)	-7.770	-7.987	-7.941	-7.770	-7.988	-8.038	-7.713	-7.989	-8.039
(2,0)	NA	NA	-6.611	NA	NA	NA	-7.713	-7.960	0.010
(2,1)	-7.768	-7.986	-8.034	-7.771	-7.987	-8.036	-7.771	-7.987	-8.037
(0,2)	-7.585	-7.885	-6.212	-7.585	-7.889	-7.954	NA	-7.886	31.281
(1,2)	-7.768	-7.986	-8.037	-7.769	-7.987	-8.037	-7.7671	-7.988	-8.037
(2,2)	-7.767	-7.986	-8.038	-7.778	-7.986	-8.037	-7.769	-9.986	-8.035

Table 5: MAPE and RMSE for Best Fitted Models

Error Measure	Model	MAPE	Rank	RMSE	Rank
In-Sample	GARCH _{GED} (2,2)	0.0559	3	0.2356	3
	EGARCH _{GED} (1,1)	0.0556	2	0.2344	2
	TGARCH _{GED} (1,1)	0.0555	1	0.2336	1
Out-of-Sample	GARCH _{GED} (2,2)	0.1277	3	0.2182	3
	EGARCH _{GED} (1,1)	0.1274	2	0.2036	2
	TGARCH _{GED} (1,1)	0.1269	1	0.2023	1

The parameter estimates of the best fitted model TGARCH_{GED} (1,1) are shown in Table 6. The p-values shows that all parameters given in the TGARCH_{GED} (1,1) are statistically significant at 5% level except μ and γ_1 .

Table 6: Estimated Parameters of TGARCH_{GED} (1, 1)

Parameter	Estimate	Standard Error	T-Value	P-Value
μ	0	0.0000	-0.0008	0.99939
ω	0.0007	0.0002	3.3003	0.00096
α_1	0.2368	0.0401	5.9005	<0.0001
β_1	0.7212	0.0565	12.7722	<0.0001
γ_1	-0.0774	0.0809	-0.9561	0.3390
ν	0.7489	0.0446	16.7972	<0.0001

Table 7 shows the diagnostics on the best fitted model (TGARCH_{GED} (1,1)). The weighted Ljung-Box test for the null hypothesis of squared standardized residuals was not rejected at a 5% significance level. This implies that the squared standardized residuals are indicative of white noise. Additionally, the weighted ARCH-LM test implies the absence of an ARCH effect in the model. Taken together, these tests indicate that the TGARCH_{GED} (1,1) model is appropriate for modelling the conditional variance.

Table 7: Model Diagnostics

Test	Weighted Ljung-Box Test on Standardised Squared Residual		Weighted ARCH LM	
	H ₀ : Squared Residuals are independent	H ₁ : Squared Residuals are dependent	H ₀ : No ARCH effect	H ₁ : ARCH effect
lag	1		3	
Statistic	0.6754		0.3823	
P-Value	0.4112		0.5364	

4. Conclusion

In conclusion, this study found that the threshold GARCH (TGARCH) model with a generalized error distribution outperformed the EGARCH and standard GARCH models in forecasting the volatility of the Ghana Stock Exchange Composite Index (GSE-CI). The performance was evaluated using RMSE, MAPE, and Akaike Information Criterion (AIC). These findings provide valuable insights for investors, traders, and policymakers in Ghana's financial market.

In terms of future directions, further research could investigate the effectiveness of other models such as Stochastic Volatility (SV) and Markov-Switching GARCH (MS-GARCH) in forecasting the volatility of the GSE-CI. Finally, researchers could explore the possibility of using machine learning techniques to develop more advanced models for predicting stock market volatility in Ghana.

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